6.02 Recitation R12 (10 and 11 am) 10/22/2013

Frequency Response: $H(\Omega)$

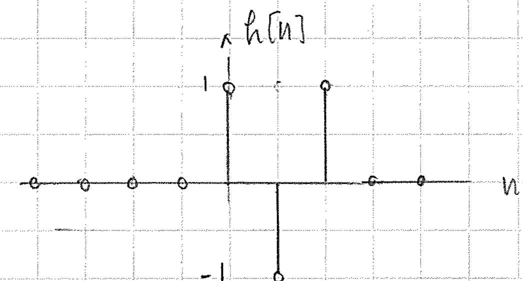
$$x[n] = e^{j\Omega n} \longrightarrow \boxed{h[\cdot]} \longrightarrow y[n] = H(\Omega)\, e^{j\Omega n}$$

where $$\boxed{H(\Omega) = \sum_m h[m]\, e^{-j\Omega m}}$$

- same frequency
- continuous
- periodic w/ period 2π
- expressed as $|H(\Omega)|$ & $\measuredangle H(\Omega)$

[note]:
$$\begin{cases} H(0) = \sum_m h[m] \\ H(\pi) = \sum_m h[m]\, e^{-j\pi m} = \sum_m (-1)^m h[m] \end{cases}$$

[Example] Quiz 2, Fall 2012



$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2]$$

a) $H(\Omega) = 1 - e^{-j\Omega} + e^{-j2\Omega}$

b) let $H(\Omega) = A(\Omega)\, e^{j\alpha(\Omega)}$, where $A(\Omega)$ & $\alpha(\Omega)$ are real, $A(\Omega)$ could be negative

$$H(\Omega) = e^{-j\Omega}\left(e^{j\Omega} - 1 + e^{-j\Omega}\right)$$
$$= e^{-j\Omega} \cdot (2\cos\Omega - 1)$$

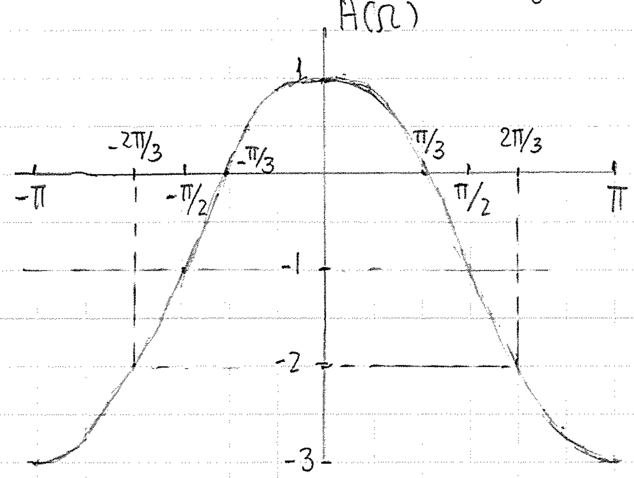
$$A(\Omega) = 2\cos\Omega - 1 \quad ; \quad \alpha(\Omega) = -\alpha$$

c) evaluate $A(\Omega)$ and $\alpha(\Omega)$ explicitly for $\Omega = \pi/3$ and $\Omega = 2\pi/3$

for $\Omega = \pi/3$; $A(\Omega) = 0$; $\alpha = -\pi/3$

for $\Omega = 2\pi/3$; $A(\Omega) = -2$; $\alpha = -2\pi/3$

d) Plot and label $A(\Omega)$: (scale by 2, shift by -1)



Check answers:

$H(\pi/3) = 0$; $H(2\pi/3) = -2$

$H(0) = \sum_m h[m] = 1$

$H(\pi) = \sum (-1)^m h[m] = -3$

e) suppose $x_2[n] = \cos\left(\frac{2\pi}{3} n + \theta_0\right)$

$y_2[n] = H\left(\frac{2\pi}{3}\right) x_2[n]$ expressed in terms of $A(\Omega)$ & $\alpha(\Omega)$

