6.02 Recitation R16 (10, 11 a.m) 11/06/12

Discrete Representation of Continuous Signals

$x(t)$ is sampled every T sec, where the sampling frequency $f_s = 1/T$

$x[n] = x(nT)$

if $x(t) = \cos 2\pi f_0 t$

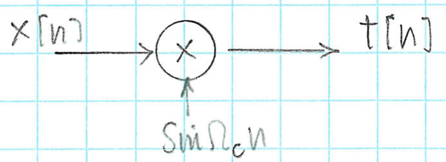
Then $x[n] = \cos(2\pi f_0 nT) = \cos(2\pi \frac{f_0}{f_s} n) = \cos \Omega_0 n$

Relationship between CT and DT frequencies are $\boxed{\Omega_0 = 2\pi \frac{f_0}{f_s}}$

note: if $\Omega_0 = \pi$, then $f_0 = \frac{f_s}{2}$ (sampling thm)

work w/ Ω

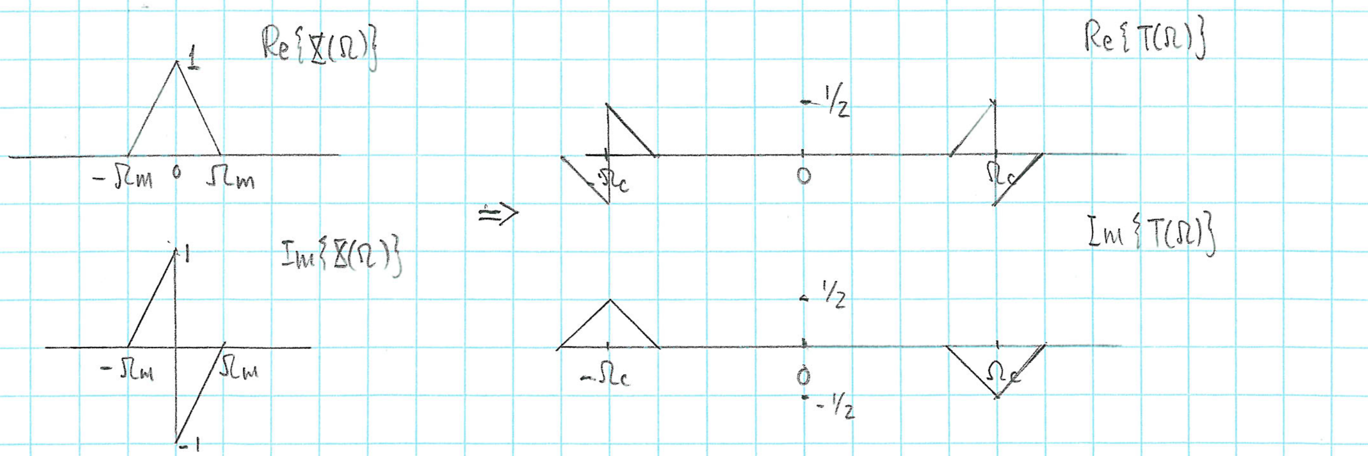
Modulation by $\sin \Omega_c n$ (from last recitation)



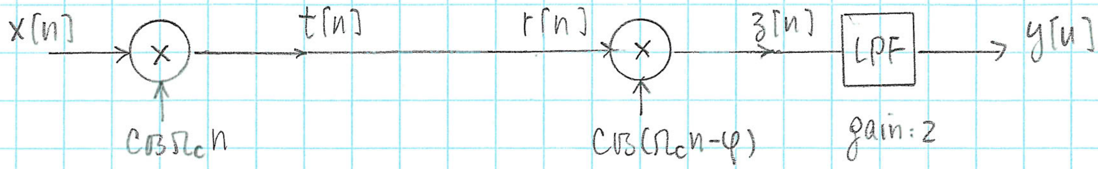
recall: $T(\Omega) = \frac{j}{2}\{X(\Omega+\Omega_c) - X(\Omega-\Omega_c)\}$

$Re\{T(\Omega)\} + j\,Im\{T(\Omega)\} = \frac{j}{2}(Re\{X(\Omega+\Omega_c)\} + j\,Im\{X(\Omega+\Omega_c)\} - Re\{X(\Omega-\Omega_c)\} - j\,Im\{X(\Omega-\Omega_c)\}$

$= \frac{1}{2}(Im\{X(\Omega-\Omega_c)\} - Im\{X(\Omega+\Omega_c)\}) + \frac{j}{2}(Re\{X(\Omega+\Omega_c)\} - Re\{X(\Omega-\Omega_c)$



