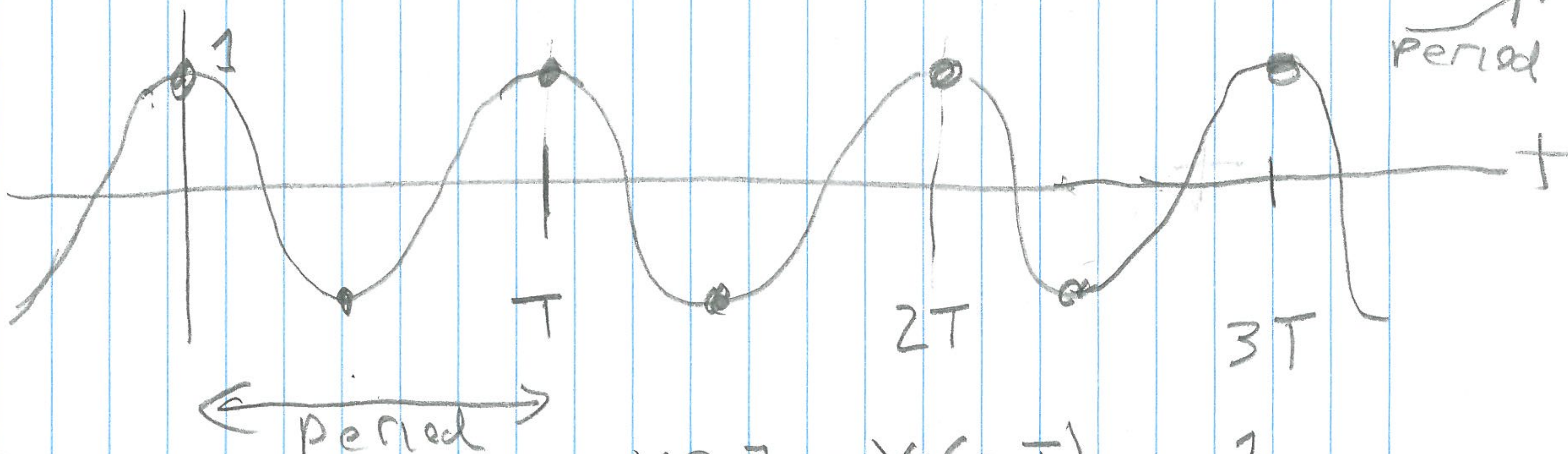
51 Sample Rate & π

<u>Continuous Time Cosine</u> $x(t) = \cos(2\pi f t) = \cos(2\pi \frac{t}{T})$

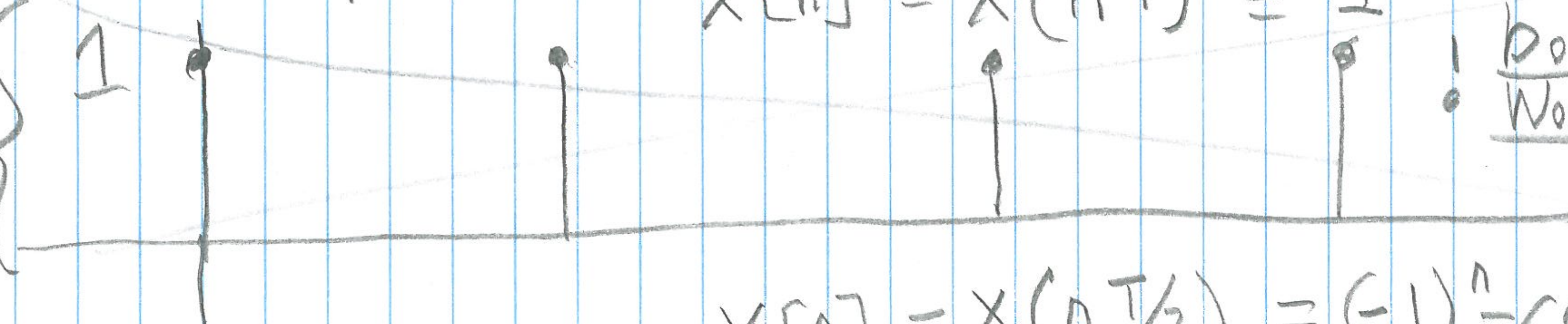
frequency

period



$x[n] = x(nT) = 1$

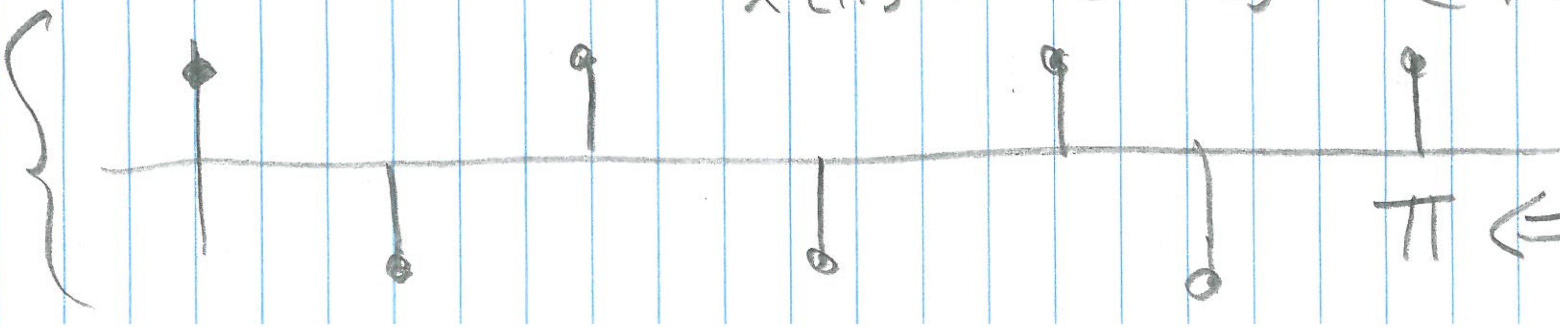
Sampling rate $f_s = f$



Doesn't Work!

$x[n] = x(nT/2) = (-1)^n = \cos \pi n$

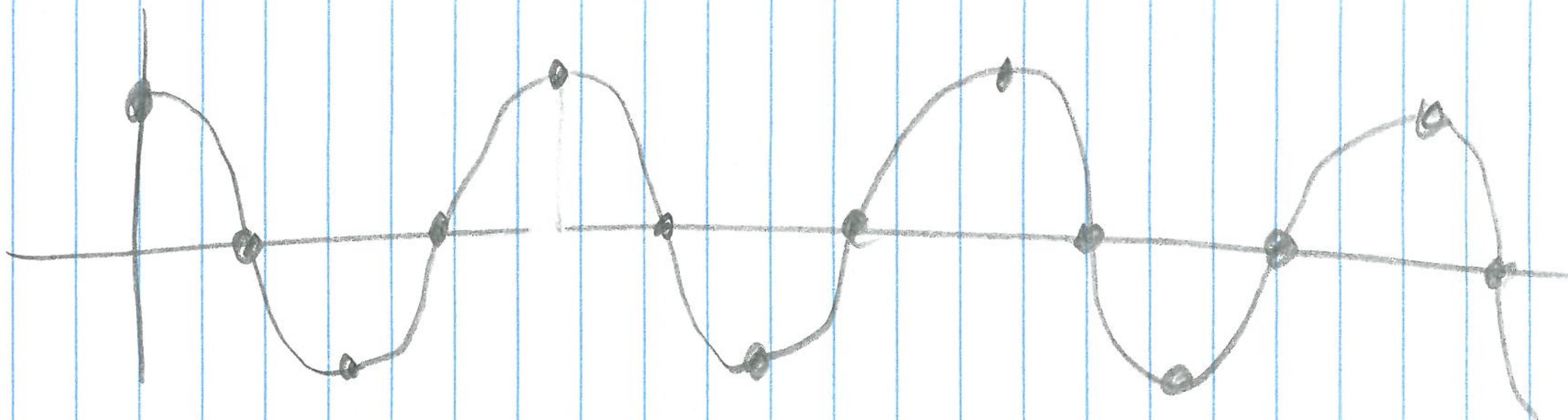
Sampling rate $f_s = 2f$



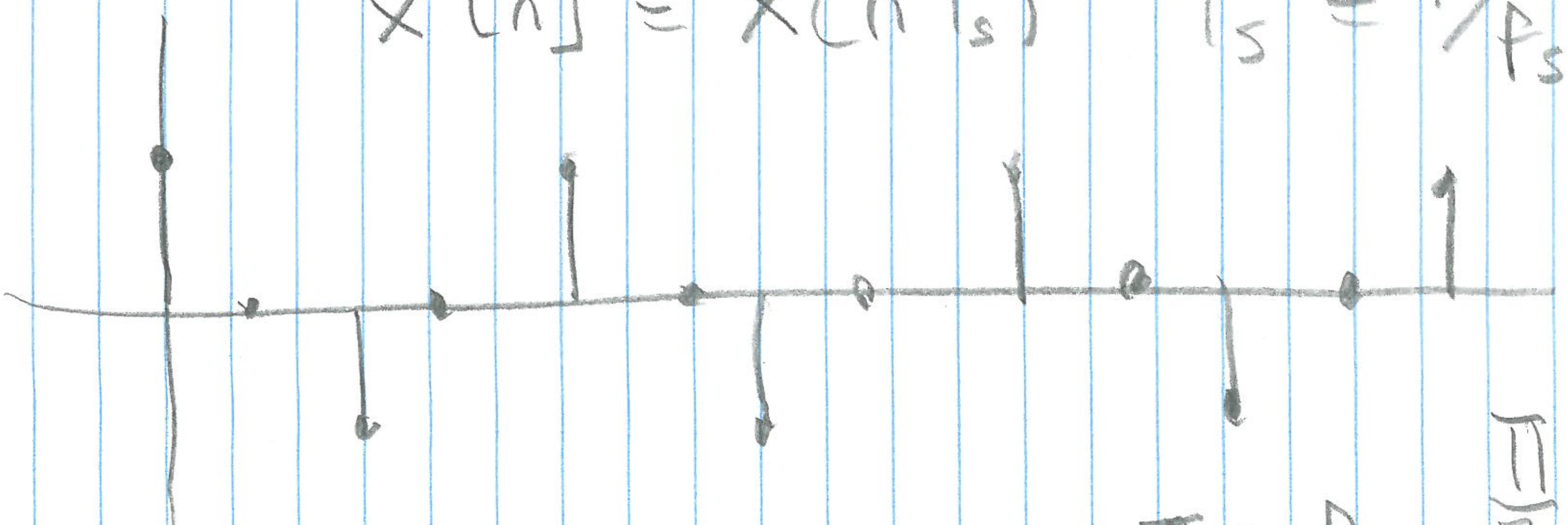
$\pi \Leftrightarrow f_s/2$

52

$\cos 2\pi f_o t \qquad f_o = f_s/4$



$x[n] = x(nT_s) \qquad T_s = 1/f_s$



$x[n] = \cos \pi/2\, n \qquad \frac{\pi}{2} \Leftrightarrow \frac{f_s}{4}$

53

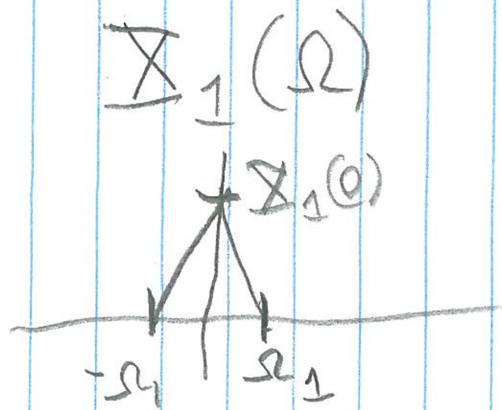
In general $\frac{f}{f_s} = \frac{\Omega_0}{2\pi}$

