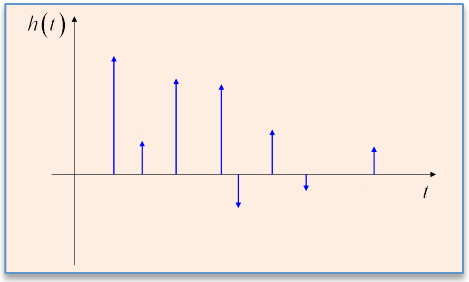
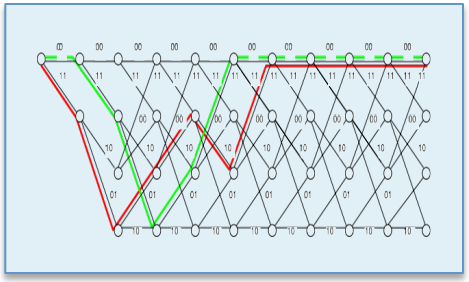
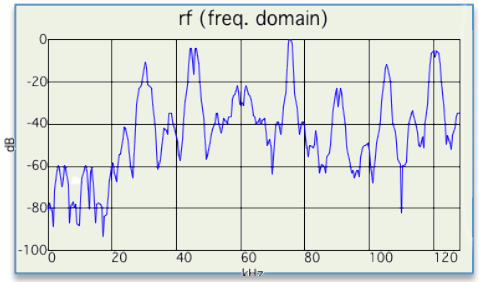
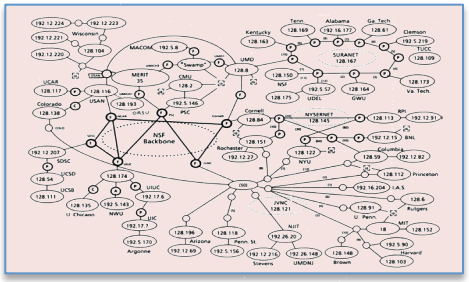








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2013
Lecture #1

- Digital vs. analog communication
- The birth of modern digital communication
- Information and entropy
- Codes, Huffman coding

Course Staff

Alex Megretski
George Verghese

Babak Ayazifar
Mujdat Cetin
Lizhong Zheng
Victor Zue

Leighton Barnes
SungWon Chung
Max Dunitz
Xue Feng
Elaine Han
Lenin Ravindranth Sivalingam
Eduardo Sverdlin Lisker (Head TA)
Lyne Tchapmi Petse

+ LAs and Graders!

Course Ethos*

Great material …
a direct and tangible line of development from 200 years ago to systems of importance today (and tomorrow!), including many links to MIT

Lots to learn (and teach) …
collaboratively

Individual effort …
we have to be seeing your own work on anything submitted for evaluation, with all collaboration fully acknowledged

*Animating principles

Lectures and Recitations

… are your best and most efficient entry to the subject. Even the stuff that's confusing or not clear in lecture/recitation is useful in letting you know what to get clarified. So please attend!!

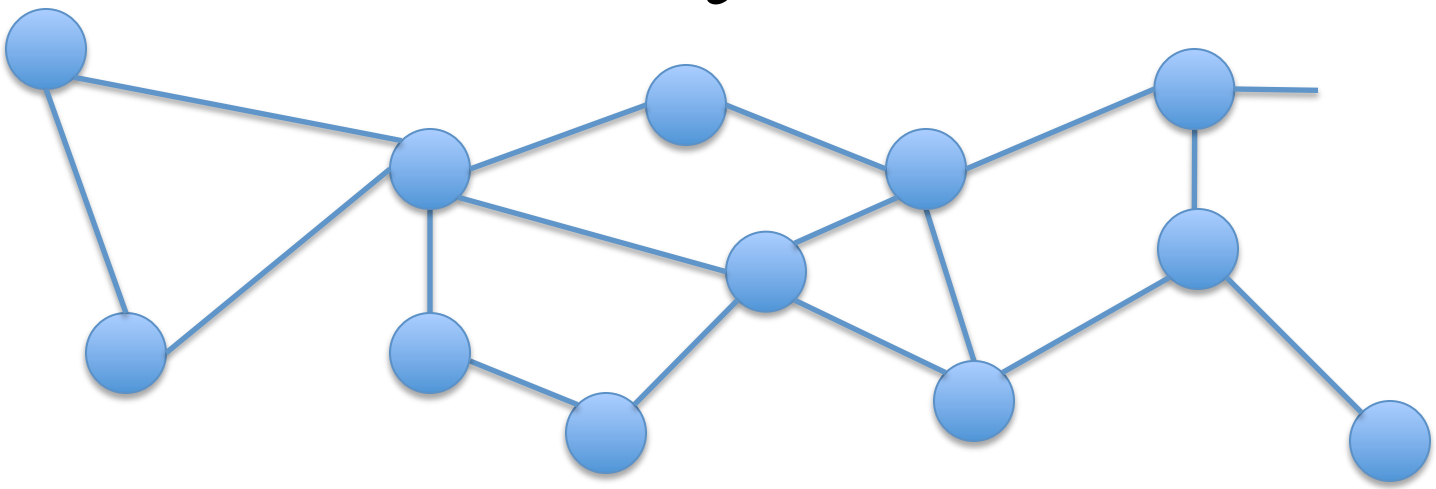
Your "contract" in this subject is not with notes or slides, but with the staff, and specifically with what's developed in lectures and recitations.

