6.02 Fall 2011 Quiz 1 Review

Noise and Bit Error Problem 1

- Suppose a channel has both noise and intersymbol interference, and further suppose the voltage at the receiver is:
  - 8.0 + *noise* volts when the transmitter sends a '1' bit preceded by a '1' bit,
  - 6.0 + *noise* volts when the transmitter sends a '1' bit preceded by a '0' bit,
  - 2.0 + *noise* volts when the transmitter sends a '0' bit preceded by a '1' bit,
  - 0.0 + *noise* volts when the transmitter sends a '0' bit preceded by a '0' bit,
- Use 4.0 volts as the threshold for deciding the bit value
- Assume the *noise* is Gaussian with standard deviation:

$$\sigma = 1$$

- Assume all bit patterns are equally likely.
- What is the probability of error?

Answer: See problem 3 of Spring 2010 Quiz 1

x	$\text{efrc}(x)$
$\frac{4}{\sqrt{2}}$	3.2×10^{-5}
$\frac{2}{\sqrt{2}}$	0.023

Noise and Bit Error Problem 2

- Again suppose *noise* is Gaussian with standard deviation $\sigma = 1$ and suppose that for a particular set of transmitted data, which we will refer to as *checkerboard* data, there is an increased probability of unequal contiguous bits. That is, for checkerboard data, the probability of two '0' bits in a row is 1/6, the probability of two '1' bits in a row is 1/6, the probability of a '1' bit preceded by a '0' bit is 1/3, and the probability of a '0' bit preceded by a '1' bit is 1/3. Approximately what is the ratio of the probability of bit error for the checkerboard case to the probability of bit error for the uniform (all bit patterns equally likely) case.

Answer: See problem 3 of Spring 2010 Quiz 1

Linear Error Correcting Codes Problem 3

- Given a code with parameters (15,12,4), determine if a linear code is possible.

Answer: Using the bound:

$$\sum_{i=0}^{t} \binom{n}{i} \leq 2^{n-k} \quad \text{where} \quad t = \lfloor \frac{d-1}{2} \rfloor$$

$$\sum_{i=0}^{1} \binom{15}{i} = \binom{15}{0} + \binom{15}{1} = 1 + 15 \nleq 2^{15-12}$$

A linear code is therefore not possible.
Note: As pointed out in the review session, this is a necessary bound but it is not sufficient.

Linear Error Correcting Codes Problem 4

- Given a code with parameters (11,6,3), determine if a linear code is possible.

Answer: Using the bound:

$$\sum_{i=0}^{1} \binom{11}{i} = \binom{11}{0} + \binom{11}{1} = 1 + 11 \leq 2^{11-6}$$

A linear code may exist.

- How many errors can this code correct/detect?

Answer: A linear code with the parameters (11,6,3) can detect:

$d - 1 = 2$ errors and correct: $t = \lfloor \frac{d-1}{2} \rfloor = 1$ errors.

Linear Error Correcting Codes Problem 4

- Can a rectangular parity code be generated with these parameters? If so, show how to generate it and give all possible values of r and c.

Answer: A rectangular parity code can be generated with r = 2 and c = 3 or r = 3 and c = 2.

- What are the syndromes for this code?

Answer: Each syndrome is the sum of all message bits and parity bits in either a column or row. For this case, we would have 5 syndrome equations.

- What is the rate of the code?

Answer: The rate is: $r = \frac{k}{n} = \frac{rc}{rc+r+c} = \frac{6}{11}$

Linear Error Correcting Codes Problem 5

- Given the parameters (7,4), can we generate a Hamming code? If so, show how to generate it. Answer: See Section 6.4.4 in the course notes
- What are the syndrome equations for this code? Answer: See Section 6.4.3 in the course notes
- What are the corrective actions for the following syndrome combinations:

$$E_1E_2E_3 = \{010, 110, 111\}$$

Answer: See Section 6.4.3 in the course notes

Linear Error Correcting Codes Problem 6

- Given the following codewords in a linear block code:

1010
0101

Can you determine if there are any additional codewords? If so, what are they?

Answer: Since the sum of any two codewords is also a codeword, the other possible codewords are: 0000 and 1111.

Linear Error Correcting Codes Problem 7

- Given that a minimum, non-zero, weight codeword in some linear block code is:

000100101001100

What is the Hamming distance of this code?

Answer: Since the minimum, non-zero, weight codeword is the Hamming distance of the code, the answer is the number of 1's in the codeword above (i.e., 5)

Linear Error Correcting Codes
Problem 7

- Using a BSC model with probability of a bit error:

$$\epsilon < \frac{1}{2}$$

Is the codeword given above the maximum likelihood estimate of a received codeword:

$$000101001001100$$

Answer: Since the ML estimate corresponds the codeword with minimum Hamming distance from the received codeword and we know that the Hamming distance of the code is 5, there can only be 2 bit flips for the codeword given on the previous slide to be the ML estimate of the received codeword.

$$000101001001100$$
$$000100101001100$$

So the codeword given on the previous slide is the ML estimate.

Convolutional Codes
Problem 8

- Given the following generators, what is the corresponding state diagram and first 2 steps of the Trellis diagram? Label everything.

$$g_0 = (1, 0, 1)$$
$$g_1 = (0, 1, 1)$$

Answer: Each generator produces the following parity equations:

$$p_0 = x[n] + x[n-2]$$
$$p_1 = x[n-1] + x[n-2]$$

The state diagram is shown in Figure 7-3 in the course notes. The only thing that needs to be done is to change the parity bits labels to correspond to the parity equations shown above (i.e., {message bit}/p_0 p_1}

Convolutional Codes
Problem 8

- What is the transmitted bit sequence if the following messages is encoded using the above generators?

101110101

Answer: The transmitted bit sequence would be:

10 01 01 00 00 10 01 01 01

Convolutional Codes
Problem 9

- Consider the trellis (see problem 3 of Quiz 2, Spring 2011) showing the operation of the Viterbi algorithm using a hard branch metric at the receiver as it processes a message encoded with a convolutional code *C*. Answer: See problem 3 of Spring 2011 Quiz 2
- What is the code rate and constraint length of the convolutional code *C*? Answer: See problem 3 of Spring 2011 Quiz 2
- What bits would be transmitted if the message "1011" were encoded using *C*? Answer: See problem 3 of Spring 2011 Quiz 2
- Computer the missing path metrics in the top two boxes of the rightmost column and draw the transition arrow showing the predecessor state for each metric.

Answer: See problem 3 of Spring 2011 Quiz 2

Convolutional Codes Problem 10

- The received parity bits for time 5 are missing from the trellis diagram. What values for the parity bits are consistent with the other information in the trellis?

  Answer: See problem 3 of Spring 2011 Quiz 2

- Determine the most-likely path through the trellis. What is the decoded message?

  Answer: See problem 3 of Spring 2011 Quiz 2

- Based on the last answer, how many bit errors were detected in the received transmission and at what time(s) did those error(s) occur?

  Answer: See problem 3 of Spring 2011 Quiz 2