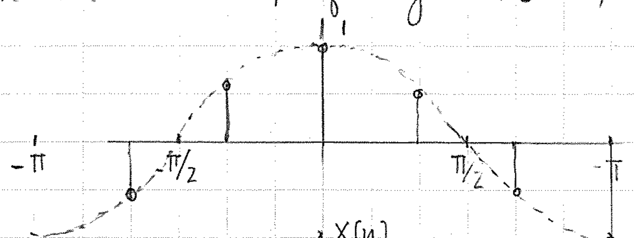
$= (-2) \cos\left(\frac{2\pi}{3} n + \theta_0 - \frac{2\pi}{3}\right) = 2 \cos\left(\frac{2\pi}{3} n + \theta_0 + \frac{\pi}{3}\right)$

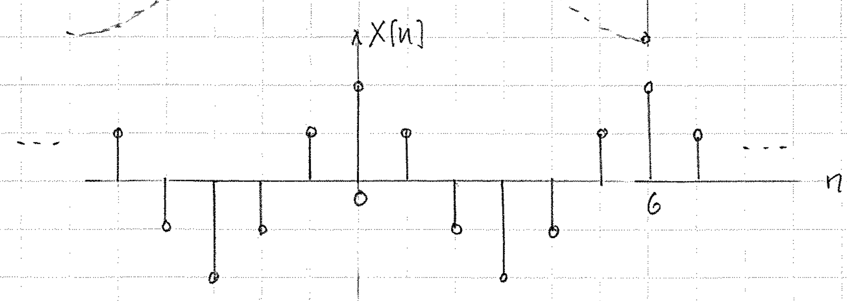
expressed in terms of $|H(\Omega)|$ & $\measuredangle H(\Omega)$

f) suppose $x_3[n] = \cos\left(\frac{\pi}{3} n + \theta_0\right)$

$y_3[n] = H\left(\frac{\pi}{3}\right) x_2[n] = 0$

Check answer: frequency $= \pi/3$; period $= 6$





By convolution: $y_3[n] = h[n] * x_3[n] = 0$

(do this to convince yourself!)

[note]: $H(\Omega)$ provides a frequency-shaping characteristics; it's a "filter"

$$H(\Omega) = \sum_m h[m] e^{-j\Omega m}$$

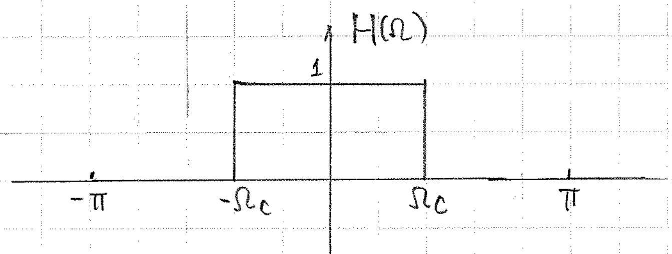
From yesterday's lecture, multiply both side by $e^{j\Omega n}$; integrate over a contiguous 2π interval; only one term, $m = n$, survives:

$$\boxed{h[n] = \frac{1}{2\pi}\int_{\langle 2\pi \rangle} H(\Omega) e^{j\Omega n} d\Omega}$$

- given a desired frequency-shaping characteristics, we can now determine the unit-sample response.

[Example] <u>Ideal lowpass Filter</u>

$H(\Omega)$ even $\Rightarrow h[n]$ real



$$h[n] = \frac{1}{2\pi}\int_{\langle 2\pi \rangle} H(\Omega) e^{j\Omega n} d\Omega$$

$$= \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} e^{j\Omega n} d\Omega = \frac{1}{2\pi}\cdot\frac{1}{nj}\left(e^{j\Omega_c n} - e^{-j\Omega_c n}\right)$$

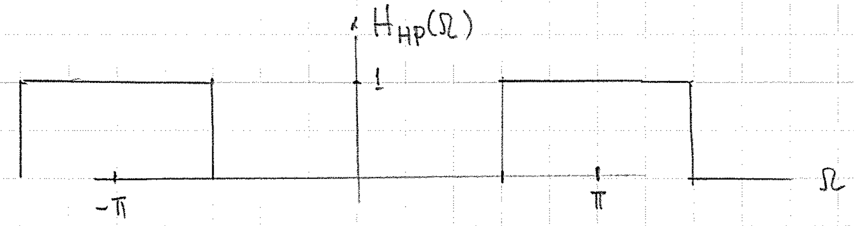
$$= \frac{1}{\pi n}\cdot\frac{1}{2j}\left(e^{j\Omega_c n} - e^{-j\Omega_c n}\right)$$

$$h[n] = \begin{cases} \dfrac{\sin \Omega_c n}{\pi n} & n \neq 0 \\ \dfrac{\Omega_c}{\pi} & n = 0 \end{cases}$$