Digital vs. Analog Communication

- ANALOG Communicating a continuous-time waveform (e.g., voltage from a microphone), via amplitude modulation (AM) or frequency modulation (FM) or ...
  - Analog electronics
  - Fidelity to the waveform

- DIGITAL Communicating a message comprising a discrete-time sequence of symbols from some source alphabet
  - Often coded onto some other sequence of symbols that's adapted to the communication channel, e.g., binary digits, 0 and 1.
  - Often involving analog communication across the physical channel
  - Fidelity to the message
  - Well suited to riding the staggering growth in computational power, storage, big data, …

6.02 Syllabus



Point-to-point communication channels (transmitter→receiver):

- Measuring and appropriately encoding information BITS
- Transmission on physical channels SIGNALS
- Noise, bit errors, error correction
- Sharing a channel

Multi-hop networks:

- Packet switching, efficient routing PACKETS
- Reliable delivery on top of a best-efforts network

Samuel F.B. Morse
1791-1872

UNITED STATES PATENT OFFICE.

SAMUEL F. B. MORSE, OF NEW YORK, N. Y.

IMPROVEMENT IN THE MODE OF COMMUNICATING INFORMATION BY SIGNALS BY THE APPLICATION OF ELECTRO-MAGNETISM.

Specification forming part of Letters Patent No. **1,647**, dated June 20, 1840.

To all whom it may concern:

Be it known that I, the undersigned, SAMUEL F. B. MORSE, of the city, county, and State of New York, have invented a new and useful machine and system of signs for transmitting intelligence between distant points by the means of a new application and effect of electro-magnetism in producing sounds and signs, or either, and also for recording permanently by the same means, and application, and effect of electro-magnetism, any signs thus produced and representing intelligence, transmitted as before named between distant points; and I denominate said invention the "American Electro-Magnetic Telegraph," of which the following is a full and exact description, to wit:

It consists of the following parts—first, of a circuit of electric or galvanic conductors from any generator of electricity or galvanism and of electro-magnets at any one or more points in said circuit; second, a system of signs by which numerals, and words represented by numerals, and thereby sentences of words, as well as of numerals, and letters of any extent and combination of each, are communicated from any generator of electricity or galvanism, to one or more electro-magnets placed at any point or points in said circuit, the magnetic power thus concentrated in such magnet or magnets is used for the purposes of producing sounds and visible signs, and for permanently recording the latter at any and each of said points at the pleasure of the operator and in the manner hereinafter described—that is to say, by using the system of signs which is formed of the following parts and variations, viz:

Signs of numerals consist, first, of ten dots or punctures, made in measured distances of equal extent from each other, upon paper or any substitute for paper, and in number corresponding with the numeral desired to be represented. Thus one dot or puncture for the numeral 1, two dots or punctures for the numeral 2, three of the same for 3, four for 4, five for 5, six for 6, seven for 7, eight for 8, nine for 9, and ten for 0, as particularly represented on the annexed drawing marked Example 1, Mode 1, in which is also included a second character, to represent a cipher, if prefered.

