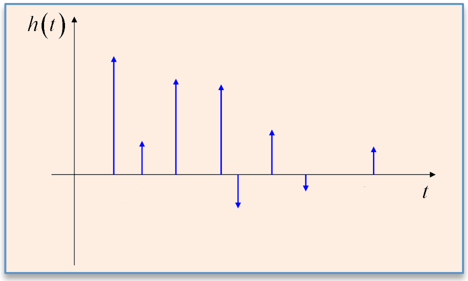
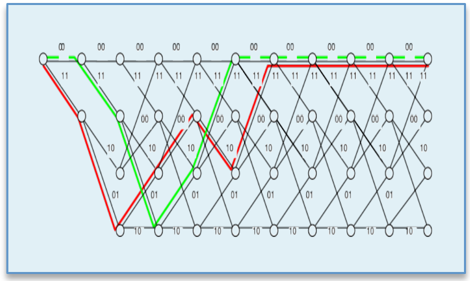
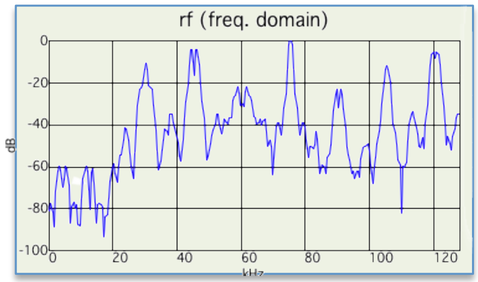
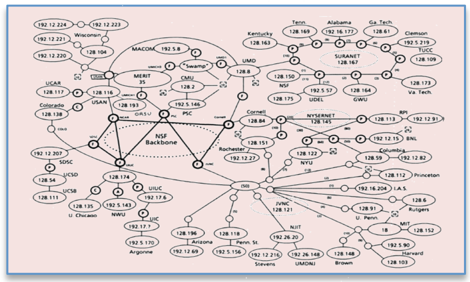








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2012
Lecture #9

- A small postscript on multiple random variables
- Introduction to modulation and demodulation
- Input/output descriptions of systems
- Linear time-invariant (LTI) models

Dealing with Multiple Random Variables

- PDF of a random variable X:

$$f_X(x) \geq 0 \quad \text{and} \quad \int f_X(x)\, dx = 1$$

(this integral is over the entire real line)

- The natural extension to the case of two random variables X and Y is the **joint PDF of *X* and *Y*:**

$$f_{X,Y}(x,y) \geq 0 \quad \text{and} \quad \iint f_{X,Y}(x,y)\, dx\, dy = 1$$

(2D-integral covers the entire x,y plane)

- Expected value of a function of X,Y:

$$E[g(X,Y)] = \iint g(x,y)\, f_{X,Y}(x,y)\, dx\, dy$$

- And similarly for more random variables

In our Signal Detection Setting

- Last lecture we discussed averaging multiple random variables: $A = (w[1] + w\{2] + .. + w[M]) / M$

  and wanted the mean and variance of A.

Here each $w[n]$ was the additive noise component of a received sample in a fixed bit slot, and assumed to be a zero-mean Gaussian of variance σ^2, independent of all other $w[.]$. These $w[.]$ constitute "additive white Gaussian noise" (AWGN) --- "white" here=zero-mean iid

- Strictly speaking, we should have been working with the joint PDF of the M random variables, in an M-dimensional space. However, the following facts suffice to get us through with just 1D PDFs:

Two Important Facts

We write these for two random variables X and Y, but the results extend to M random variables. Also, $g(.)$ and $h(.)$ below are arbitrary functions.

1. **Expectation is always additive**, i.e.,

$$E[g(X,Y) + h(X,Y)] = E[g(X,Y)] + E[h(X,Y)]$$

- Follows from the fact that integration is additive; needs no assumptions (apart from existence of the expected value)
- In particular, $E[g(X) + h(Y)] = E[g(X)] + E[h(Y)]$

  The RHS only needs 1D PDFs, not joint PDFs!

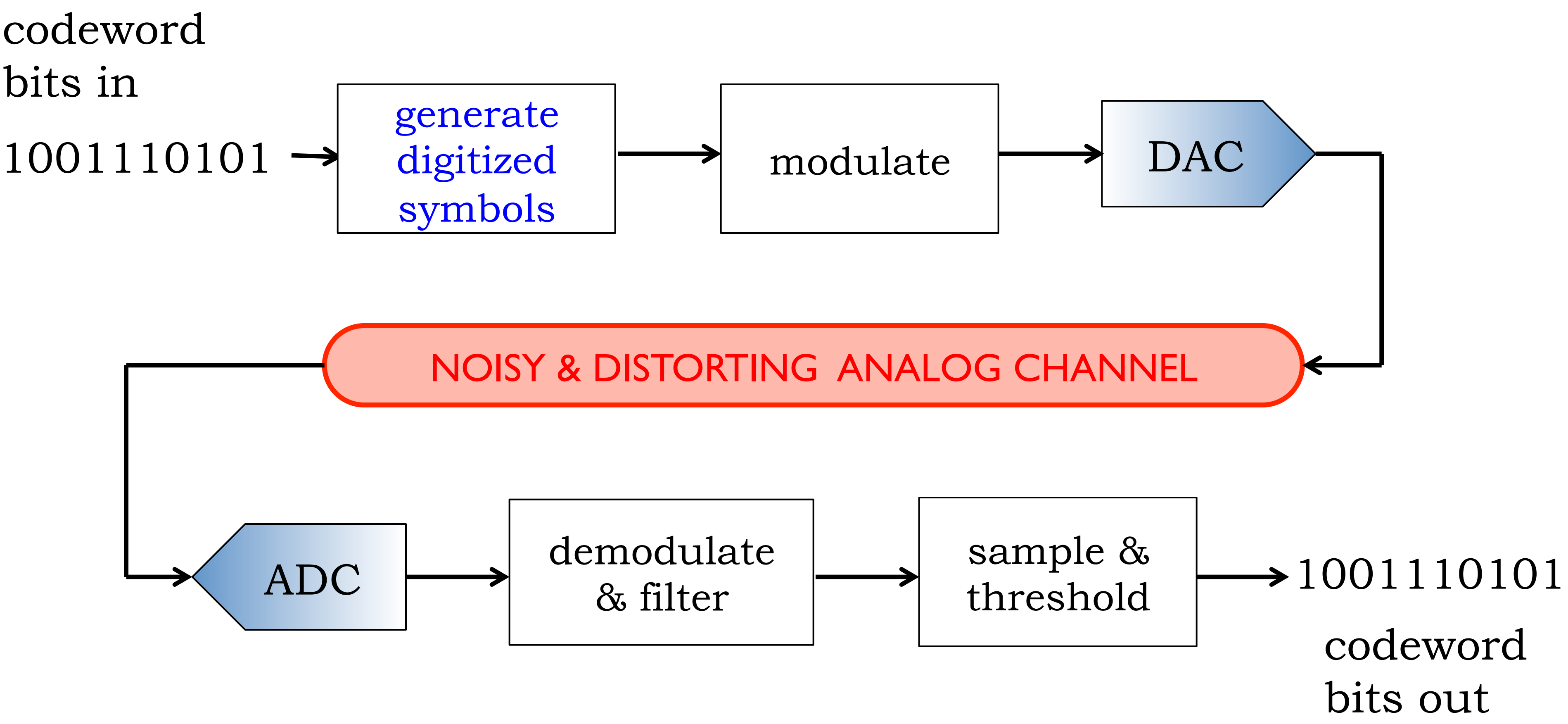
2. **For INDEPENDENT random variables, expectation is always multiplicative**. In fact, X and Y are independent if and only if