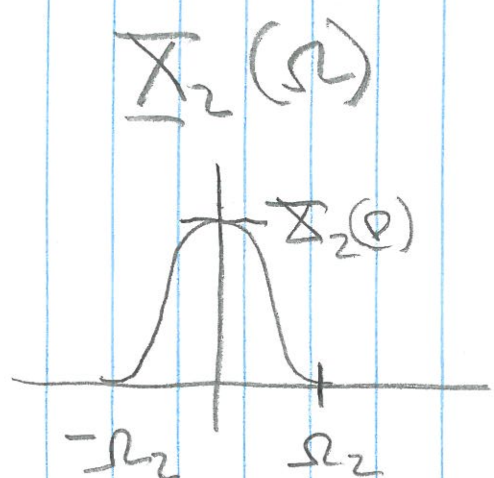
$f_s = 1 \cdot 10^6 \quad f = 1 \cdot 10^5 \quad \frac{f}{f_s} = \frac{1}{10}$

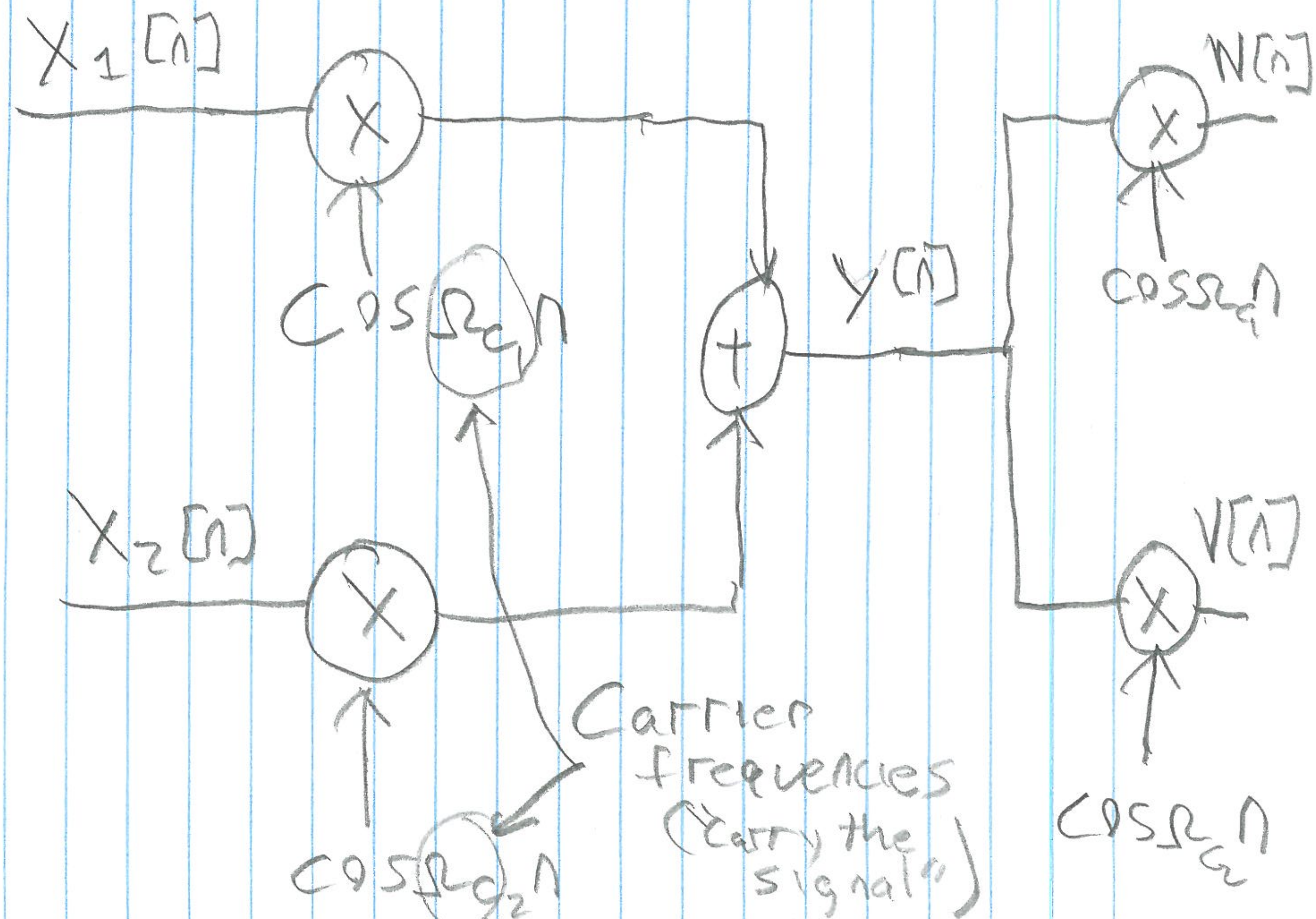
$\Rightarrow \Omega_0 = \frac{\pi}{5}$

Basic Modulation

BM1





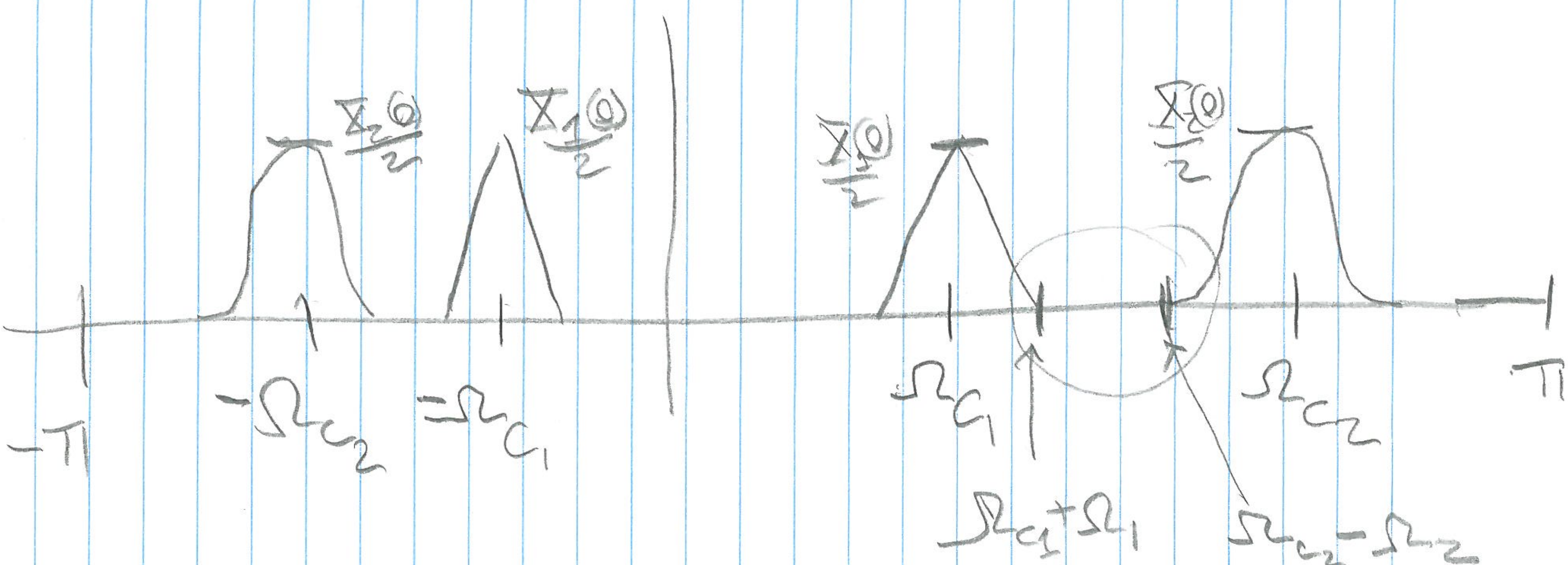


What is $Y(\Omega)$, $W(\Omega)$, $V(\Omega)$

BM2

Assume $\pi >> \Omega_{c_2} > \Omega_{c_1}$

$Y(\Omega)$



<u>No interference if</u>

$(\Omega_{c_1}+\Omega_1) < (\Omega_{c_2}-\Omega_2)$

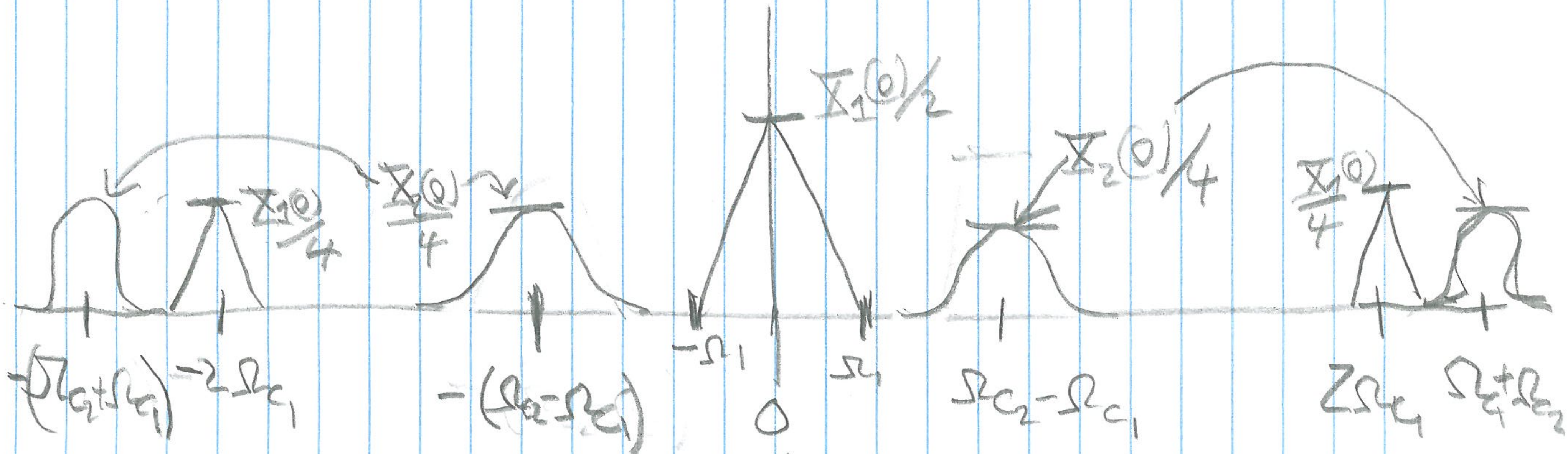
Sum of signal "Bandwidths"