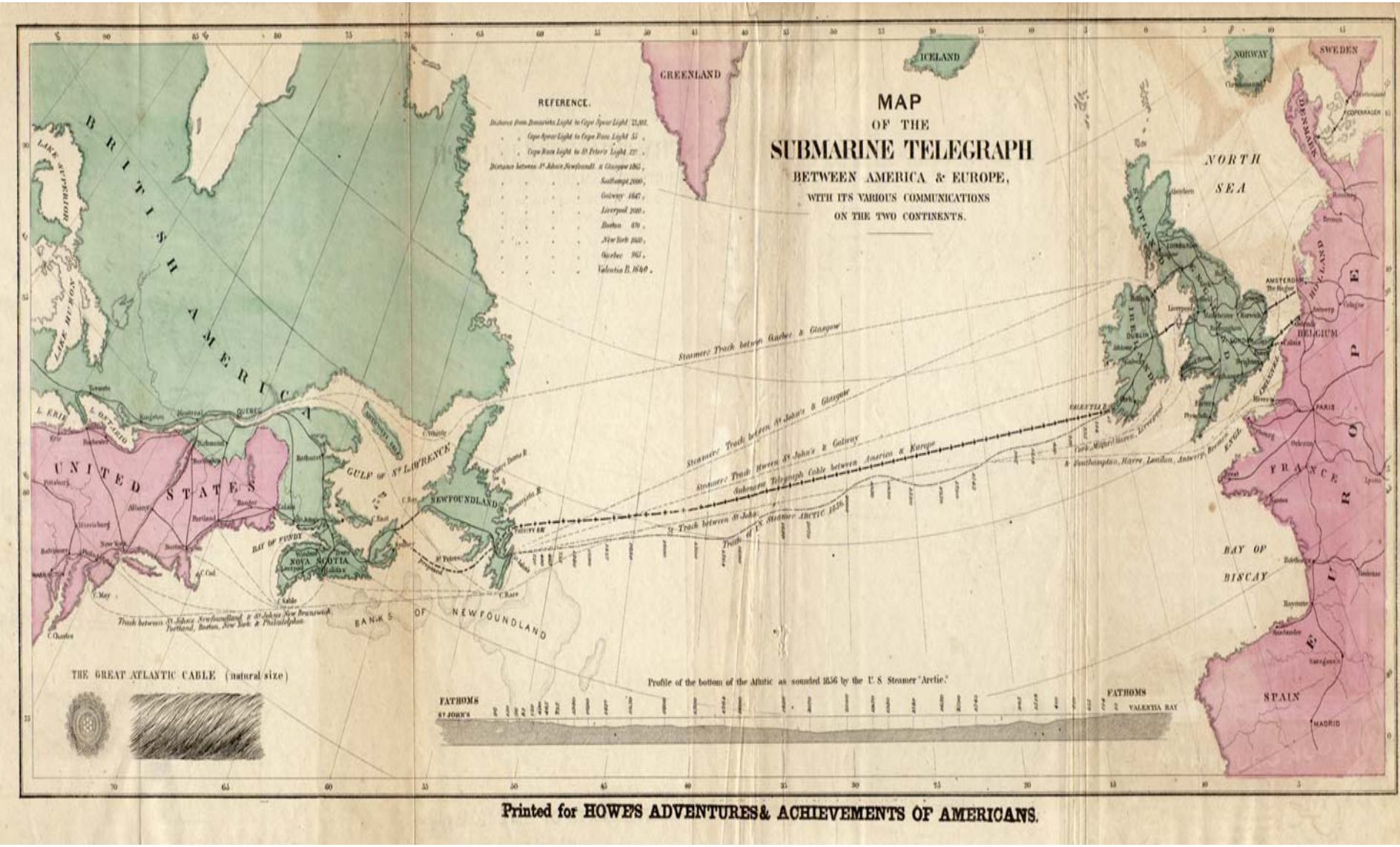
Samuel F.B. Morse

- Invented (1832 onwards, patent #1,647 in 1840) the most practical form of electrical telegraphy, including keys, wire arrangements, electromagnets, marking devices, relays, …, and Morse code!

- Worked tirelessly to establish the technology

- After initial struggles, telegraphy was quickly adopted and widely deployed
  - Trans-Atlantic cable attempts 1857 (16 hours to send 98 words from Queen Victoria to President Buchanan!), 1858, 1865, finally success in 1866 (8 words/minute)
  - Trans-continental US in 1861 (effectively ended the Pony Express)
  - Trans-Pacific 1902

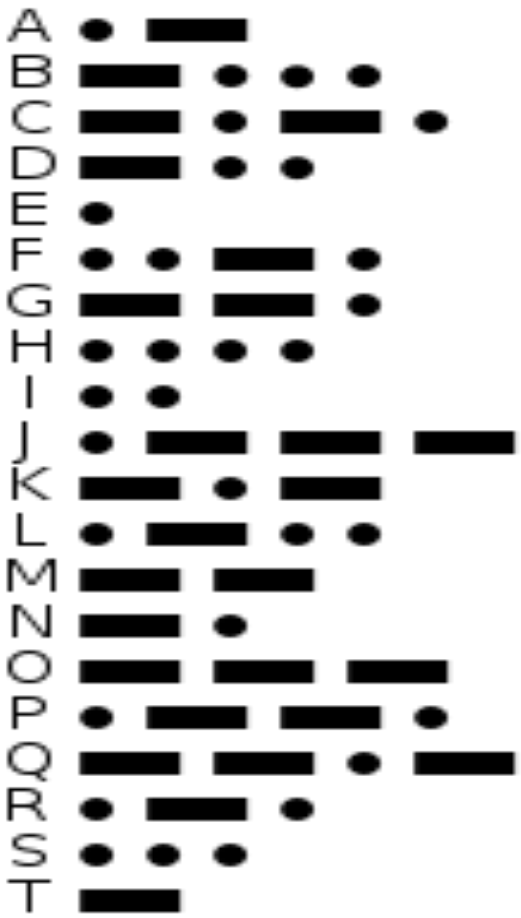
- Telegraphy transformed communication (trans-Atlantic time from 10 days by ship to minutes by telegraph) and commerce, also spurred major developments in EE theory & practice (Henry, Kelvin, Heaviside, Pupin, …)