Demodulation w/ $\cos(\Omega_c n - \varphi)$, i.e., including a phase error



$$z[n] = r[n] \cos\Omega_c n \cdot \cos(\Omega_c n - \varphi)$$

$$Z(\Omega) = \sum_m x[m] \frac{1}{2} \{ \cos(2\Omega_c n - \varphi) + \cos\varphi \} e^{-j\Omega m}$$

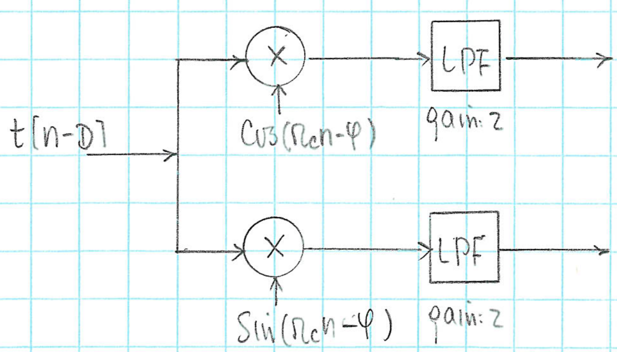
$$\rightsquigarrow \quad Y(\Omega) = \frac{1}{2} \cos\varphi \, X(\Omega)$$

$$\Rightarrow \quad y[n] = x[n] \cos\varphi \qquad\qquad y[n] = 0 \quad \text{if } \varphi = \pi/2 \ !$$

- note: phase error and delays behaves the same way

$$x[n-D] \longleftrightarrow e^{-j\Omega D} X(\Omega)$$

Solution: Quadrature Demodulation



$$I[n] = x[n-D] \cos\theta$$

where $\theta = \Omega_c D - \varphi$

$$Q[n] = x[n-D] \sin\theta$$

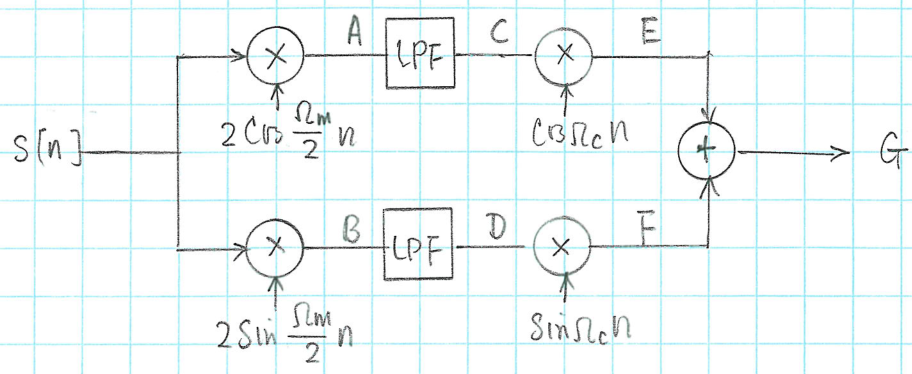
let $\quad w[n] = I[n] + j\,Q[n]$

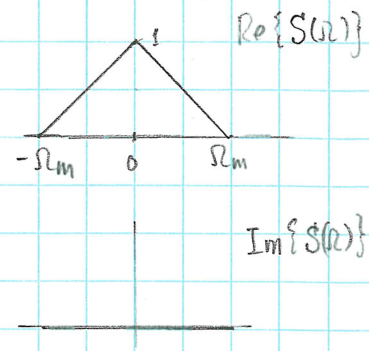
$$|w[n]| = |x[n-D]| \sqrt{\cos^2\theta + \sin^2\theta}$$

$$= x[n-D]$$

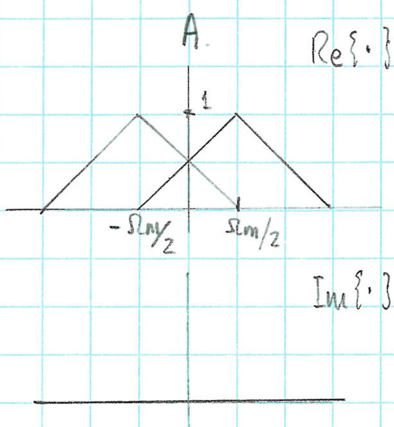
$\uparrow$ if $x[\cdot] \geq 0$

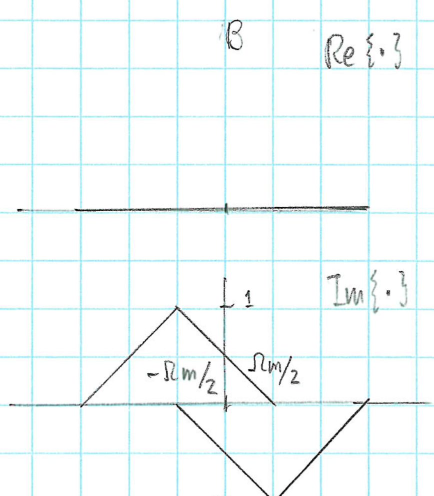
[Example]: practice problem 2 — Single Sideband (SSB) Modulation