OR $(\Omega_1+\Omega_2) < (\Omega_{c_2}-\Omega_{c_1})$

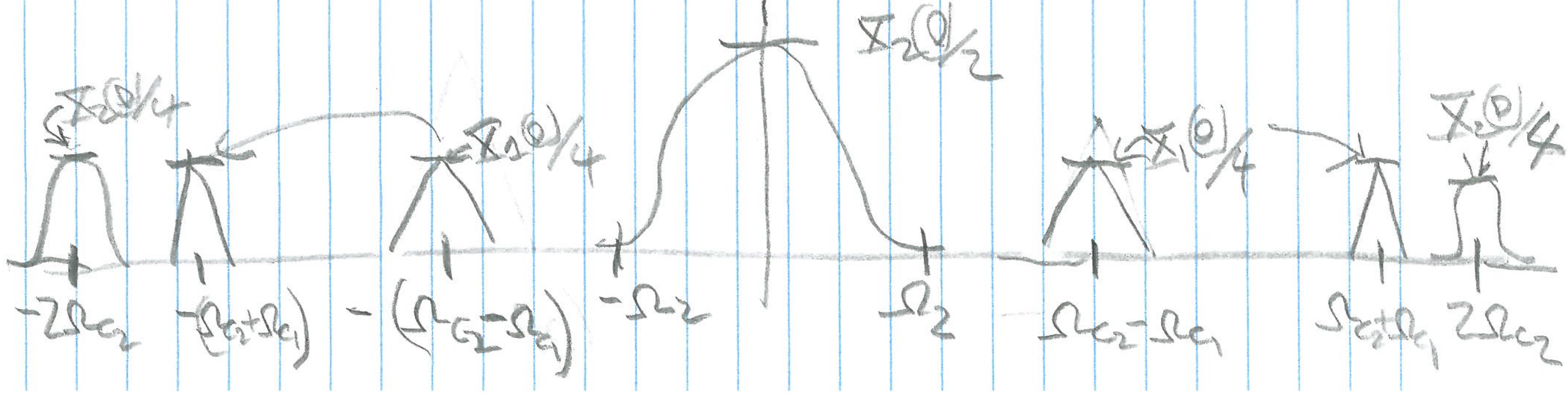
Carrier Frequency Gap.

$W(\Omega)$

BM2

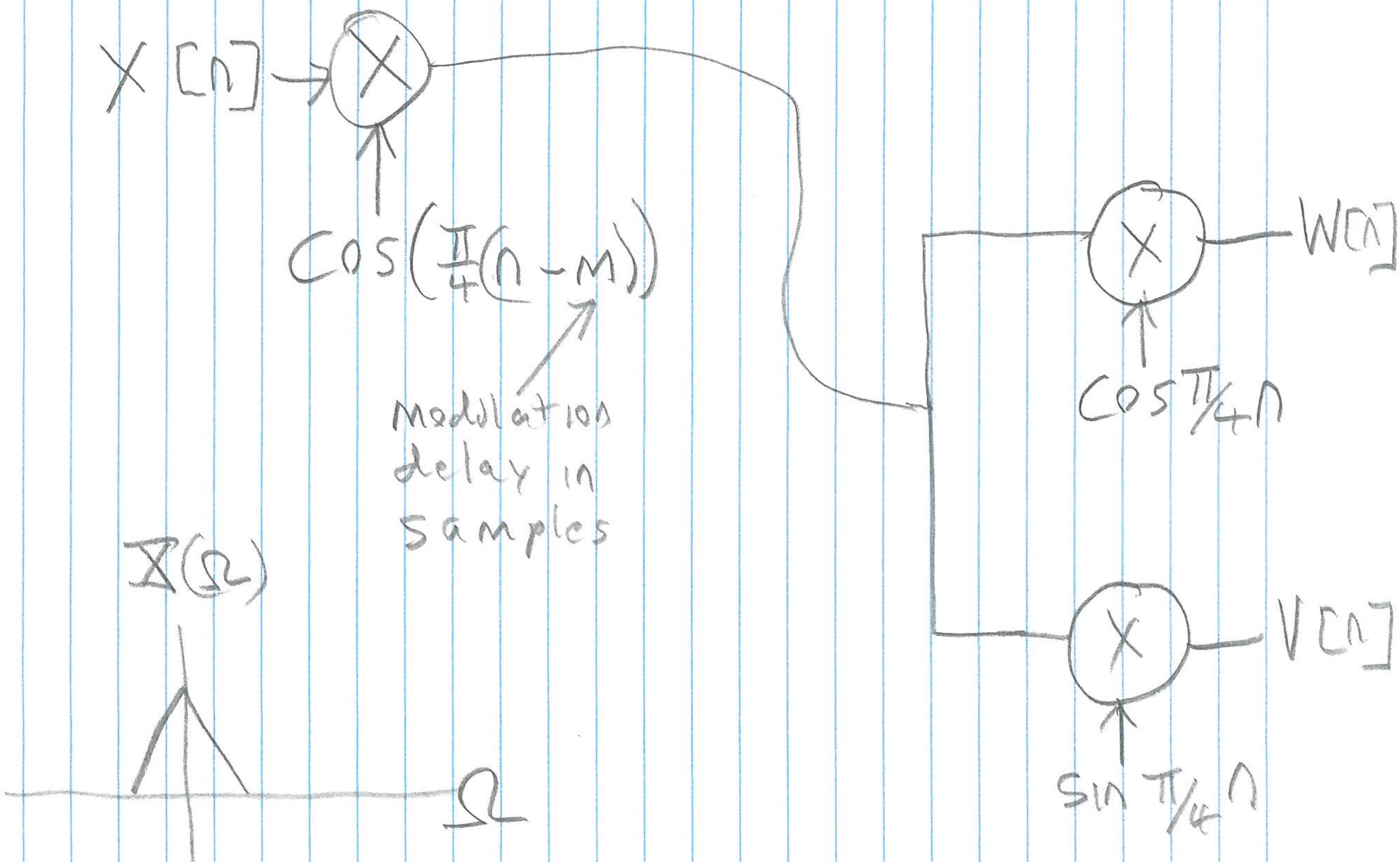


$V(\Omega)$



Transmitter Delayed Modulation

DM1



DM2

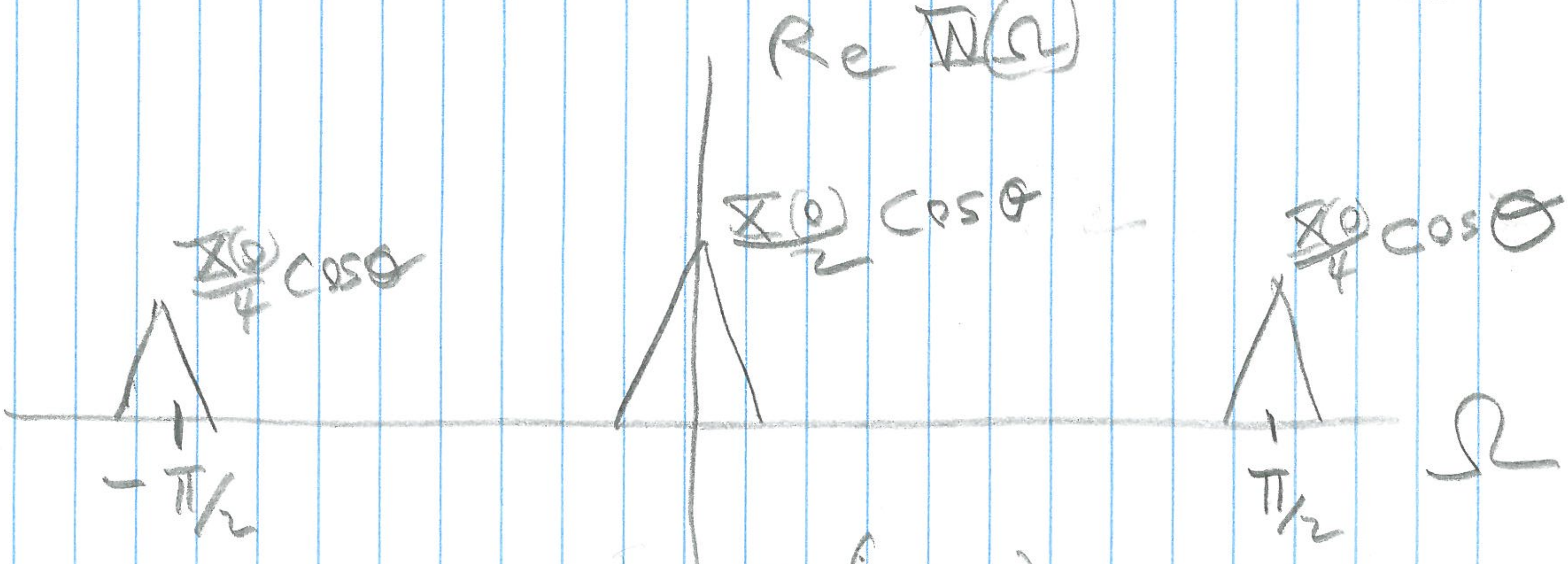
$$w[n] = \underbrace{\cos \tfrac{\pi}{4} n}_{\text{demod}} \; \underbrace{\cos \tfrac{\pi}{4}(n-m) \; x[n]}_{\text{modulation}}$$