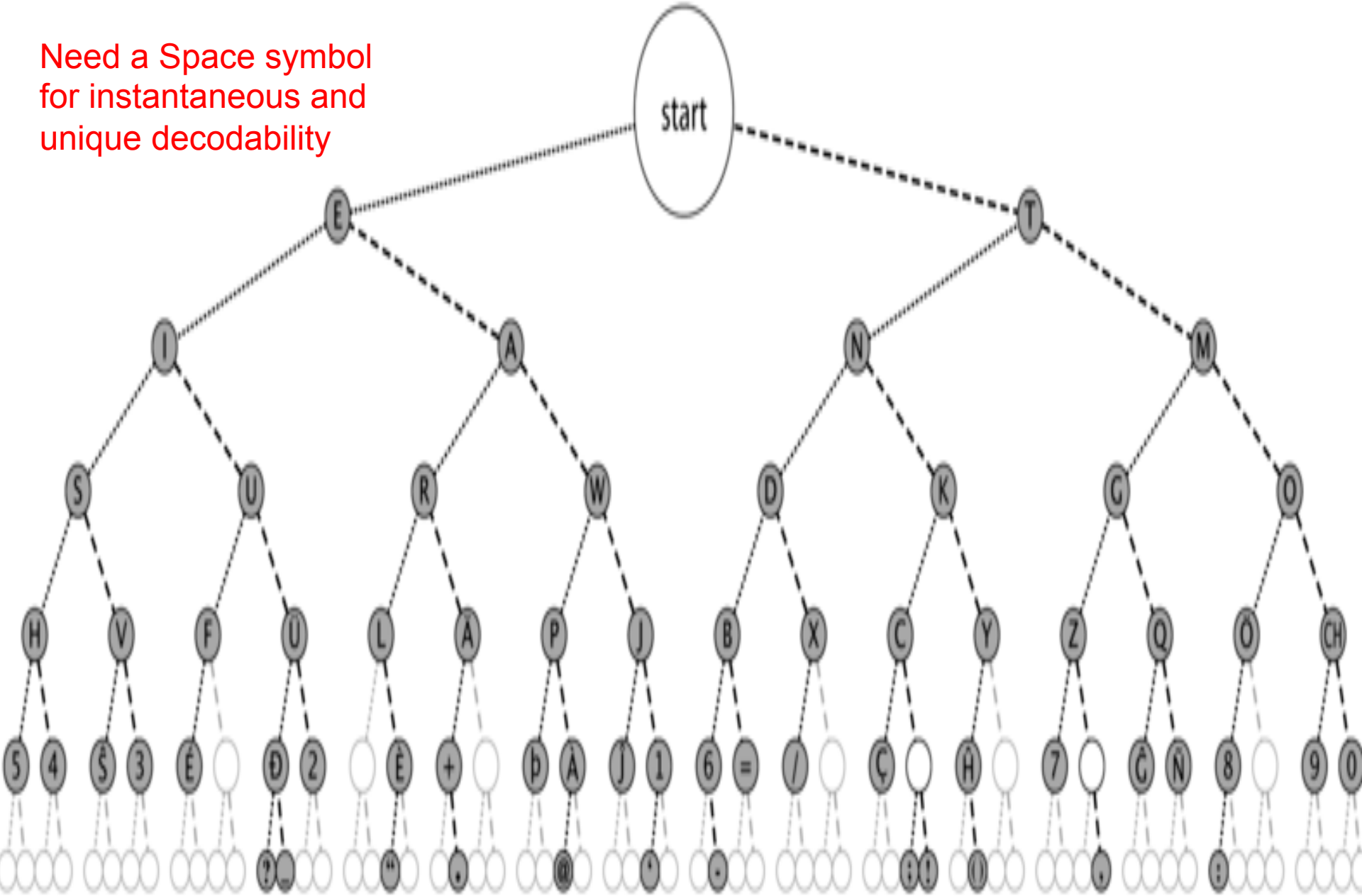
Route of the 1858 trans-Atlantic cable

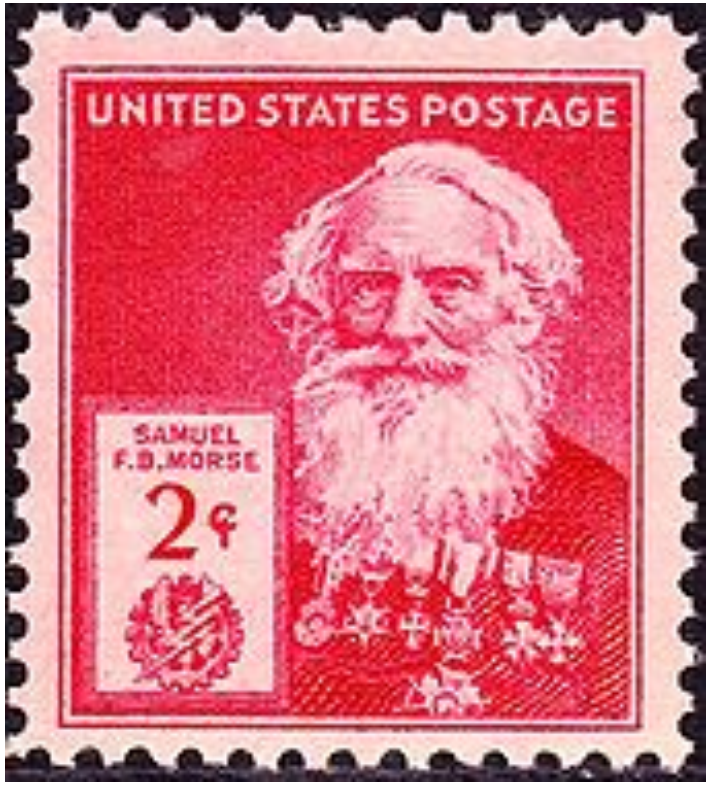
International Morse Code

1. A dash is equal to three dots.
2. The space between parts of the same letter is equal to one dot.
3. The space between two letters is equal to three dots.
4. The space between two words is equal to seven dots.



Need a Space symbol for instantaneous and unique decodability




UNITED STATES POSTAGE
SAMUEL
F.B.MORSE
2¢

Fast-forward 100 years

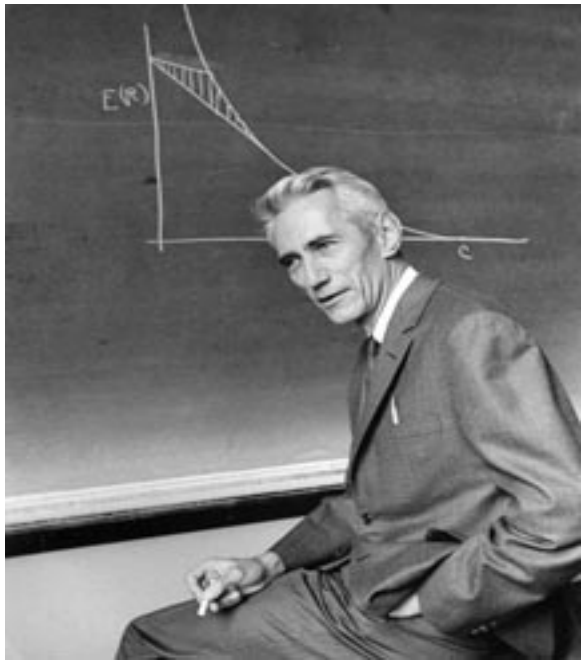
- Via
  - Telephone ("Improvement in Telegraphy", patent # 174,456, Bell 1876)
  - Wireless telegraphy (Marconi 1901)
  - AM radio (Fessenden 1906)
  - FM radio (Armstrong 1933)
  - Television broadcasting by the BBC (1936)
- Mostly back to analog for a while!
- Bell Labs galaxy of researchers
  - Nyquist, Bode, Hartley, …

Claude E. Shannon, 1916-2001

1937 Masters thesis, EE Dept, MIT
A symbolic analysis of relay and switching circuits
Introduced application of Boolean algebra to logic circuits, and vice versa.
Very influential in digital circuit design.
"Most important Masters thesis of the century"