$$E[g(X)h(Y)] = E[g(X)].E[h(Y)]$$

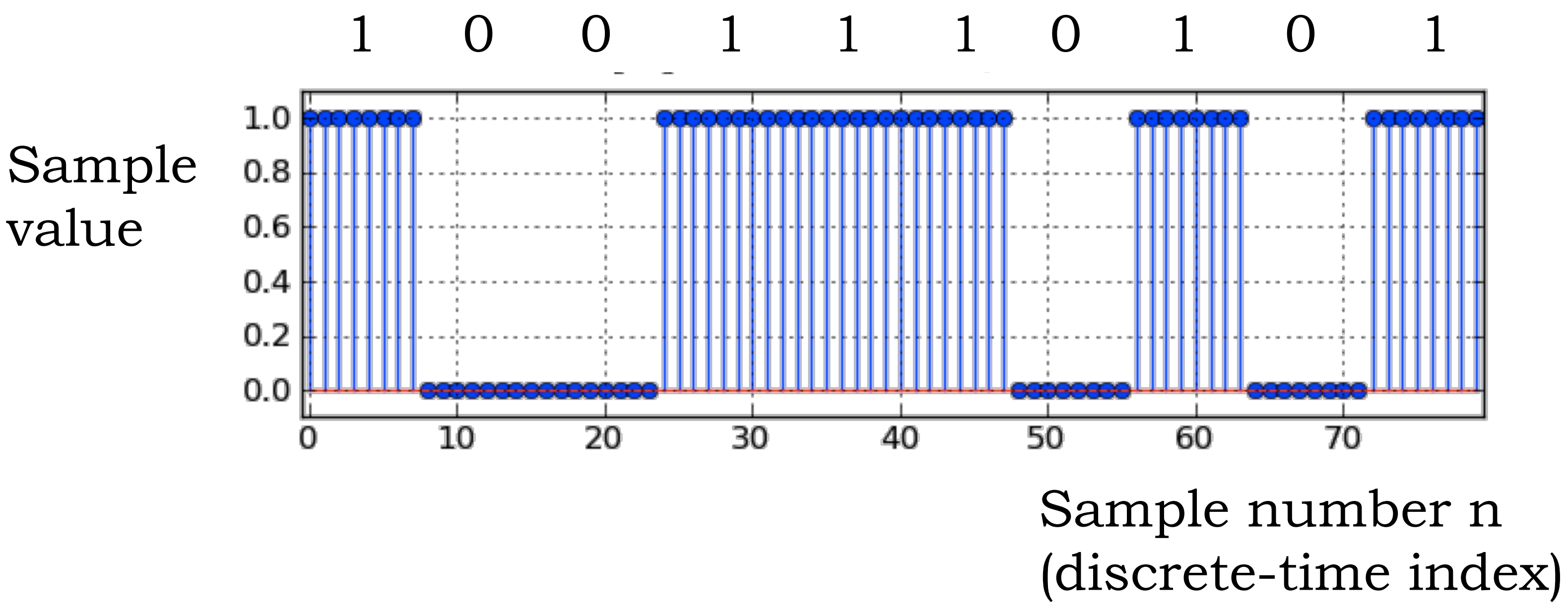
for *all* choices of functions $g(.)$ and $h(.)$.

Again, the RHS needs only 1D PDFs, not joint PDFs!