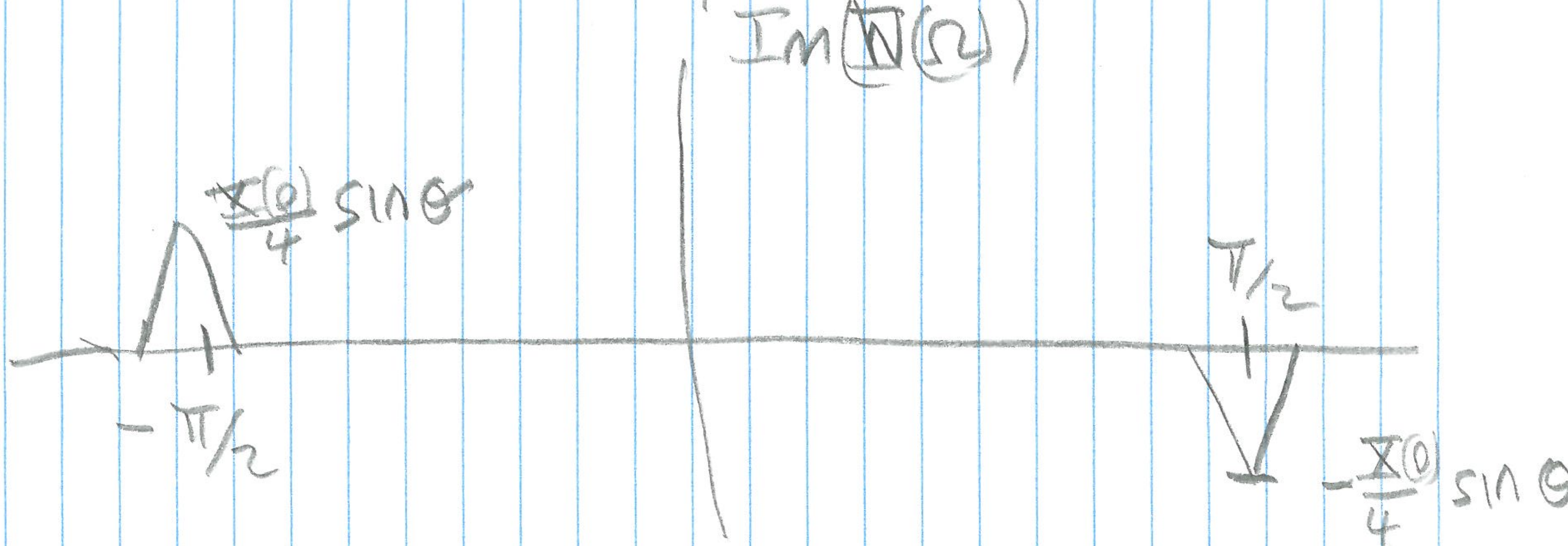
$$w[n] = \frac{1}{2}\left(e^{j\pi/4\, n} + e^{-j\pi/4\, n}\right)\frac{1}{2}\left(e^{j\pi/4 (n-m)} + e^{-j\pi/4(n-m)}\right) x[n]$$

$$= \frac{x[n]}{4}\left(e^{j\frac{\pi}{2} n} e^{-j\pi/4\, m} + \underbrace{e^{j\pi/4\, m} + e^{-j\pi/4\, m}}_{2\cos \frac{\pi}{4} m} + e^{j\frac{\pi}{2} n} e^{j\pi/4\, m}\right)$$

$$\frac{\pi}{4} m \equiv \Theta$$

$W(\Omega)$ DM3

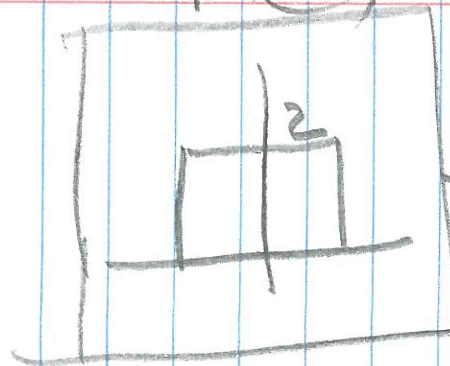

Re $W(\Omega)$
$\frac{X(0)}{2}\cos\theta$
$\frac{X(0)}{4}\cos\theta$
$\frac{X(0)}{4}\cos\theta$
$-\pi/2$
$\pi/2$
Ω


$Im(W(\Omega))$
$\frac{X(0)}{4}\sin\theta$
$\pi/2$
$-\pi/2$
$-\frac{X(0)}{4}\sin\theta$

DM4

$H(\Omega)$

$W[n] \rightarrow$



LPF

$\underbrace{x[n]\cos\theta}_{I[n]}$

"in phase"

DM5

$$V[n] = \sin \tfrac{\pi}{4} n \cos \tfrac{\pi}{4}(n-m)\, x[n]$$

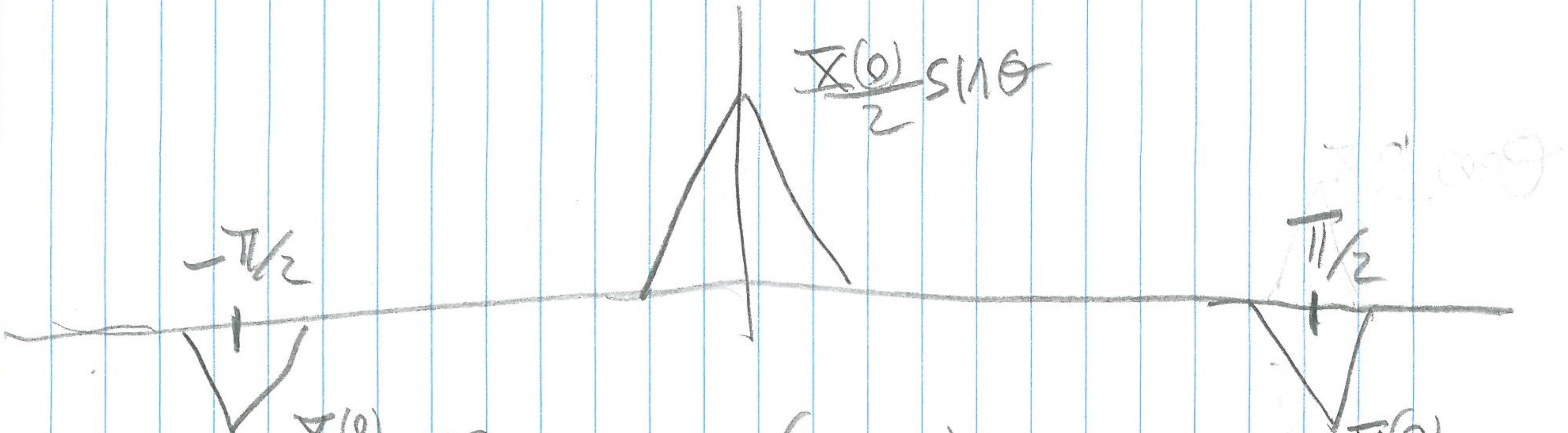
$$V[n] = \frac{-j}{2}\left(e^{j\pi/4 n} - e^{-j\pi/4 n}\right)\frac{1}{2}\left(e^{j\pi/4 (n-m)} + e^{-j\pi/4 (n-m)}\right) x[n]$$

$$= \frac{-j\,x[n]}{4}\left(e^{j\pi/2 n} e^{-j\pi/4 m} + \left(e^{j\pi/4 m} - e^{-j\pi/4 m}\right) - e^{-j\pi/2 n} e^{+j\pi/4 m}\right)$$

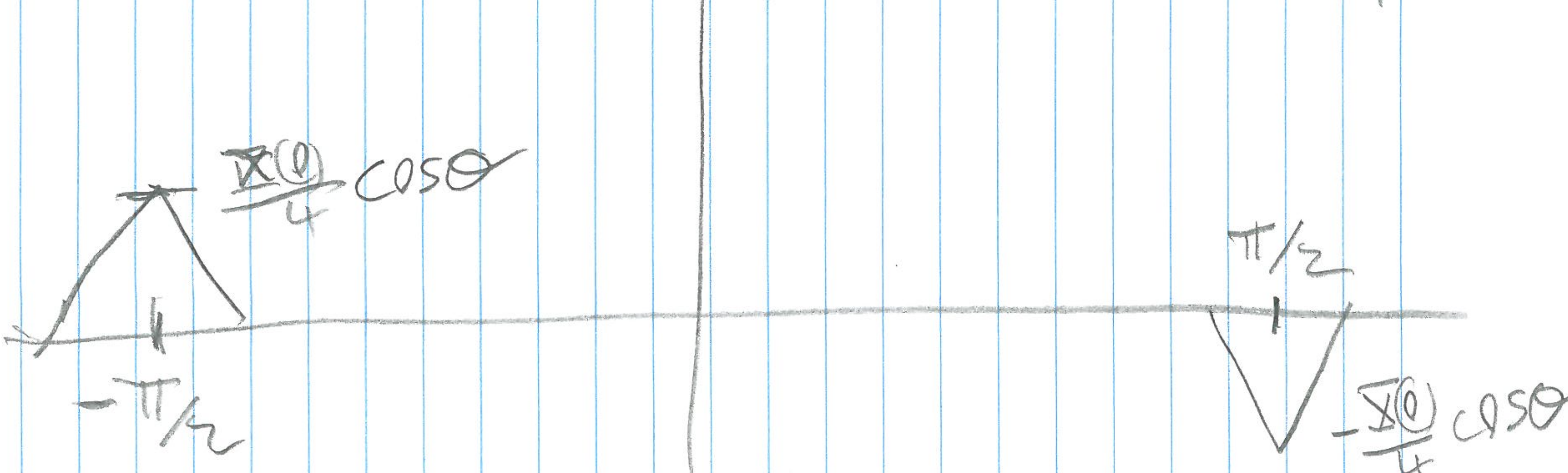
$$\pi/4\, m \equiv \theta$$

DM6

$Re(V(\Omega))$



$Im(V(\Omega))$

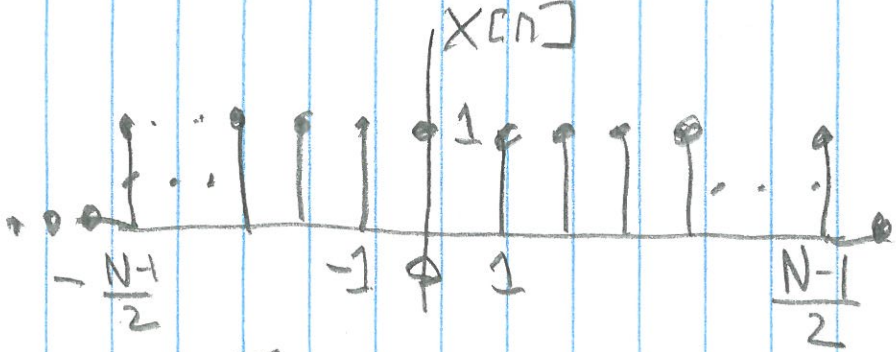


$$V[n] \rightarrow \boxed{\text{LPF}} \rightarrow \underbrace{X[n]\sin\theta}_{\substack{Q[n] \\ \text{"quadrature"}}}$$