1940 PhD, Math Dept, MIT
An algebra for theoretical genetics
To analyze the dynamics of Mendelian populations.

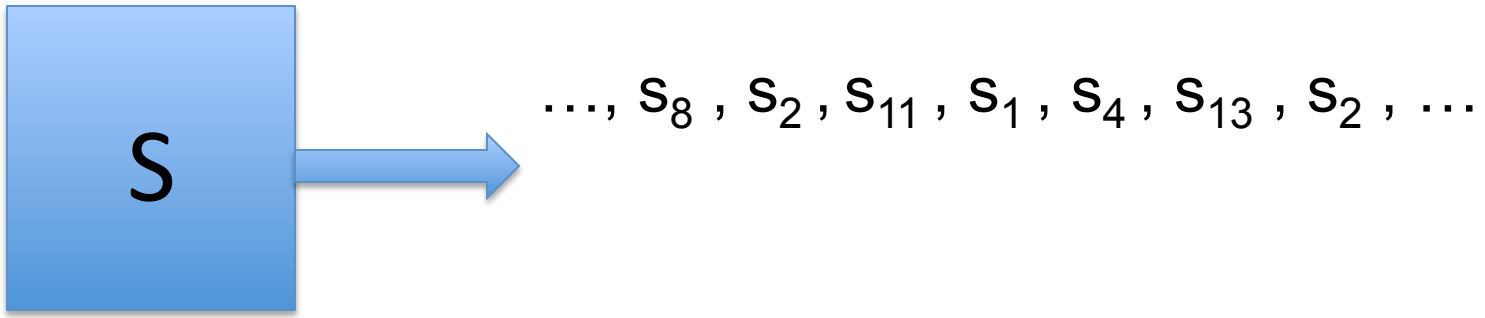
Joined Bell Labs in 1940.



MIT faculty
1956-1978

"A mathematical theory of cryptography" 1945/1949
"A mathematical theory of communication" 1948

A Probabilistic Source



Emitting symbols sequentially in time, with the symbol at each time chosen independently of the choices at other times, but using the same probabiity distribution, i.e., an independent, identically distributed or i.i.d. symbol stream

Probabilistic Models (checklist)

- Universe **U** of elementary outcomes e_1, e_2, … . One and only one outcome in each experiment or run of the model.
- Events A, B, C, … are subsets of **U**. We say event A has occurred if the outcome of the experiment lies in A.
- Events form an "algebra" of sets, i.e., A *or* B (union, also written A+B) is an event, A *and* B (intersection, also written AB) is an event, *not* A (complement, also written A^c) is an event. So **U** and the null set **0** are also events.
- Probabilities are defined on events, such that $0 \le P(A) \le 1$, $P(\mathbf{U})=1$, and P(A+B)=P(A)+P(B) if A and B are mutually exclusive, i.e. if AB=**0**. More generally, P(A+B)=P(A)+P(B)-P(AB).
- Events A, B, C, D, E, etc., are said to be (mutually) independent if joint probability of every combination of these events factors into product of individual probabilities, so P(ABCDE)=P(A)P(B)P(C)P(D)P(E), P(ABCD)=P(A)P(B)P(C)P(D), P(ADE)=P(A)P(D)P(E), etc.
- Conditional probability P(A, given that B has occurred) = P(A|B) = P(AB)/P(B).

Random Variables, Expectation

- A random variable is a mapping from elementary outcomes e_i to numbers r_i --- or equivalently, a numerically specified outcome of a probabilistic experiment.
- The expected value or mean value of a random variable is the probability-weighted average of the (numerical) outcomes.

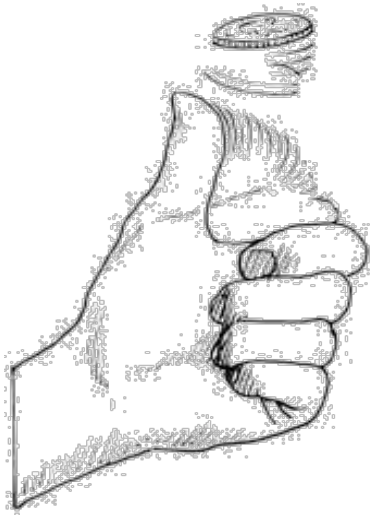
Measuring Information

Shannon's (and Hartley's) definition of the information obtained on being told the outcome s_i of a probabilistic experiment S :

$$I(S = s_i) = \log_2\left(\frac{1}{p_S(s_i)}\right)$$

where $p_S(s_i)$ is the probability of the event $S = s_i$.

The unit of measurement (when the log is base-2) is the **bit** (**b**inary **i**nformation uni**t** --- not the same as binary digit!).



1 bit of information corresponds to $p_S(s_i) = 0.5$. So, for example, when the outcome of a *fair* coin toss is revealed to us, we have received 1 bit of information.

"*Information is the resolution of uncertainty*"
Shannon

Examples

We're drawing cards at random from a standard N=52-card deck. Elementary outcome: card that's drawn, probability 1/52, information $\log_2(52/1) = 5.7$ bits.

For an event comprising M such (mutually exclusive) outcomes, the probability is M/52.