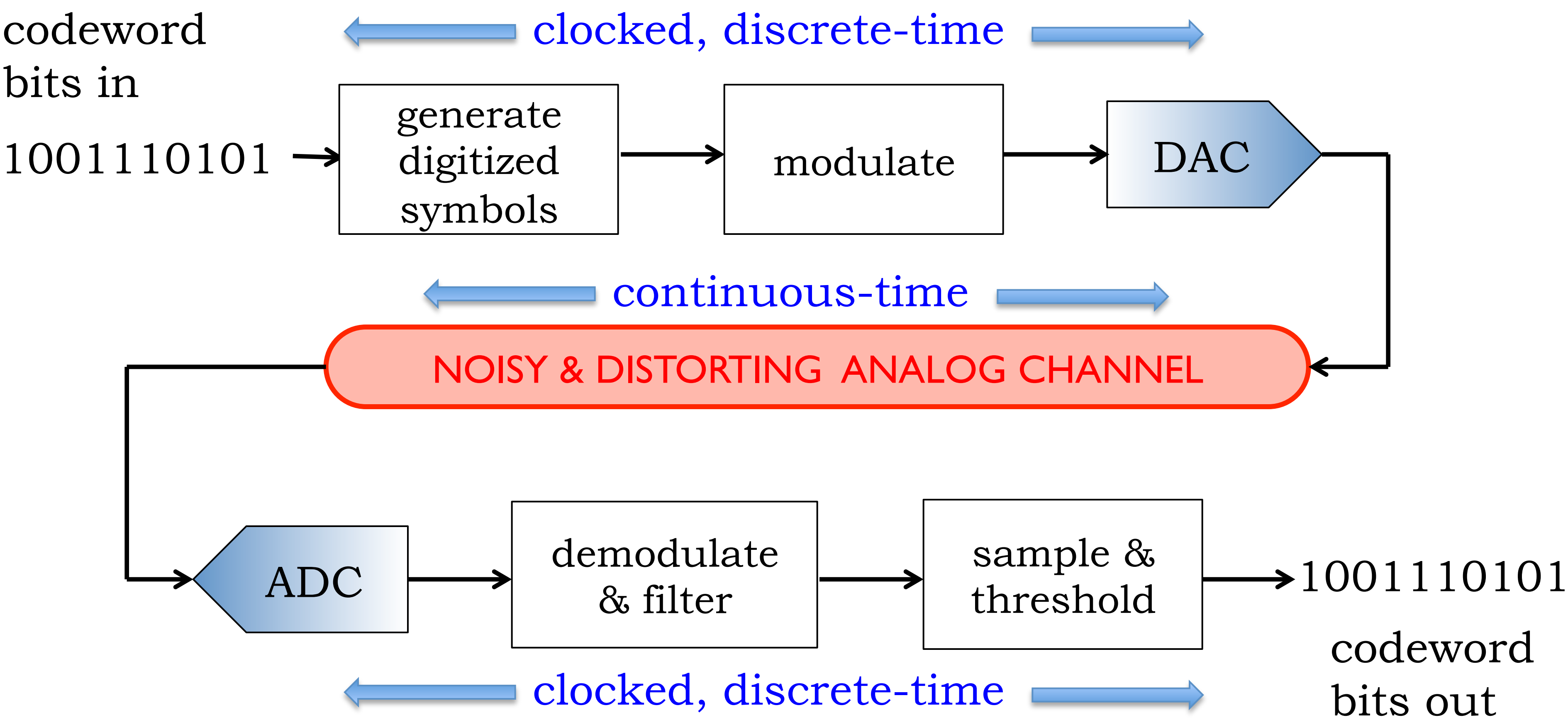
A Single Link



Digitized Symbols

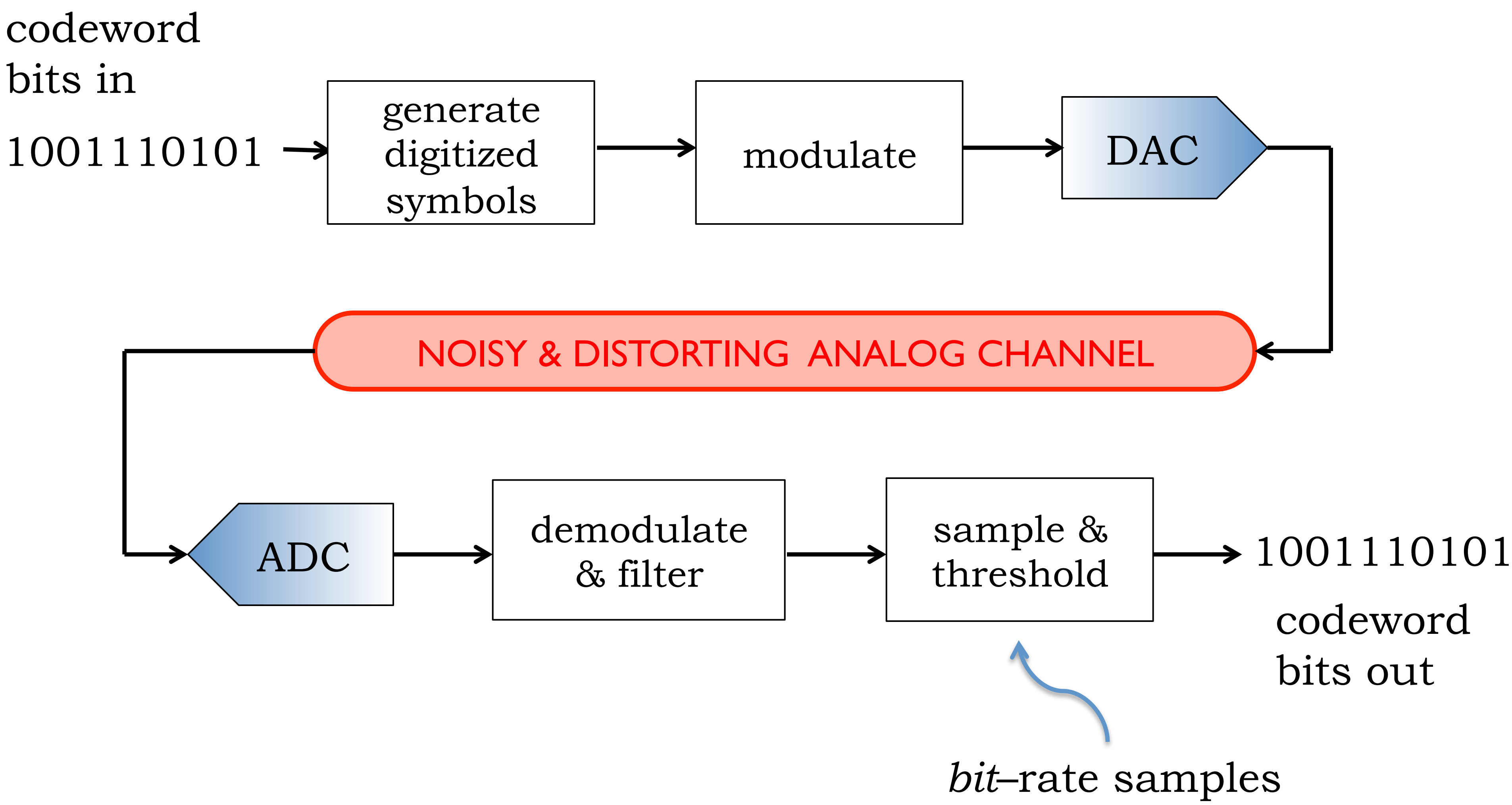


A Single Link

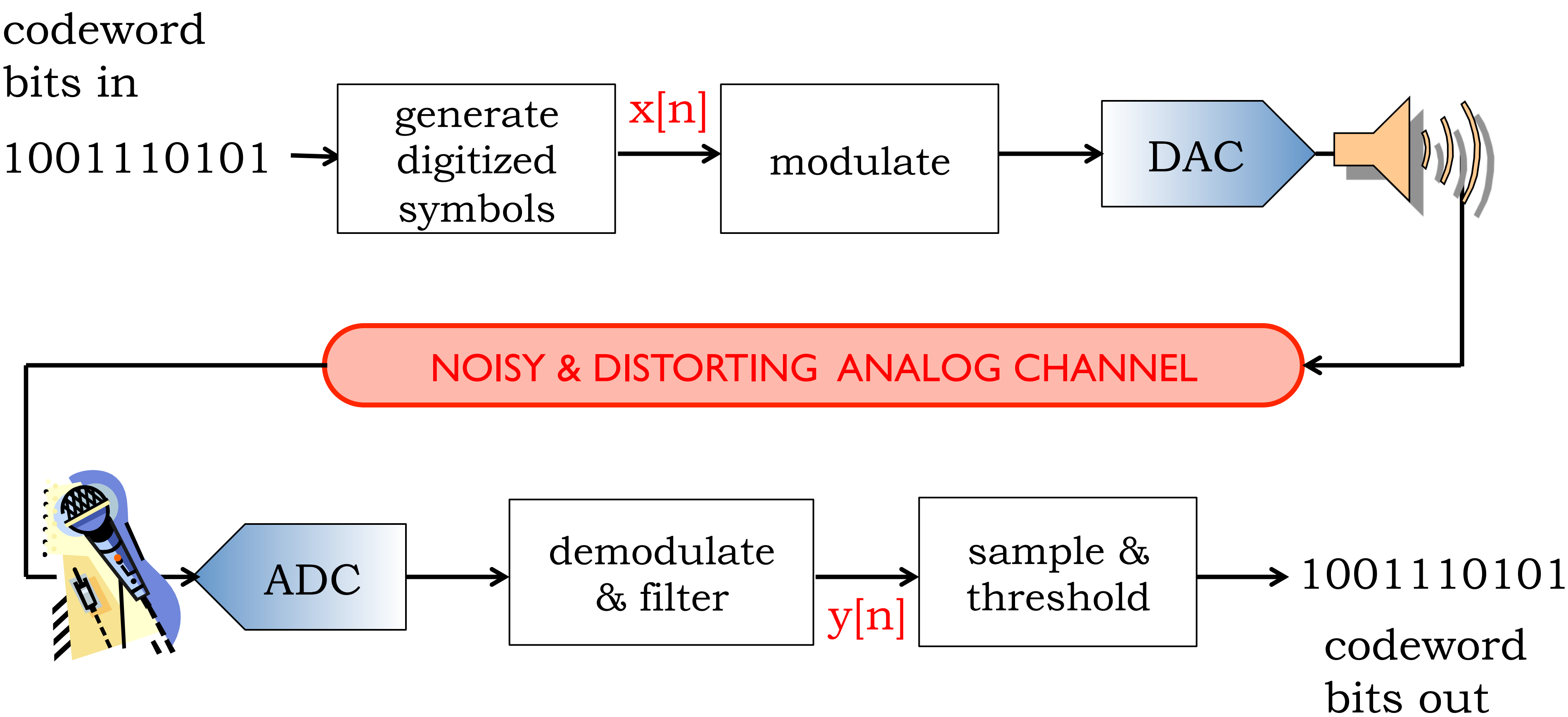


DAC: Digital-to-analog converter
ADC: Analog-to-digital converter

A Single Link



From Baseband to Modulated Signal, and Back



Modulation (at the Transmitter)

Adapts the digitized signal x[n] to the characteristics of the channel.

e.g., Acoustic channel from laptop speaker to microphone is *not* well suited to transmitting *constant* levels v_0 and v_1 to represent 0 and 1. So instead transmit **sinusoidal** pressure-wave signals proportional to speaker voltages

$$v_0 \cos(2\pi f_c t) \quad \text{and} \quad v_1 \cos(2\pi f_c t)$$

where f_c is the *carrier frequency* (e.g., 2kHz; wavelength at 340 m/s = 17cm, comparable with speaker dimensions) and

$$v_0 = 0 \qquad v_1 = V > 0$$

(on-off or *amplitude* keying)

or alternatively

$$v_0 = -V \qquad v_1 = V > 0$$

(bipolar or *phase-shift* keying)

Could also key the *frequency*.

From Brant Rock tower, radio age was sparked
By Carolyn Y. Johnson, Globe Staff | July 30, 2006

MARSHFIELD, MA -- A century ago, radio pioneer Reginald A. Fessenden used a massive 420-foot radio tower that dwarfed Brant Rock to send voice and music to ships along the Atlantic coast, in what has become known as the world's first voice radio broadcast. This week, Marshfield will lay claim to its little-known radio heritage with a three-day extravaganza to celebrate the feat -- including pilgrimages to the base of the long-dismantled tower, a cocktail to be named the Fessenden Fizz, and a dramatic reenactment of the historic moment, called ``Miracle at Brant Rock."

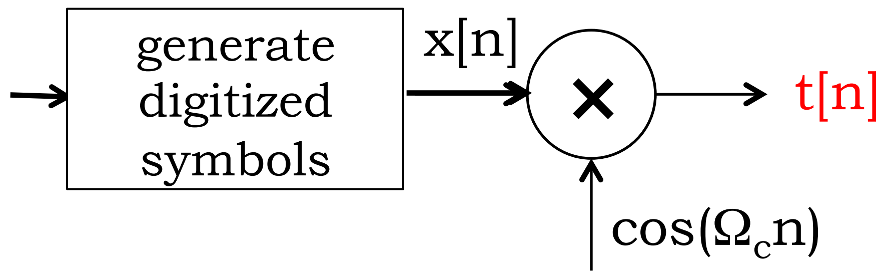
Amplitude Modulation (AM)