Modulation Notes For Boxcars

1

First: The Even function of $\sqcap$ Box car

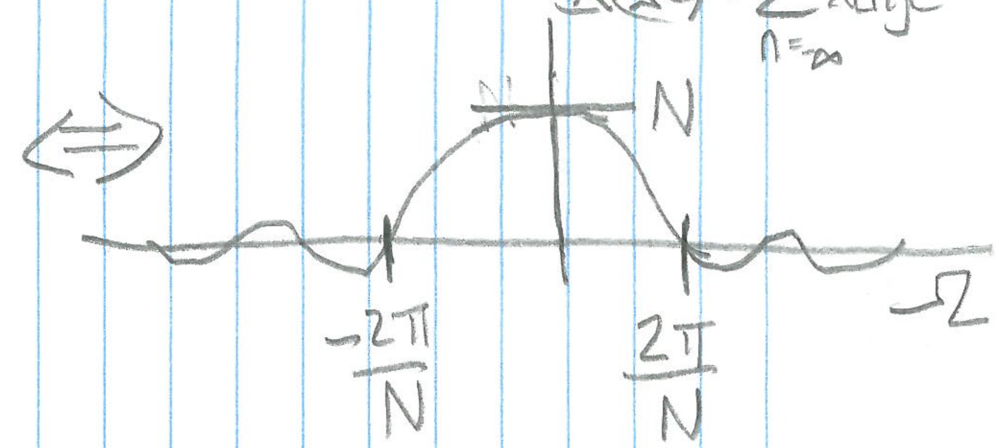


N is odd

x is an even function

$\Longleftrightarrow$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$



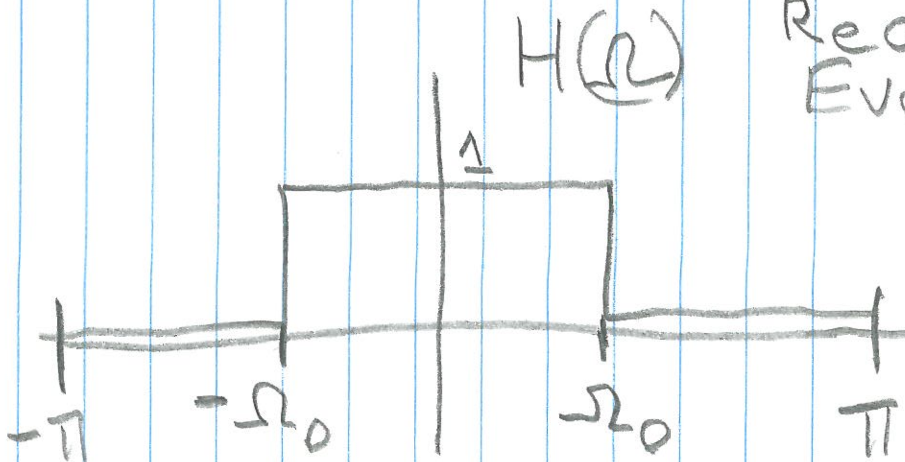
$X(\Omega)$ is real and even

$$X(0) = \sum x[n]\, e^{-j0n} = \sum x[n] = N$$

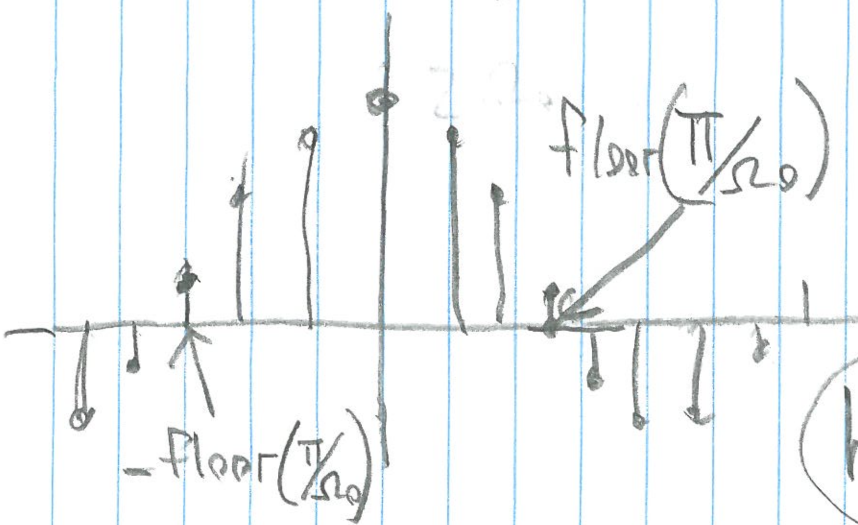
See Class Notes $\Longrightarrow$ $X(\Omega) = \dfrac{\sin(N\frac{\Omega}{2})}{\sin(\Omega/2)} = 0$ when $\Omega = 2\pi/N$

The Ideal Low Pass Filter

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\Omega)\, e^{j\Omega n}\, d\Omega$$



Real
Even



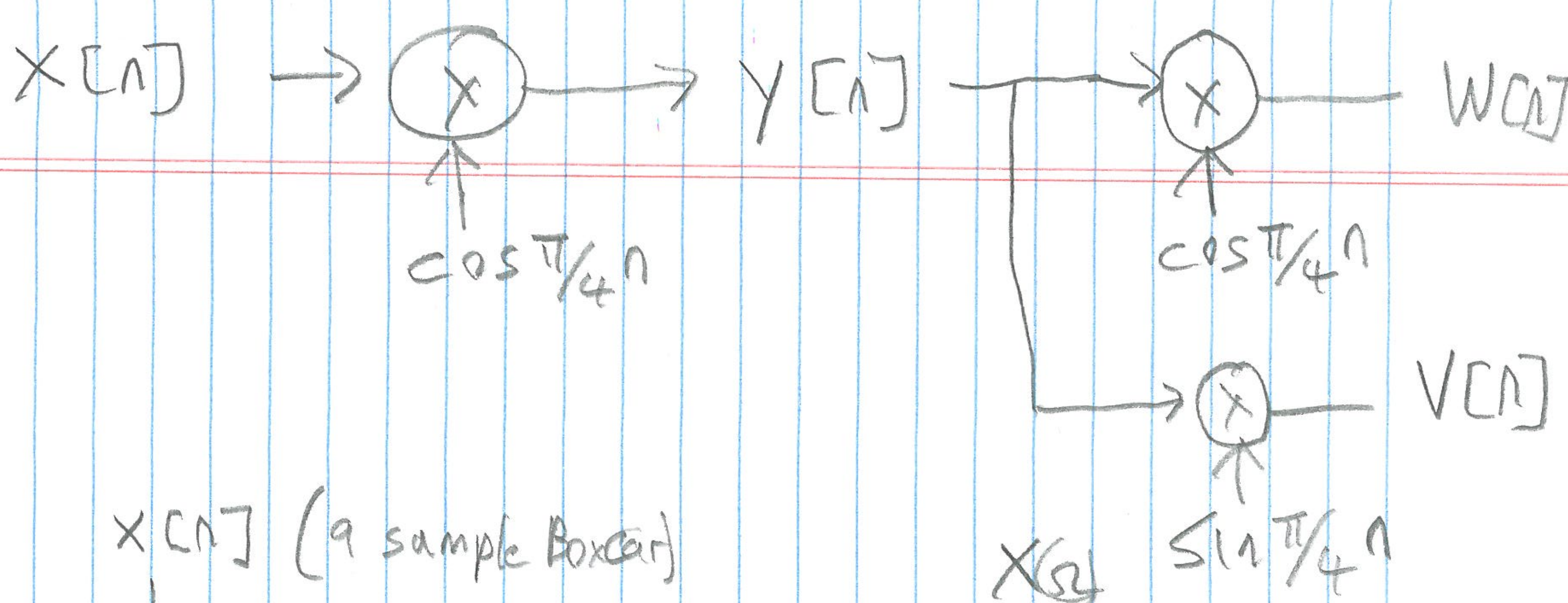
$$h[n] = \frac{\sin(\Omega_0 n)}{\pi n} \Leftarrow \text{See Class Notes}$$

$$h[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\Omega)\, e^{j\Omega(0)}\, d\Omega$$

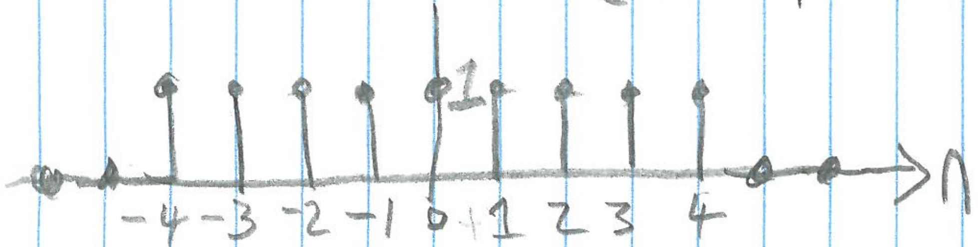
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\Omega)\, d\Omega = \frac{\Omega_0}{\pi}$$

$H(\Omega)$ is Real $\Rightarrow$ $h[n]$ is even

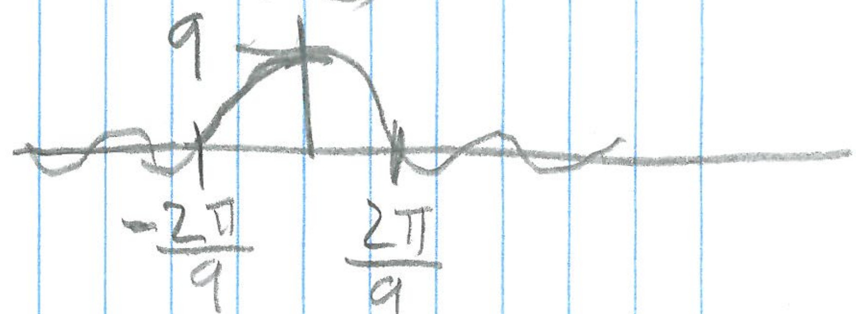
3



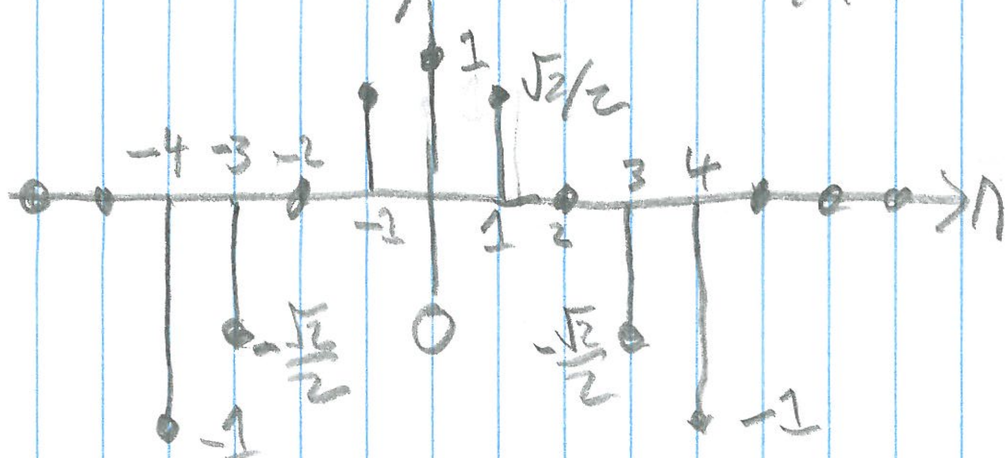
x[n] (9 sample Boxcar)



X(Ω)