Q. If I tell you the card is a spade ♠, how many bits of information have you received?
A. Out of N=52 equally probable cards, M=13 are spades ♠, so probability of drawing a spade is 13/52, and the amount of information received is $\log_2(52/13) = 2$ bits.
This makes sense, we can encode one of the 4 (equally probable) suits using 2 binary digits, e.g., 00=♡, 01=♢, 10=♧, 11=♠.

Q. If instead I tell you the card is a seven, how much info?
A. N=52, M=4, so info = $\log_2(52/4) = \log_2(13) = 3.7$ bits

Properties of Information Definition

- A lower-probability outcome, i.e., a more uncertain outcome, yields higher information

- A highly informative outcome does not necessarily mean a more valuable outcome, only a more surprising outcome, i.e., there's no intrinsic value being assessed (can think of information as degree of surprise)

- Often used fact: The information in independent events is additive.

(This is what motivates using a logarithmic measure in the first place.)

Expected Information or Average Uncertainty = Entropy

Consider a discrete random variable S, which may represent the set of possible symbols to be transmitted at a particular time, taking possible values $s_1, s_2, \ldots, s_N$, with respective probabilities $p_S(s_1), p_S(s_2), \ldots, p_S(s_N)$.

The *entropy* $H(S)$ of S is the expected (or mean or average) value of the information obtained by learning the outcome of S:

$$H(S) = \sum_{i=1}^{N} p_S(s_i) I(S = s_i) = \sum_{i=1}^{N} p_S(s_i) \log_2 \left(\frac{1}{p_S(s_i)} \right)$$

When all the $p_S(s_i)$ are *equal* (with value $1/N$), then

$$H(S) = \log_2 N \qquad \text{or} \qquad N = 2^{H(S)}$$

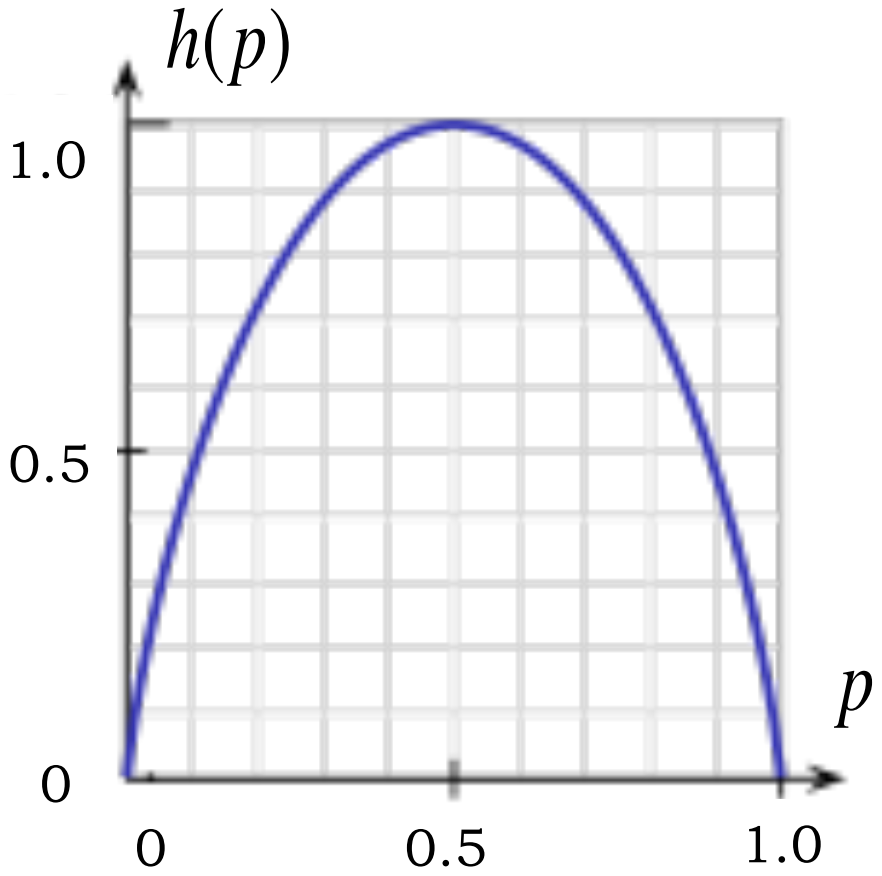
This is the maximum attainable value!

e.g., Binary entropy function $h(p)$

Heads (or C=1) with probability p

Tails (or C=0) with probability $1-p$

So the maximum information carried on average by a binary digit (0 or 1) e.g., reporting the result of a coin toss, is: 1 bit.



$$H(C) = -p\log_2 p - (1-p)\log_2(1-p) = h(p)$$

Significance of Entropy

Entropy (in bits) tells us the average amount of information (in bits) that must be delivered in order to resolve the uncertainty about the outcome of a trial. This is a lower bound on the number of binary digits that must, on the average, be used to encode our messages: $H \leq L$

Confusingly, a binary digit is also referred to as a "bit!"