Wireless Station, Brant Rock, Mass.
10103

Modulation

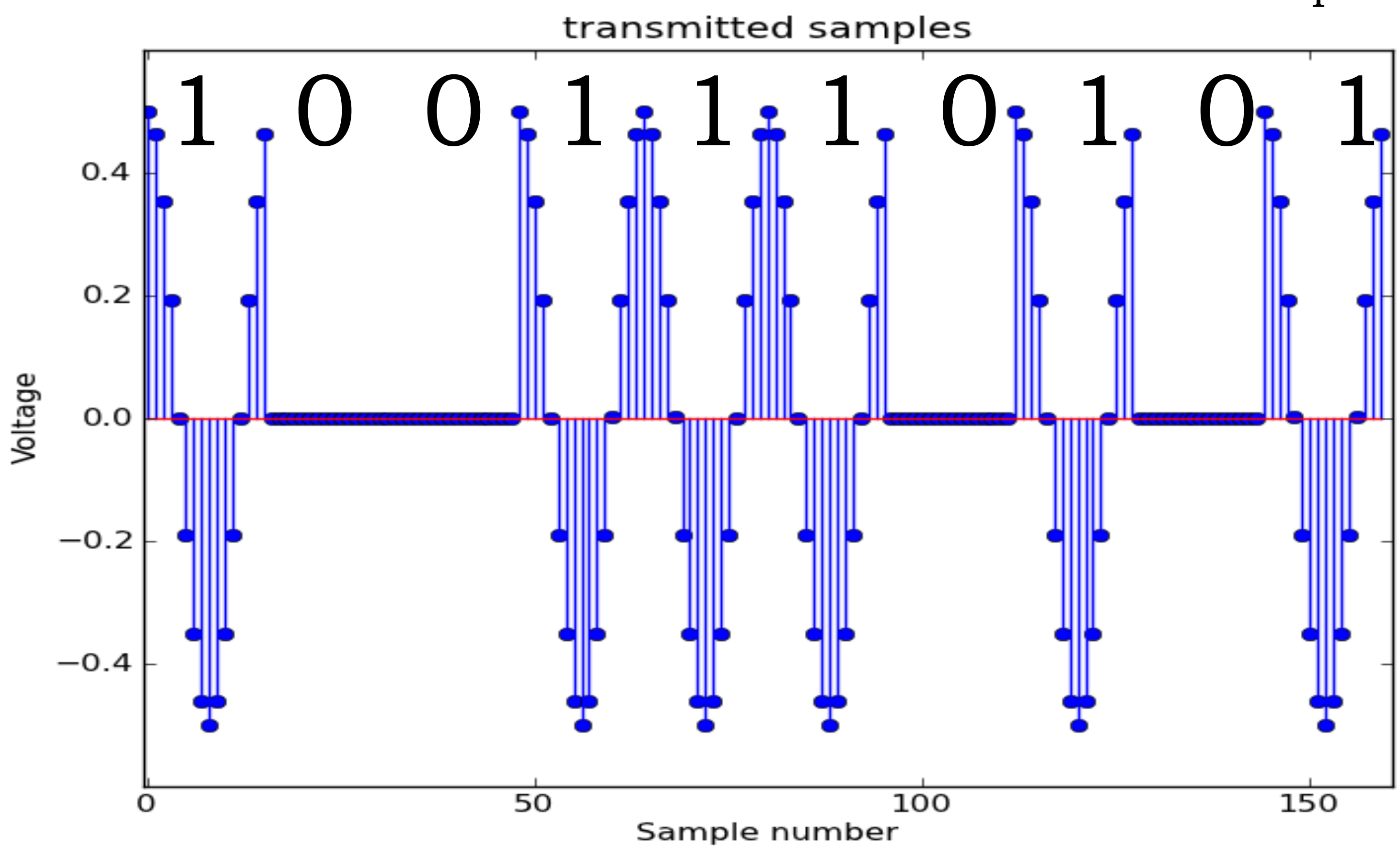
codeword bits in

1001110101



$\Omega_c = 2\pi/16$

16 samples per cycle



Ideas for Demodulation

- For on-off keying, it suffices to detect when there's signal and when there isn't, since we're only trying to distinguish $v_0 = 0$ $v_1 = V > 0$

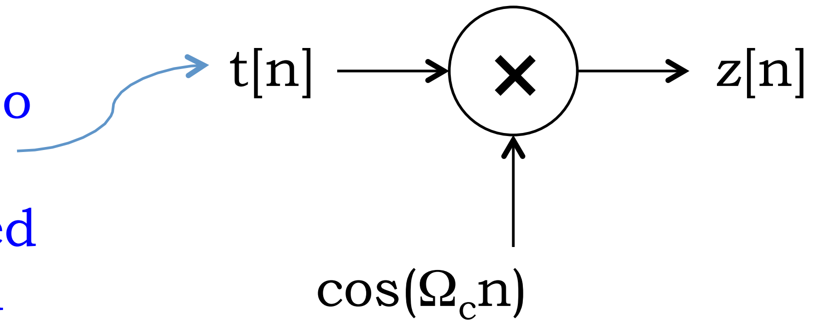
Many ways to do that, e.g., take absolute value and then local average over half-period of carrier

- For bipolar keying, we need the sign:

$$v_0 = -V \qquad v_1 = V > 0$$

Demodulation

Assuming no distortion or noise on channel, so what was transmitted is received



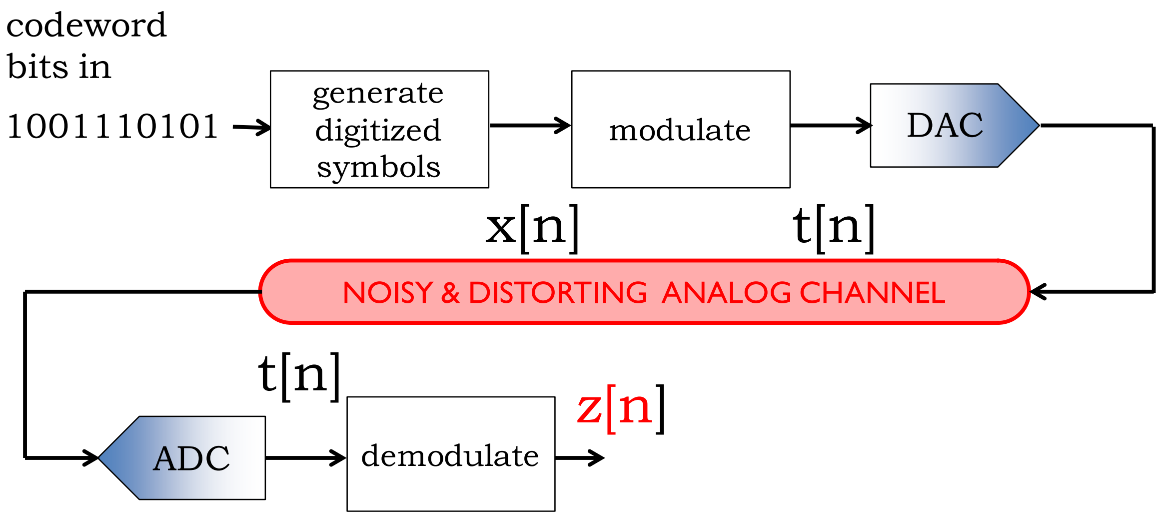
$$z[n] = t[n]\cos(\Omega_c n)$$

$$z[n] = x[n]\cos(\Omega_c n)\cos(\Omega_c n)$$

$$z[n] = 0.5x[n](1+\cos(2\Omega_c n))$$

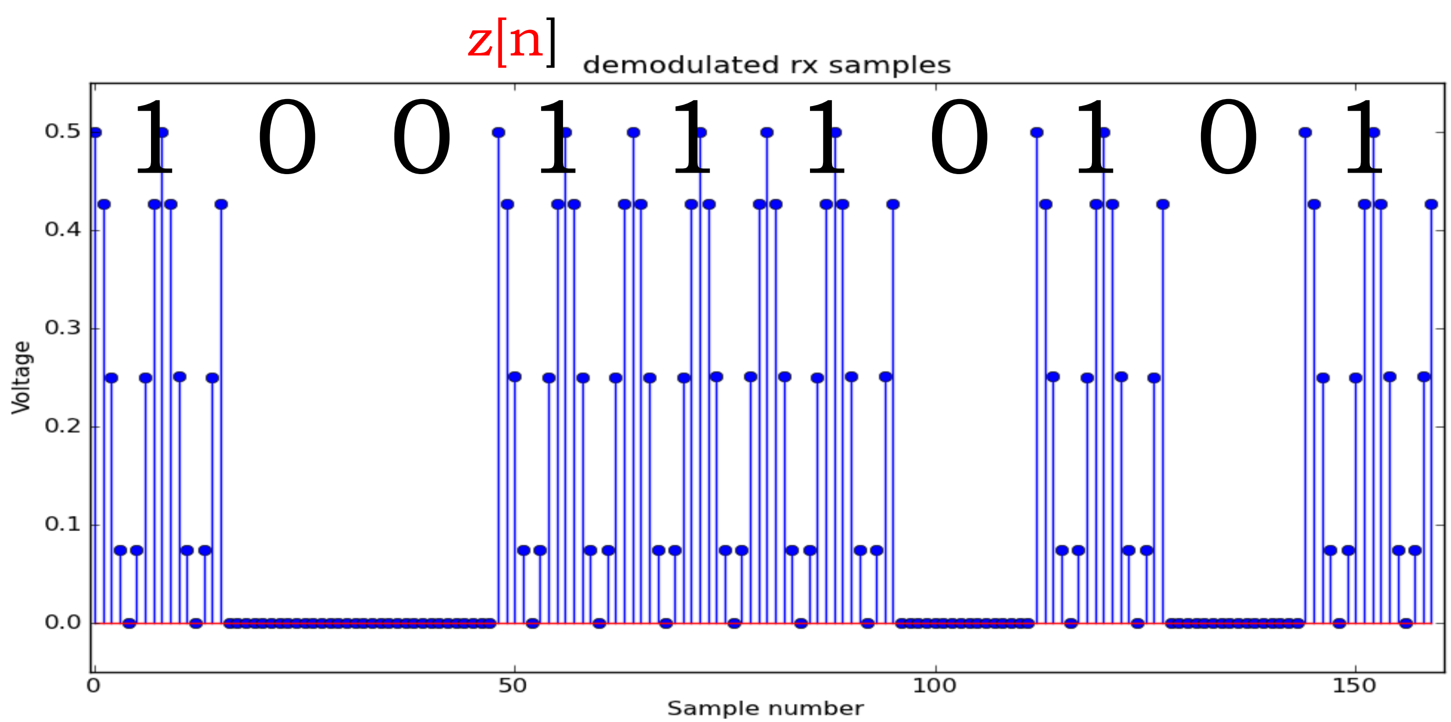
$$z[n] = 0.5x[n] + 0.5x[n]\cos(2\Omega_c n)$$