$y[n] = \cos \frac{\pi}{4} n$

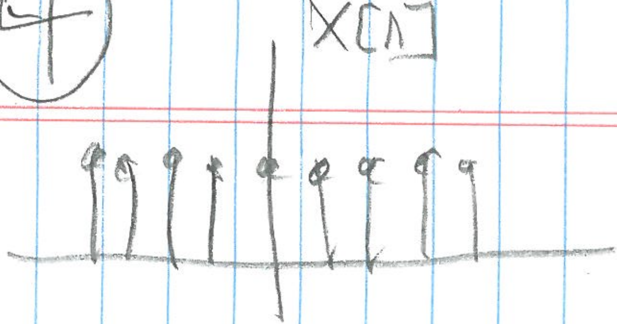


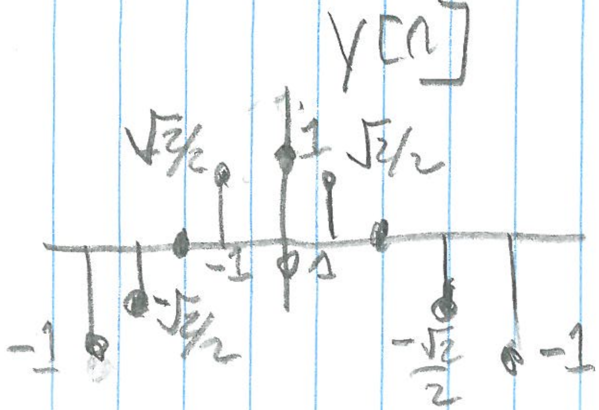
$Y(\Omega) = ?$

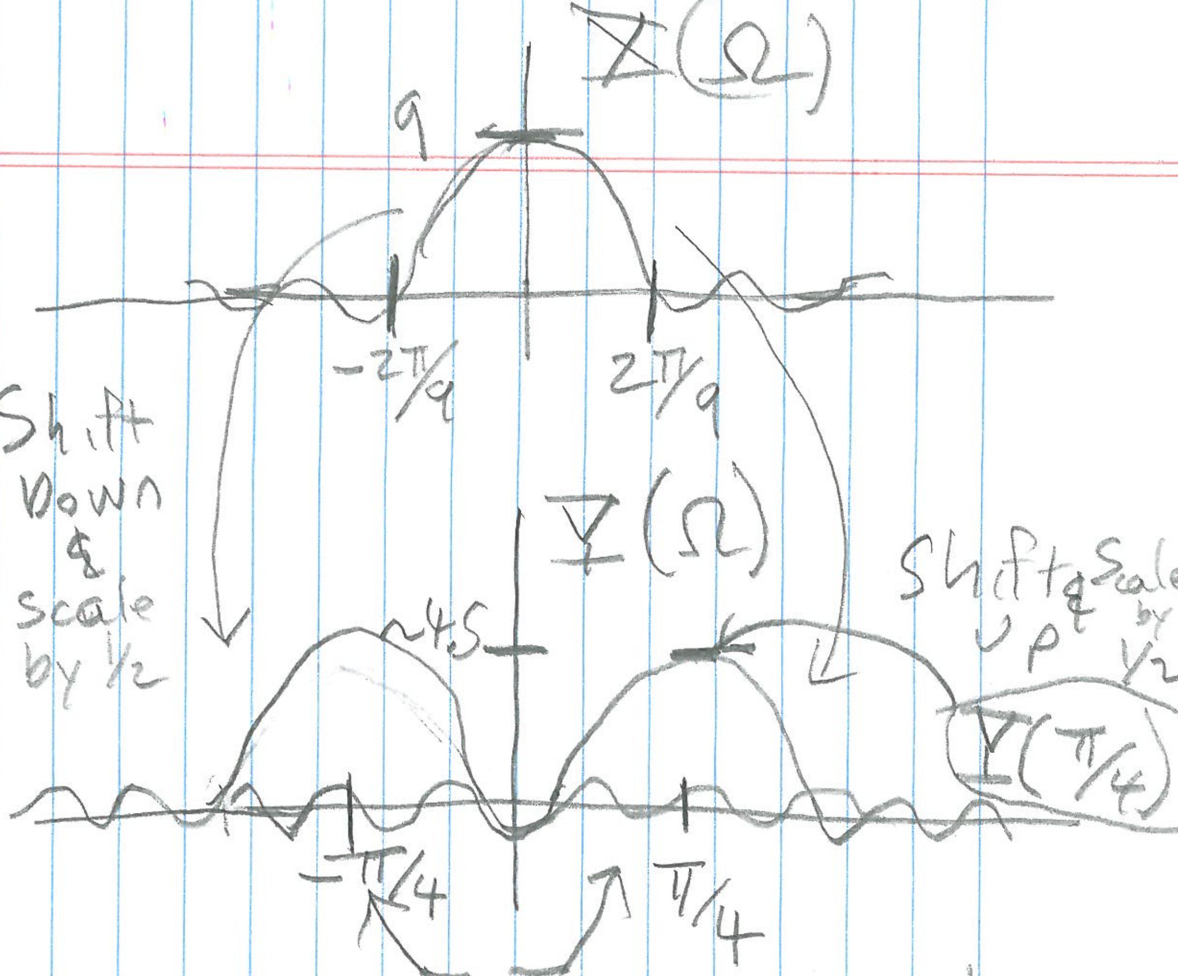
$w[n]$ & $W(\Omega)$

$v[n]$ & $V(\Omega)$

4







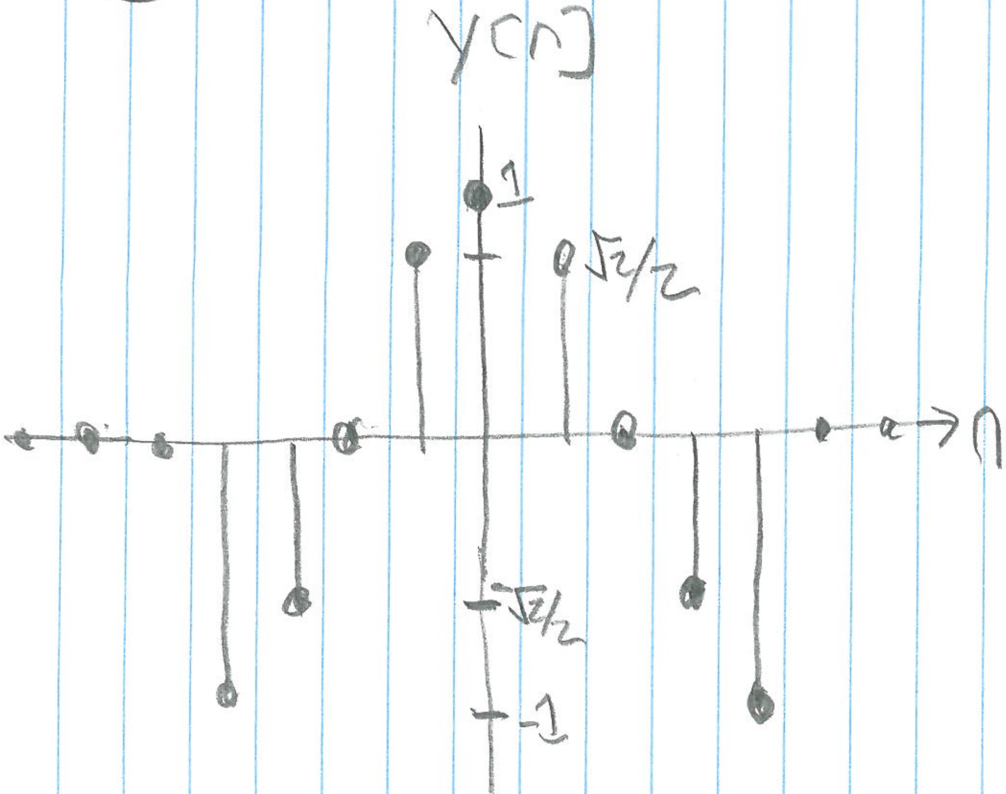
Note Interference!

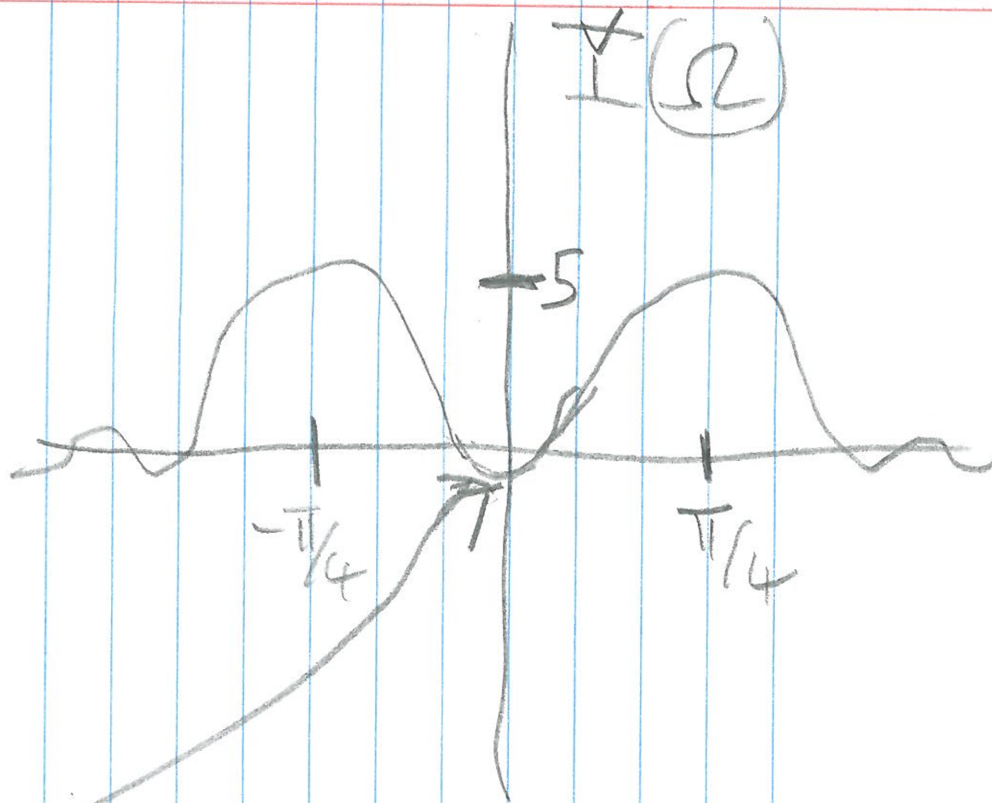
$$Y(0) = \sum y[n] = -1$$

$$Y(\pi/4) = 1e^{-j\pi/4(0)} + \frac{\sqrt{2}}{2}\underbrace{\left(e^{-j\pi/4} + e^{j\pi/4}\right)}_{2\cos\pi/4} - \frac{\sqrt{2}}{2}\underbrace{\left(e^{-j3\pi/4} + e^{j3\pi/4}\right)}_{2\cos 3\pi/4} + 2\cos\pi$$

$$= 5 \cong 4.5$$ (Difference due to interference)