- $h[n]$ is in the form of a sinc fn; $\sin X / X$
- it falls off at $1/n$; not absolutely summable ($\sum_n |h[n]| = B < \infty$), so not BIBO stable
- $h[n]$ is non-causal

- it is of ∞ length, so can't shift enough to make it causal
- to make $h[n]$ causal, need to trucate $h[n]$, then shift.
- see hand out; note Gibbs phenomenon

[Example] <u>create HPF from LPF $H(\Omega)$</u>



$$H_{HP}(\Omega) = H(\Omega - \pi) = \sum_m h[m]\, e^{-j(\Omega-\pi)m} = \sum_m e^{j\pi m}\, h[m]\, e^{-j\Omega m}$$

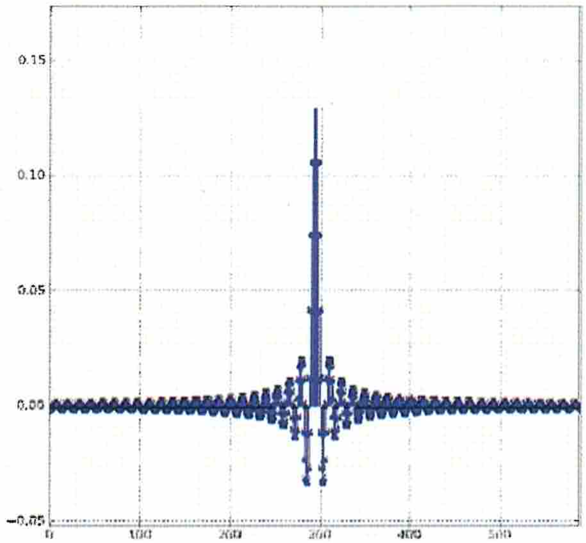
$$\Rightarrow \quad h_{HP}[n] = (-1)^n\, h[n]$$

- see handout
- $h_{HP}[n]$ flips the sign of alternate values of $h[n]$

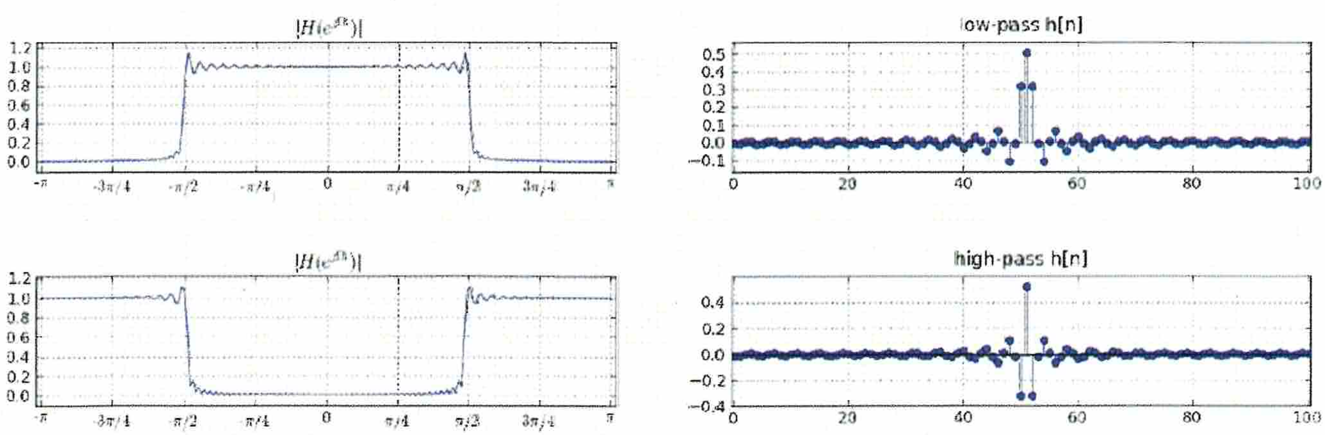
6.02 Fall 2013

10/22/2013

10, 11 a.m. Recitation



Plot of h[n] for ideal LPF



Plot of H(Ω) and h[n] for LPF and HPF