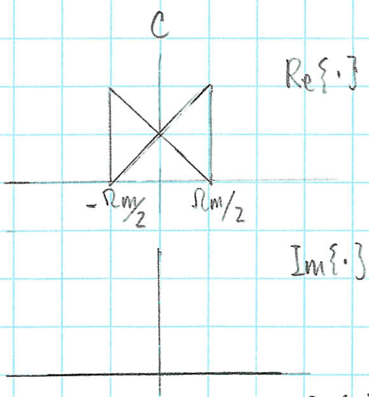


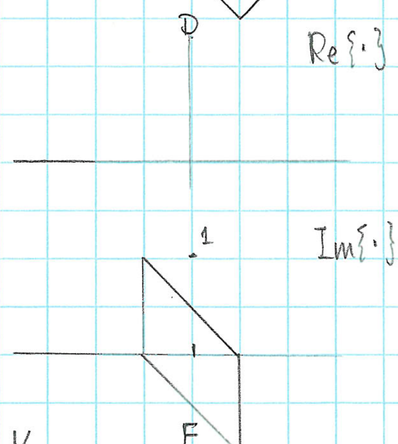


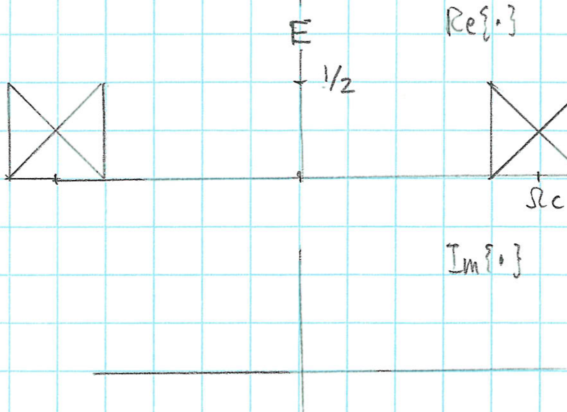
LPF cut-off @ $\pm\frac{\Omega_m}{2}$; gain = 1

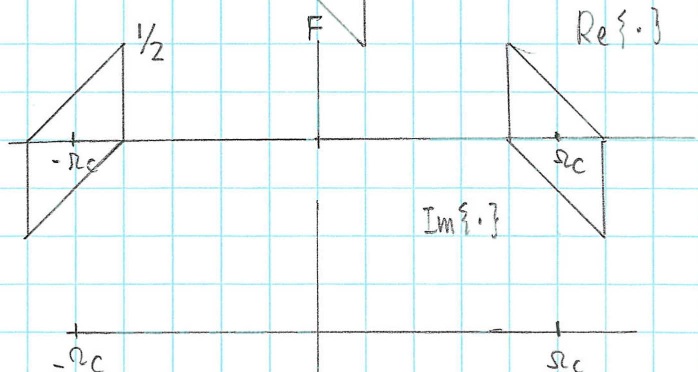


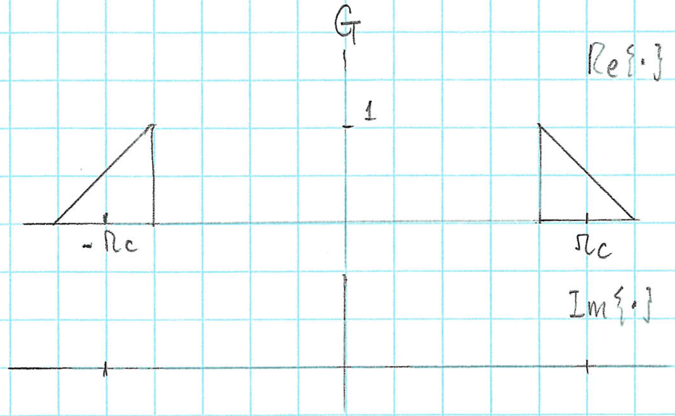








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