5





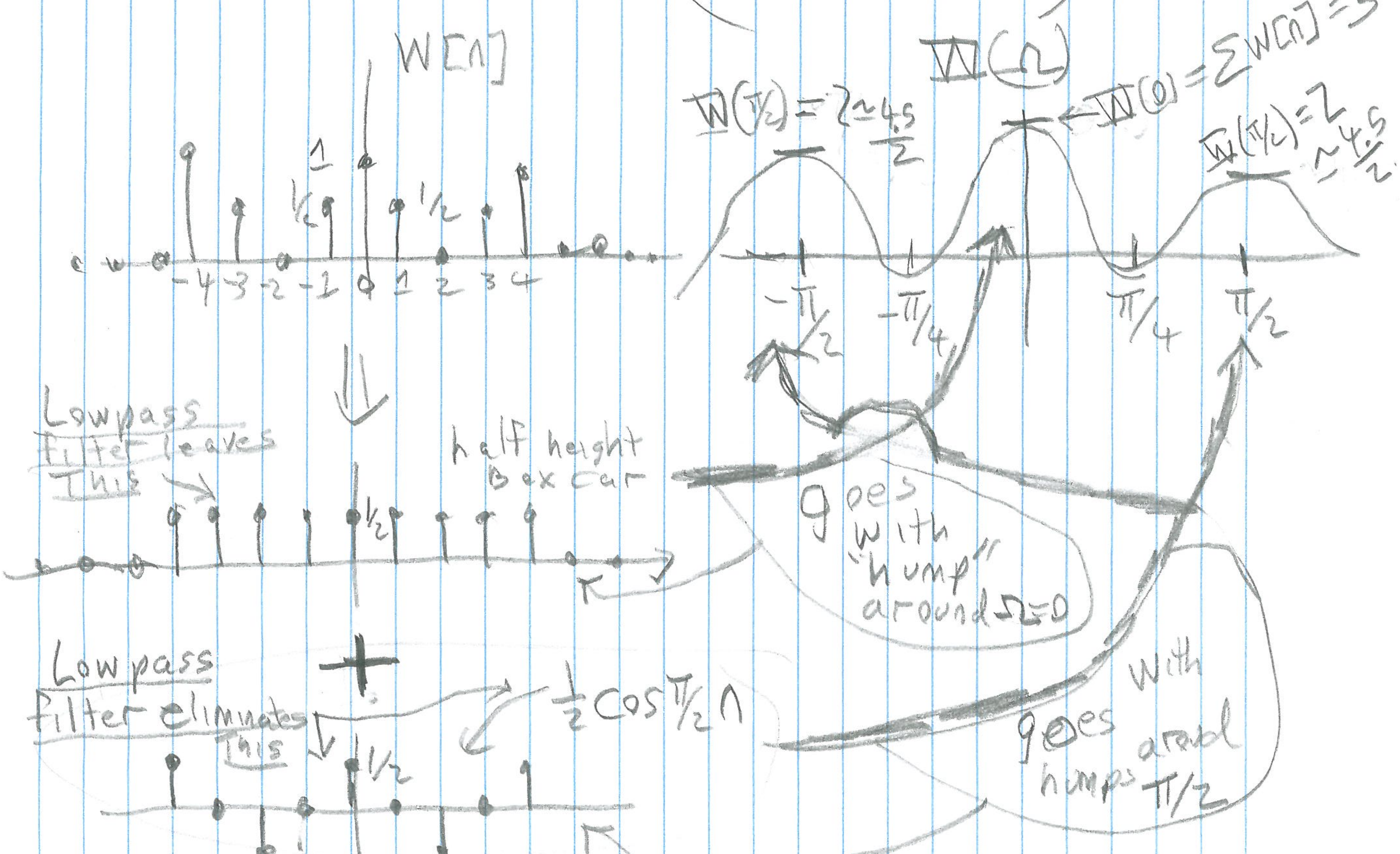
$Y(0) = \sum y[n] = -1$

<u>Note</u> $y[n]$ has a low average value $\Rightarrow Y(0) \simeq 0$

$y[n]$ "oscillates" with $\Omega = \pi/4$ $Y(\pi/4)$ is a peak

⑥ $W[n] = \cos \frac{\pi}{4} n \left(\cos \frac{\pi}{4}\ x[n] \right)$

≈ 4.5



6A $\quad \overline{W}(\pi/2)$

$$\overline{W}(\pi/2) = \sum w[n] e^{-j\pi/2 n}$$

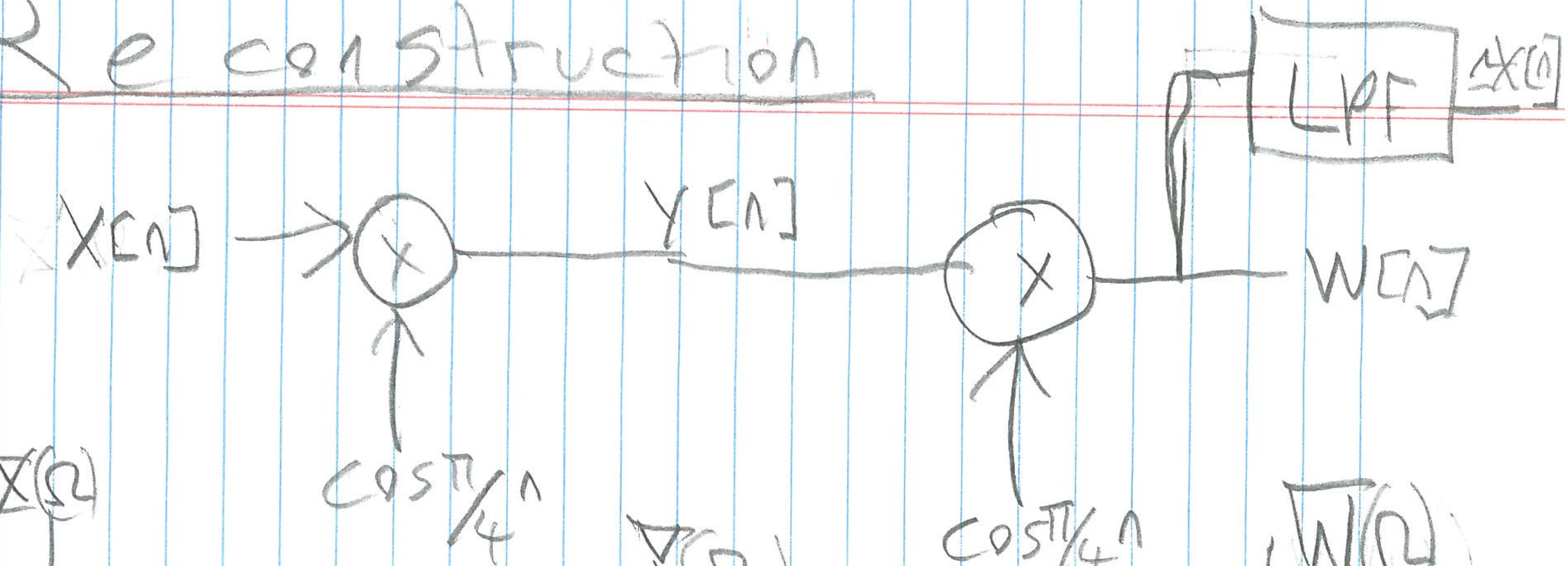
$$= 1 + \tfrac{1}{2}\cdot(2\cos\pi/2) + \tfrac{1}{2}(2\cos 3\pi/2) + \tfrac{1}{2}(2\cos(2\pi))$$

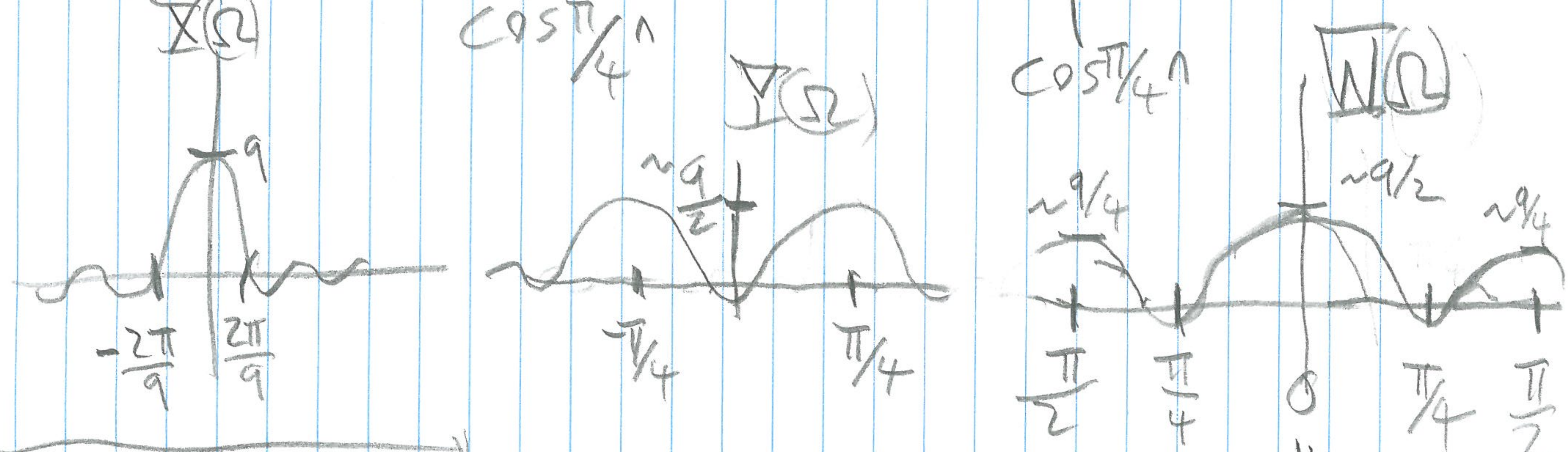
$$= 1 + 0 + 0 + 1 = 2$$

$$= \overline{W}(-\pi/2)$$

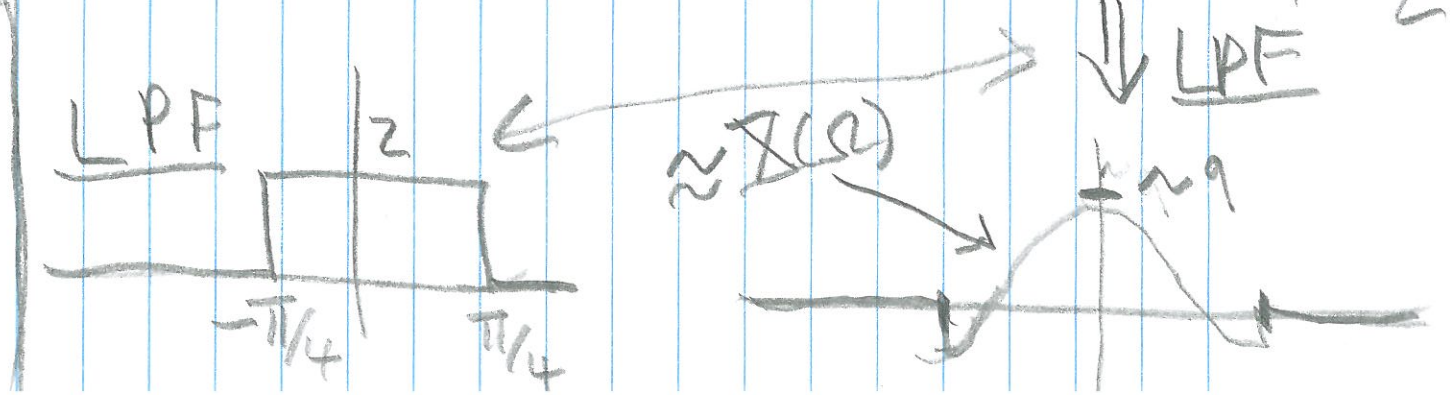
$$\cong \underbrace{\left(\underbrace{\overline{X}(0)/2}_{\text{Modulation}}\right)/2}_{\text{DeModulation}}$$