Demodulation

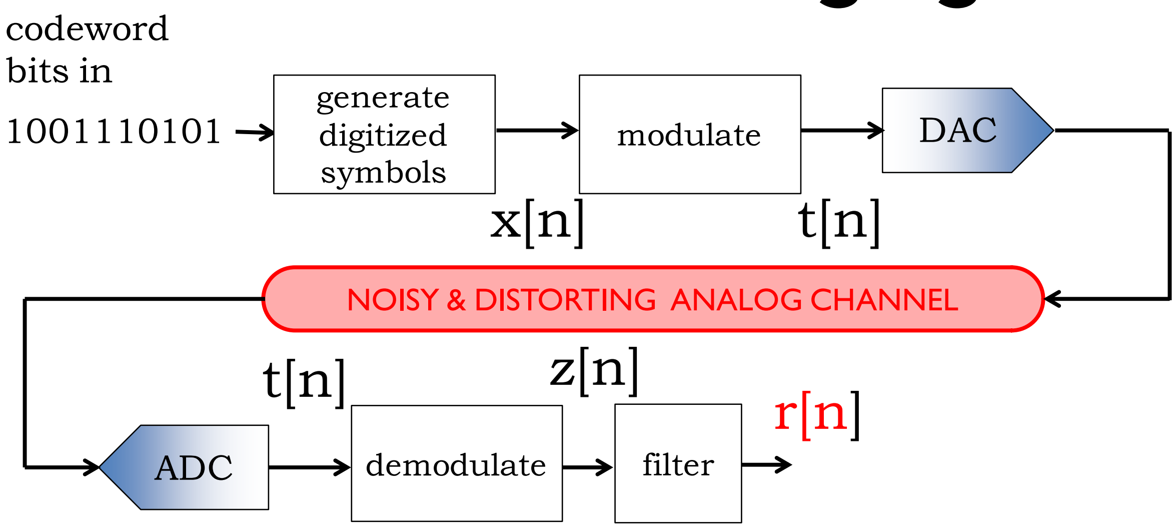


$\Omega_c = 2\pi/16$

16 samples per cycle



Averaging filter



$\Omega_c = 2\pi/16$

16 samples per cycle

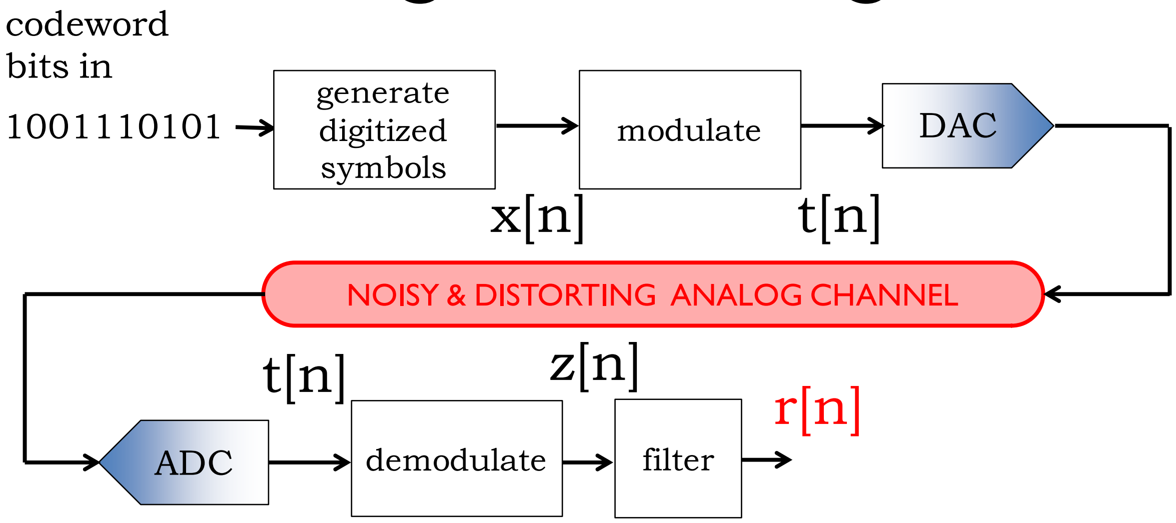
r[n] = z[n]+…+z[n-L], L+1 length of the averaging filter

For L+1=8, $2\Omega_c$ component is at $2\pi/8$, which is 8 samples per cycle

So, the $2\Omega_c$ component gets averaged out

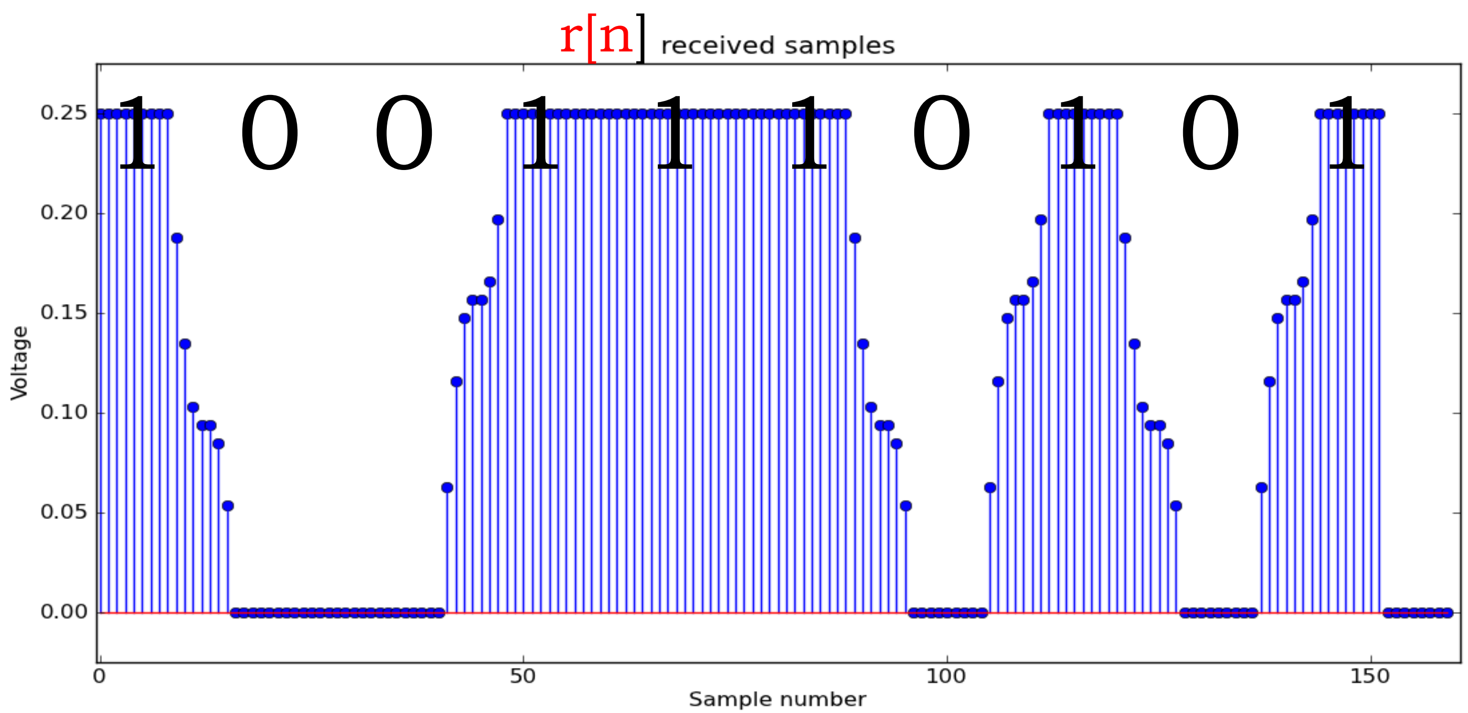
*At transitions, there is a bit of degradation, but we make decisions on the middle samples

Filtering: Removing the $2\Omega_c$ component

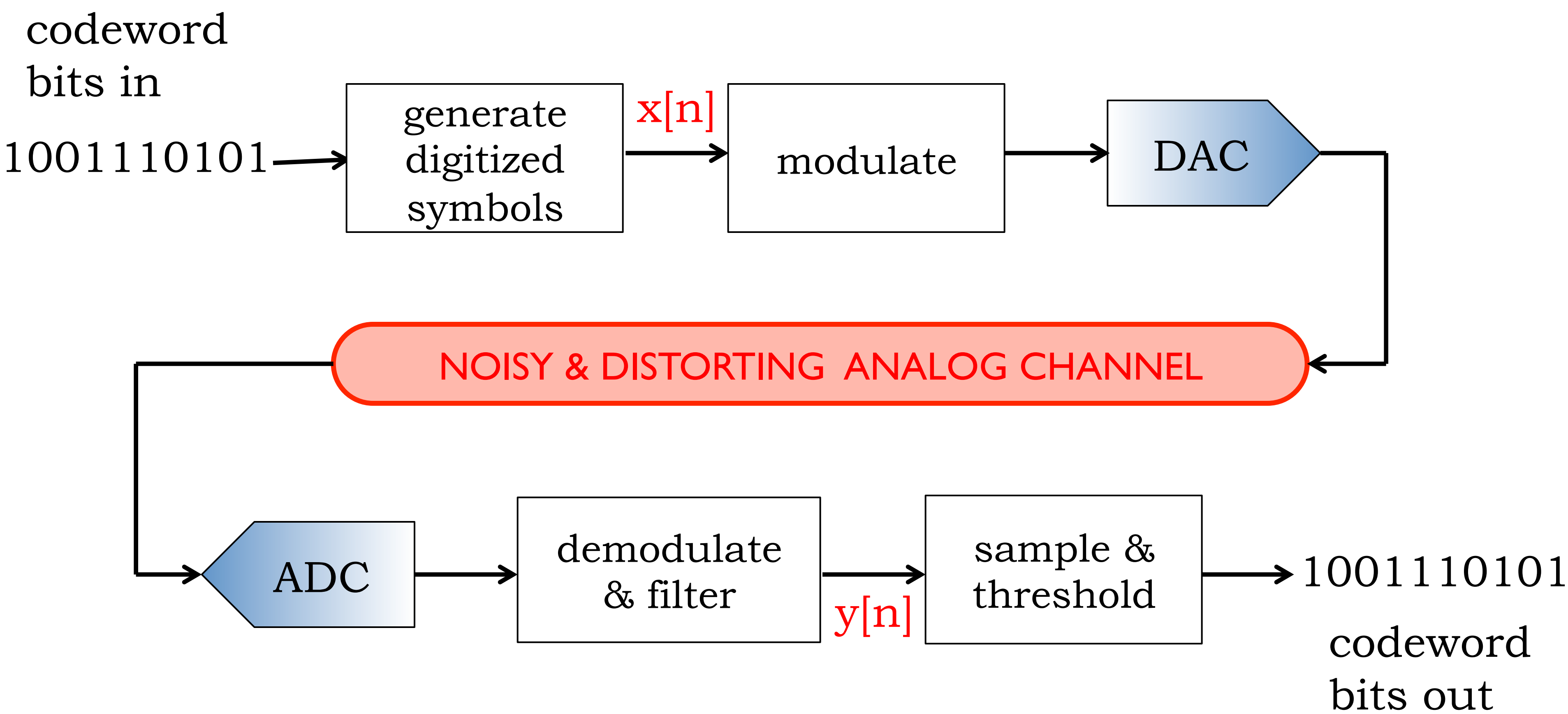


$\Omega_c = 2\pi/16$

16 samples per cycle

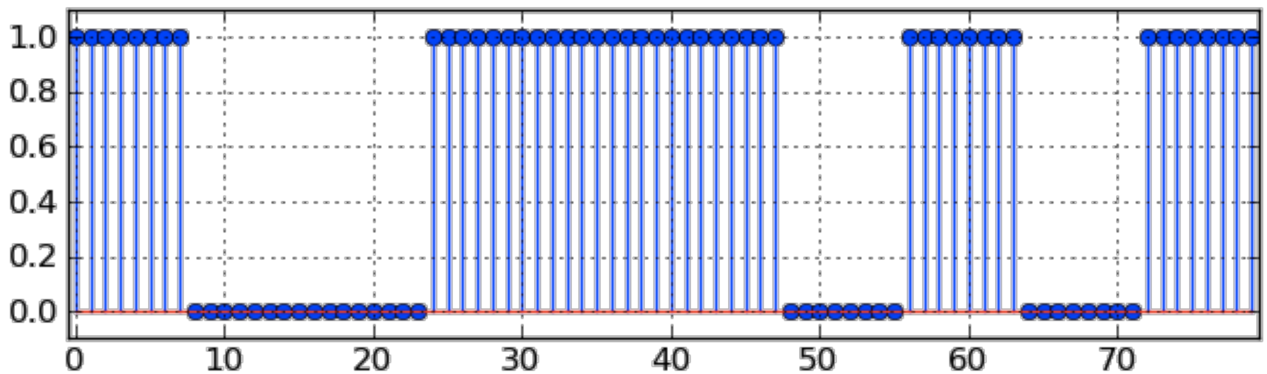


Modeling Channel Behavior

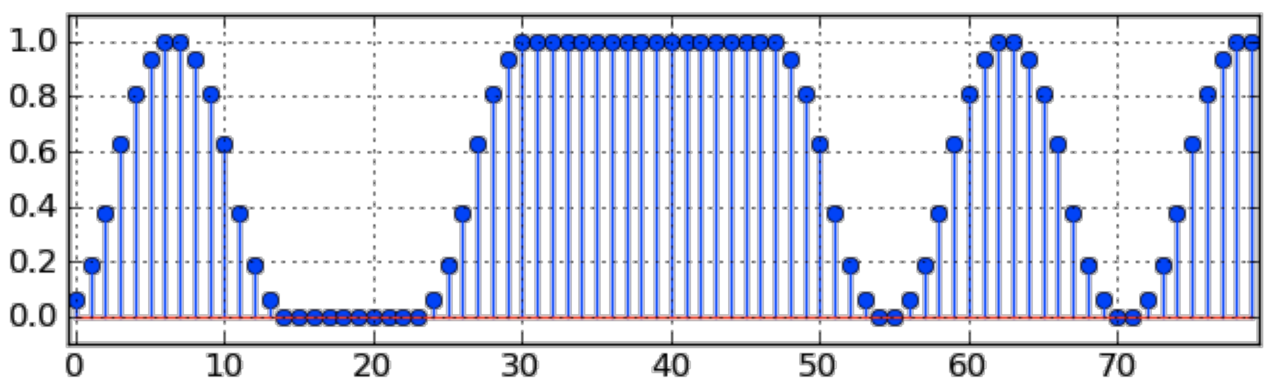


Transmission over a Channel

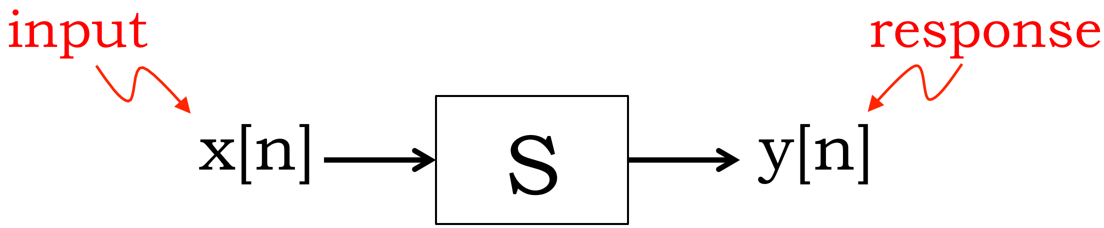
Signal x[n] from digitized symbols at transmitter



Distorted noise-free signal y[n] at receiver



System Input and Output



A discrete-time signal such as x[n] or y[n] is described by an infinite sequence of values, i.e., the time index n takes values in $-\infty$ to $+\infty$. The above picture is a snapshot at a particular time n.

In the diagram above, the sequence of *output* values y[.] is the *response* of system S to the *input* sequence x[.]

Question: Why didn't I write:

"In the diagram above, the sequence of *output* values y[n] is the *response* of system S to the *input* sequence x[n]"   ??

Notation, Notation!

--We want to be clear, but being overly explicit about things leads to a lot of notational clutter. So we take shortcuts and liberties, "abusing" and "overloading" the notation, in the hope that context and other factors will make our meaning clear.
--But poor notation can also impede, mislead, confuse! So one has draw the line carefully.

Example: our hard-working discrete-time index n (in continuous-time, it's t). Specifically, x[n] can denote

(a) the value of the signal x at a particular time n

(b) the sequence of values for n in $-\infty$ to $+\infty$, i.e., the entire signal x.