7 Reconstruction





<u>Note</u> ~ = approx
and is due
to "tails"
of spectrums
overlapping

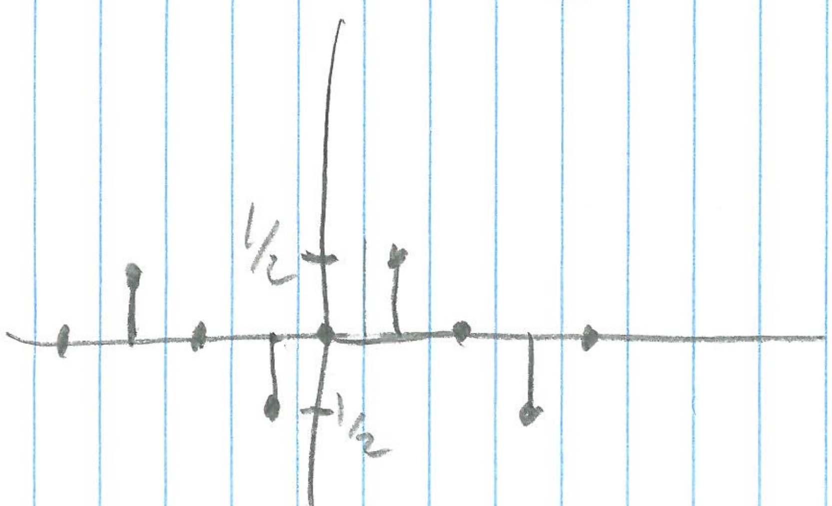


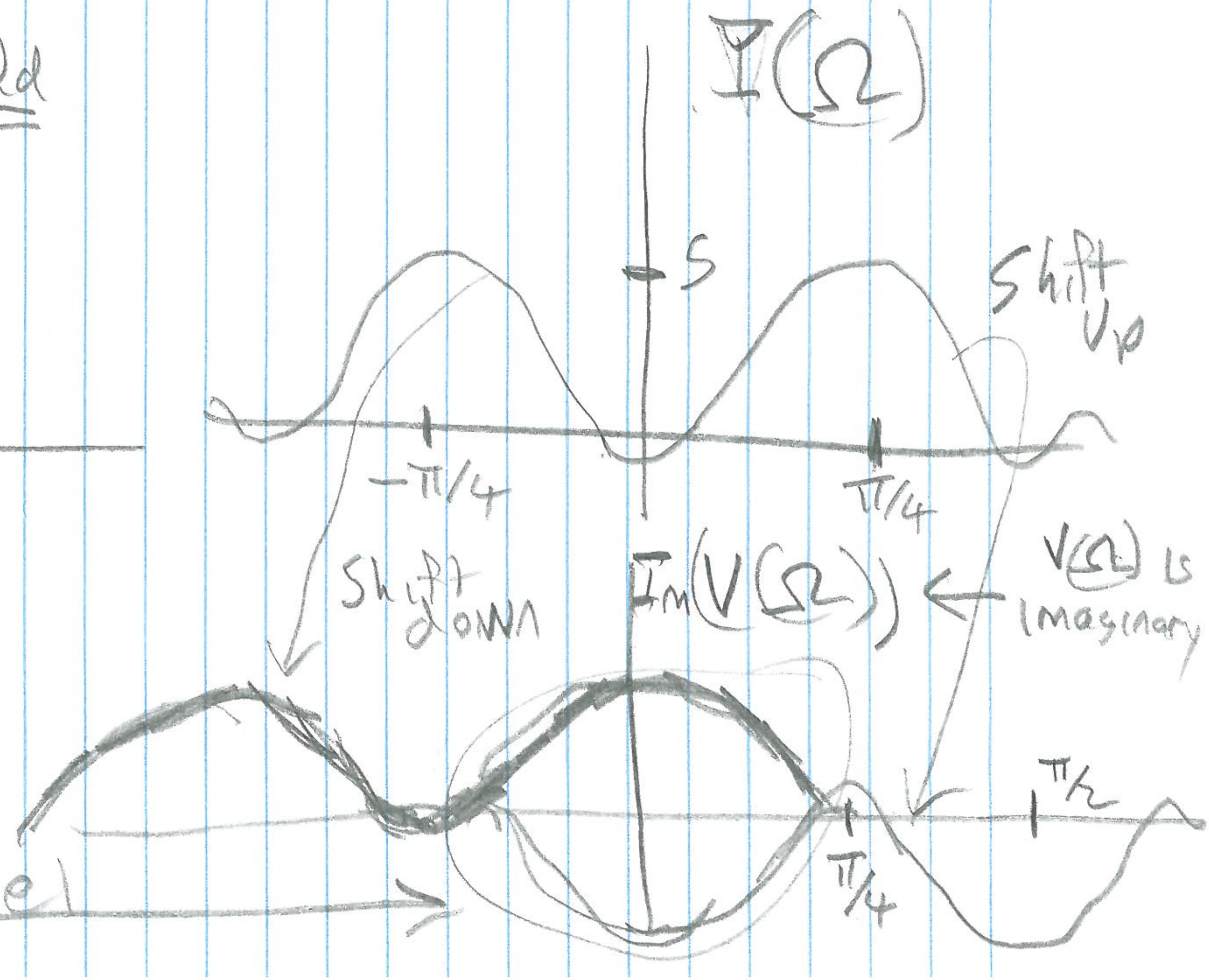
8

$$V[n] = \sin \tfrac{\pi}{4} n \underbrace{\left(\cos \tfrac{\pi}{4} n\right) x[n]}_{\text{cos modulation}}$$

sine demod

V[n] is odd

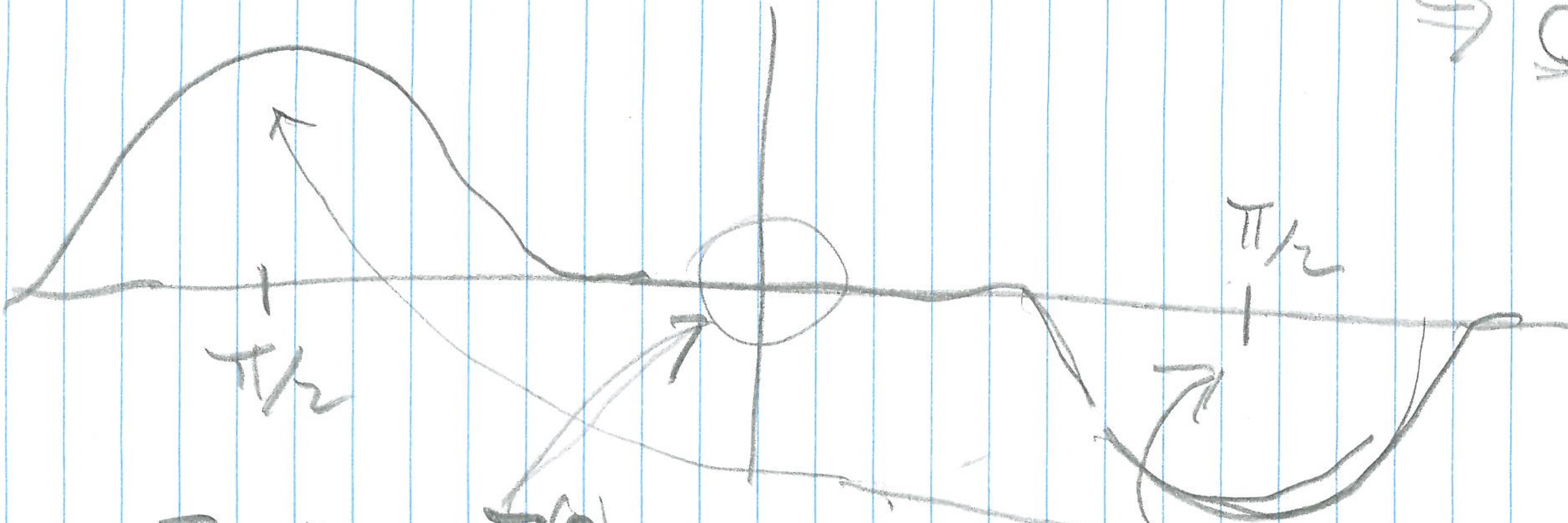




(9) $v[n]$ looks like $\sin \pi/2\, n$ $\quad V(\Omega)$ peaks at $\pi/2, -\pi/2$

$\mathrm{Im}\,(V(\Omega))$

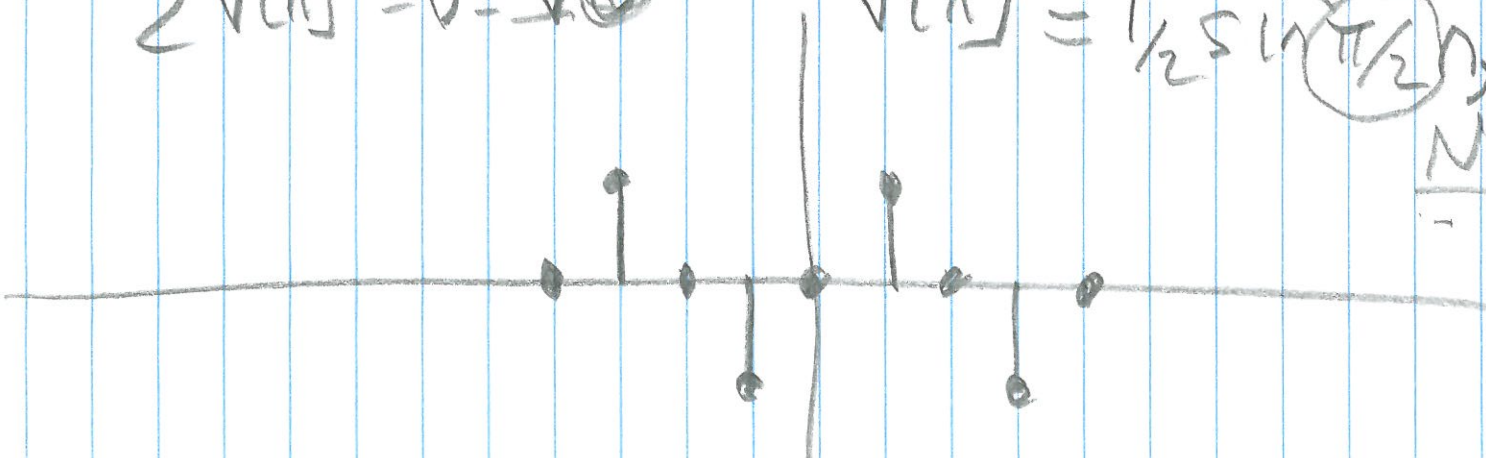
LPF on $V(\Omega)$ $\Rightarrow 0$!



$\sum v[n] = 0 = V(0)$

$v[n] = \frac{1}{2}\sin(\pi/2\, n), \; -4 \le n \le 4$

No low freq



Just like spectrum!