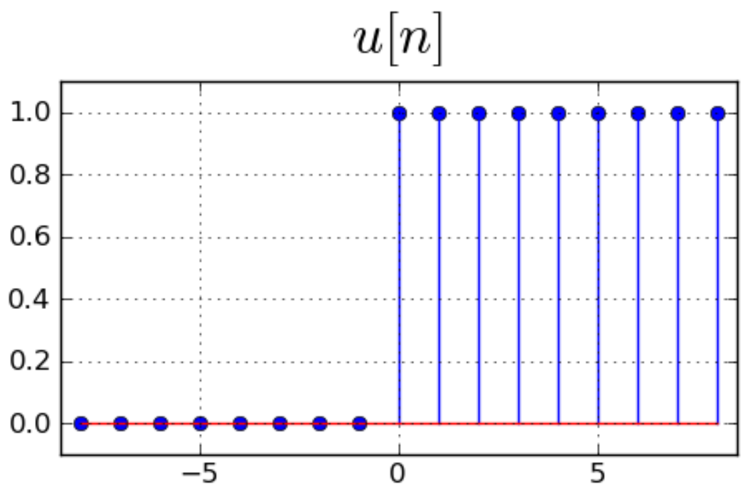
For (b), it's often clearer to write x[.] or just x --- particularly if there are multiple signals involved, because the same "dummy index" n shouldn't be used for both.

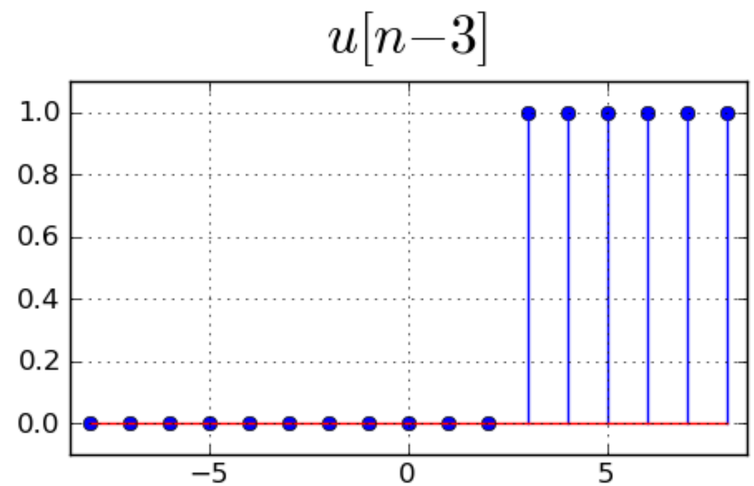
On the other hand, if you want to use x[n] for a *specific* value of time, it's sometimes clearer to write $x[n_0]$

Unit Step

A simple but useful discrete-time signal is the *unit step* signal or function, u[n], defined as

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

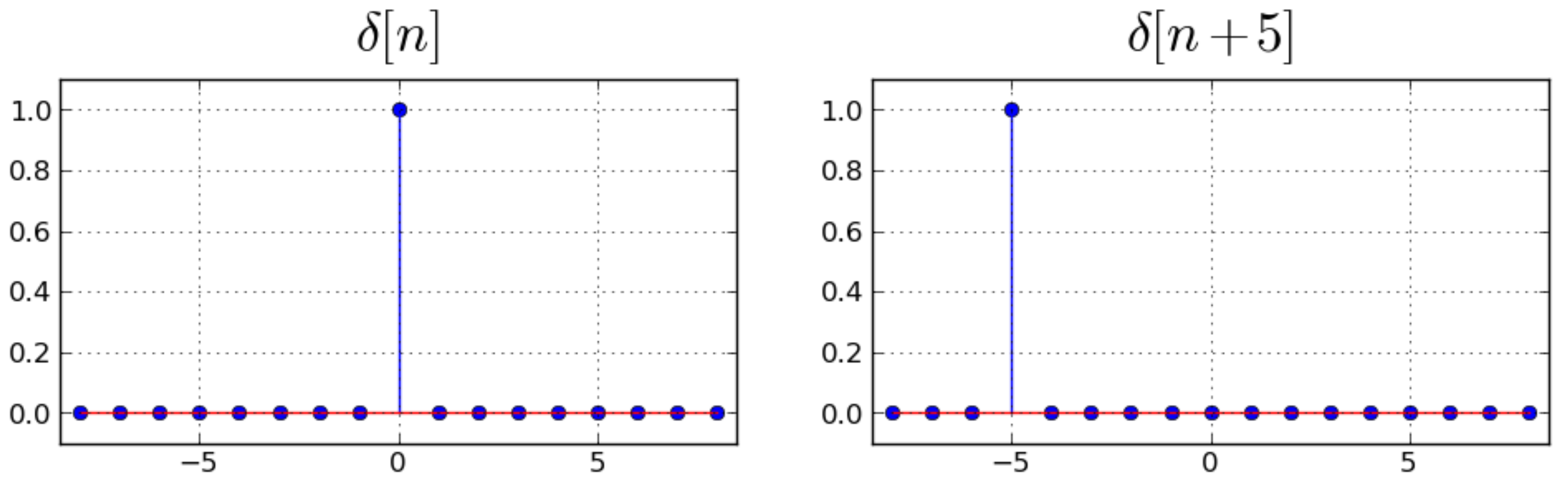




Unit Sample

Another simple but useful discrete-time signal is the *unit sample* signal or function, δ[n], defined as

$$\delta[n] = u[n] - u[n-1] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



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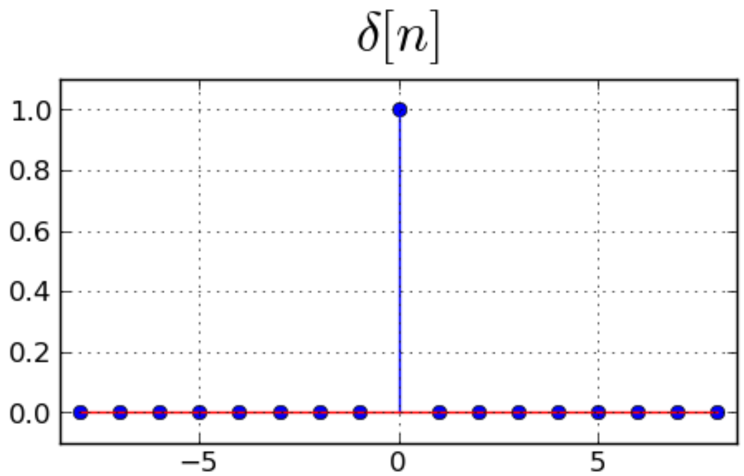
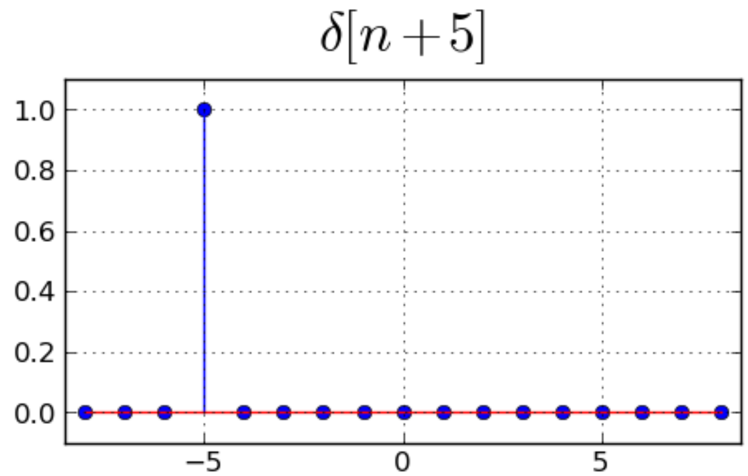
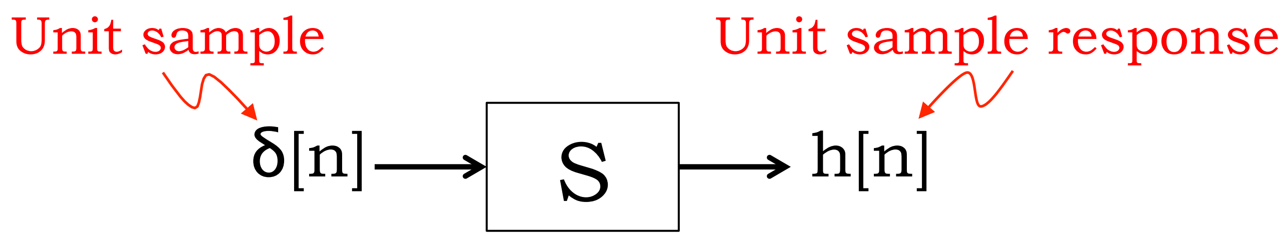




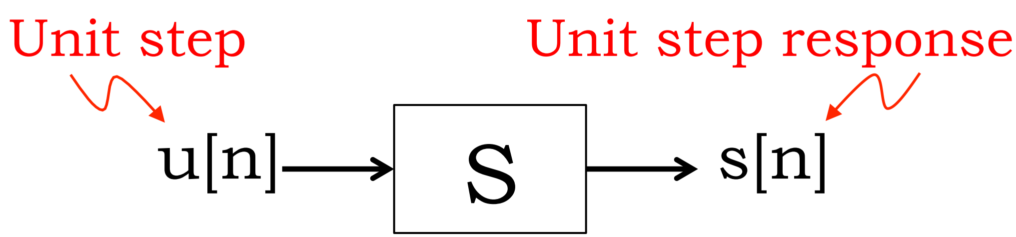
Note that standard algebraic operations on signals (e.g. subtraction, addition, scaling by a constant) are defined in the obvious way, instant by instant.

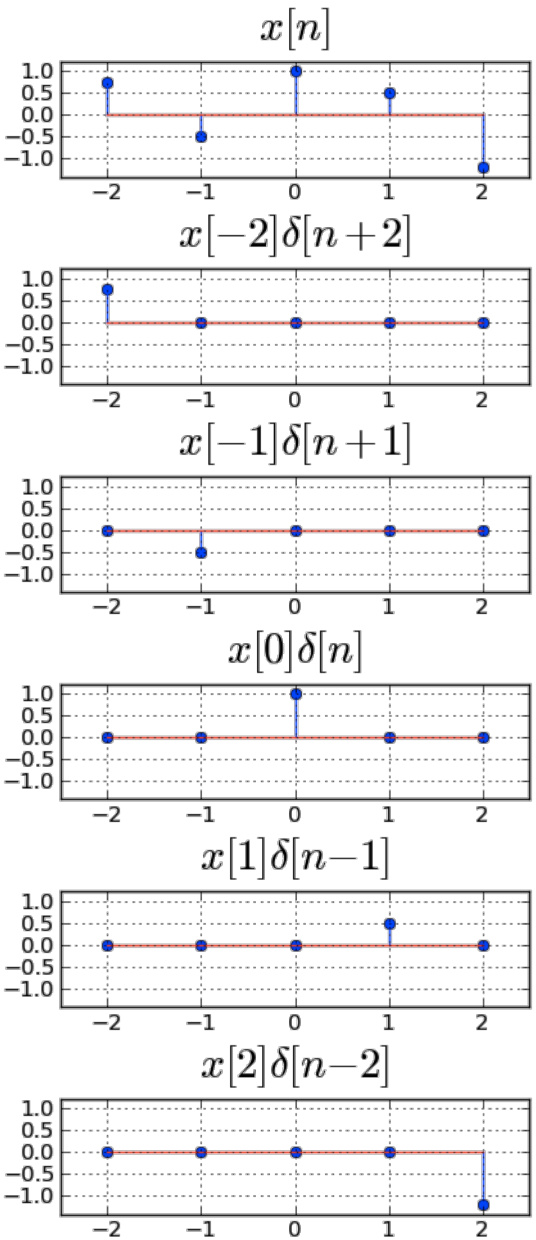
Unit Sample and Unit Step Responses



The *unit sample response* of a system S is the response of the system to the unit sample input. We will always denote the unit sample response as h[n].

Similarly, the *unit step response* s[n]:





Unit Sample Decomposition

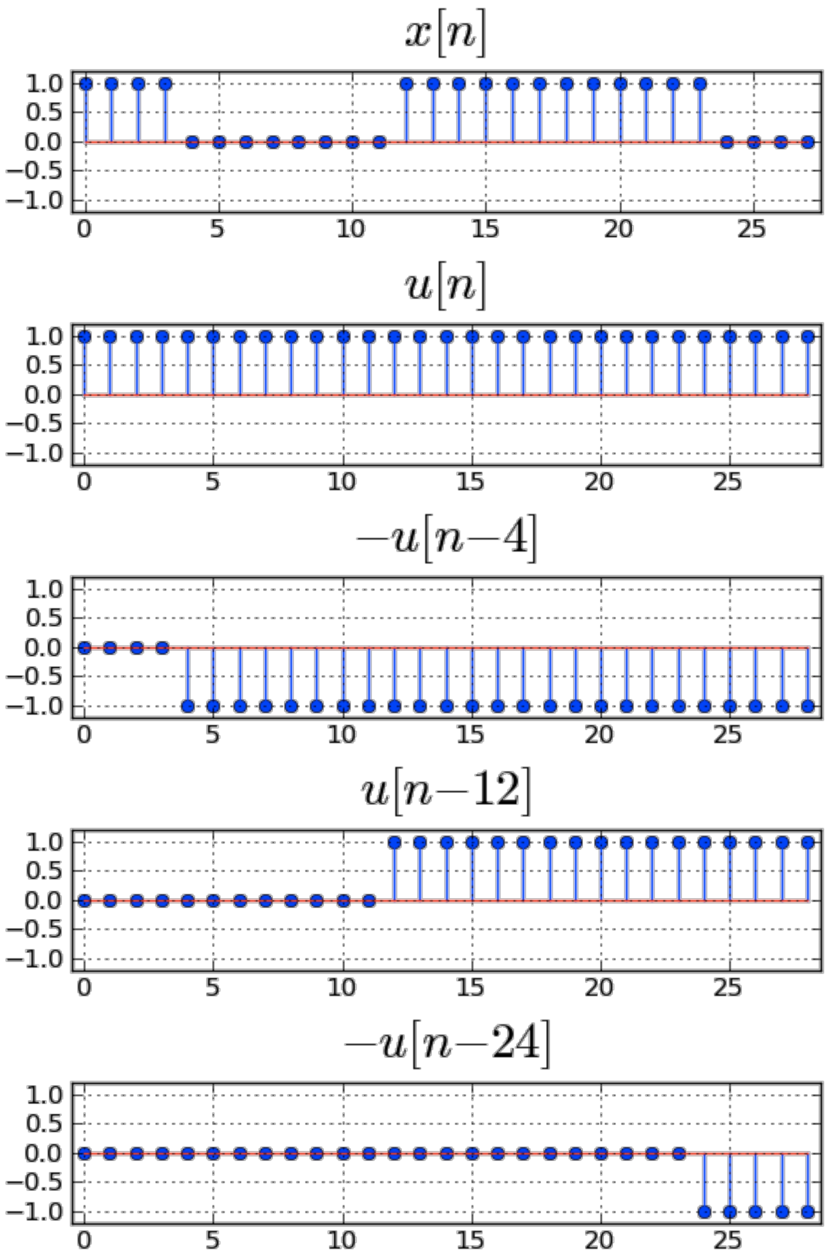
A discrete-time signal can be decomposed into a sum of time-shifted, scaled unit samples.

Example: in the figure, x[n] is the sum of x[-2]δ[n+2] + x[-1]δ[n+1] + … + x[2]δ[n-2].

In general:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

For any particular index, only one term of this sum is non-zero



Unit Step Decomposition

Digital signaling waveforms are easily decomposed into time-shifted, scaled unit steps (each transition corresponds to another shifted, scaled unit step).

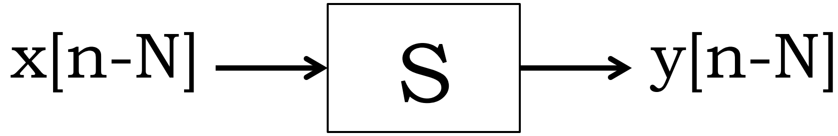
In this example, x[n] is the transmission of 1001110 using 4 samples/bit:

$$x[n] = u[n] - u[n-4] + u[n-12] - u[n-24]$$

Time Invariant Systems

Let y[n] be the response of S to input x[n].

If for all possible sequences x[n] and integers N



then system S is said to be *time invariant* (TI). A time shift in the input sequence to S results in an identical time shift of the output sequence.

In particular, for a TI system, a shifted unit sample function $\delta[n-N]$ at the input generates an identically shifted unit sample response $h[n-N]$ at the output.

Linear Systems

Let $y_1[n]$ be the response of S to an arbitrary input $x_1[n]$ and $y_2[n]$ be the response to an arbitrary $x_2[n]$.

If, for arbitrary scalar coefficients *a* and *b*, we have:

$$ax_1[n]+bx_2[n] \longrightarrow \boxed{S} \longrightarrow ay_1[n]+by_2[n]$$

then system S is said to be *linear*. If the input is the weighted sum of several signals, the response is the *superposition* (i.e., weighted sum) of the response to those signals.

One key consequence: If the input is identically 0 for a linear system, the output must also be identically 0.

Our focus will be on LTI Models

- LTI = Linear *and* Time Invariant

- Good description of time-invariant systems for small deviations from a nominal operating equilibrium

- Lots of structure, detailed analysis possible, amenable to development of good computational tools, …

- Major arena for engineering design