If we send fewer binary digits on average, the receiver will have some uncertainty about the outcome described by the message.

If we send more binary digits on average, we're wasting the capacity of the communications channel by sending binary digits we don't have to.

Achieving the entropy lower bound is the "gold standard" for an encoding (at least from the viewpoint of information compression).

Connection to (Binary) Coding

- Suppose p=1/1024, i.e., very small probability of getting a Head, typically one Head in 1024 trials. Then

$$h(p) = (1/1024)\log_2(1024/1) + (1023/1024)\log_2(1024/1023)$$

$= .0112$ bits of uncertainty or information per trial on average

- So using 1024 binary digits (C=0 or 1) to code the results of 1024 tosses of this particular coin seems inordinately wasteful, i.e., 1 binary digit per trial. Can we get closer to an average of .0112 binary digits/trial?

- Yes!

Fixed-length Encodings

An obvious choice for encoding equally probable outcomes is to choose a fixed-length code that has enough sequences to encode the necessary information

- 96 printing characters → 7-"bit" ASCII
- Unicode characters → UTF-16
- 10 decimal digits → 4-"bit" BCD (binary coded decimal)

Fixed-length codes have some advantages:

- They are "random access" in the sense that to decode the n^{th} message symbol one can decode the n^{th} fixed-length sequence without decoding sequences 1 through n-1.
- Table lookup suffices for encoding and decoding

Now consider:

$choice_i$	p_i	$log_2(1/p_i)$
"A"	1/3	1.58 bits
"B"	1/2	1 bit
"C"	1/12	3.58 bits
"D"	1/12	3.58 bits

The expected information content is given by the entropy:
= (.333)(1.58) + (.5)(1) + (2)(.083)(3.58) = 1.626 bits

Can we find an encoding where transmitting 1000 choices requires 1626 binary digits on the average?

The "natural" fixed-length encoding uses two binary digits for each choice, so transmitting the results of 1000 choices requires 2000 binary digits.

Variable-length Encodings

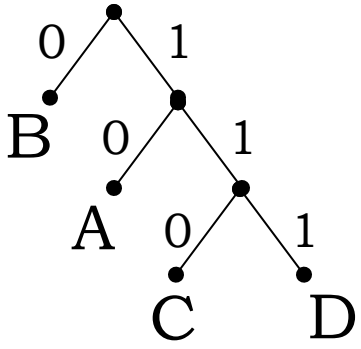
(David Huffman, in term paper for MIT graduate class, 1951)

Use shorter bit sequences for higher probability choices, longer sequences for less probable choices (back to Morse's idea!)

$choice_i$	p_i	*encoding*
"A"	1/3	10
"B"	1/2	0
"C"	1/12	110
"D"	1/12	111

B C AB A D

0110100101111



Huffman Tree

Expected length

=(.333)(2)+(.5)(1)+(2)(.083)(3)

= 1.666 bits

(≥ H=1.626)

Transmitting 1000 choices takes an average of 1666 bits… better but not optimal

Note: The symbols are at the leaves of the tree; necessary and sufficient for instantaneous (and unique) decodability.

Another Variable-length Code

Here's an alternative variable-length for the example on the previous page:

Letter	*Encoding*
A	0
B	1
C	00
D	01

Why isn't this a workable code?

The expected length of an encoded message is

(.333+.5)(1) + (.083 + .083)(2) = 1.22 bits

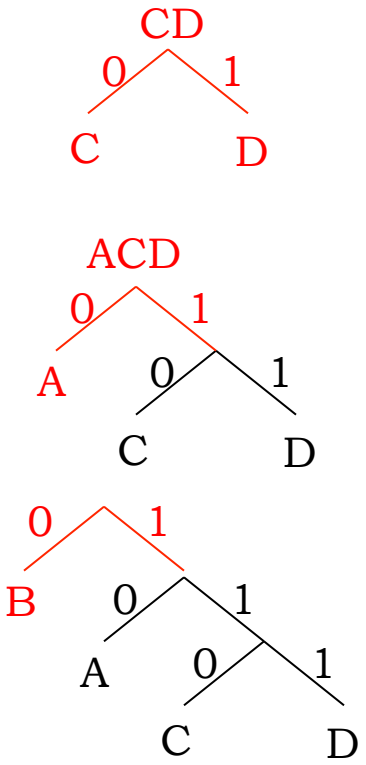
which even beats the entropy bound ☺

Huffman's Coding Algorithm

- Begin with the set S of symbols to be encoded as binary strings, together with the probability p(s) for each symbol s in S.
- Repeat the following steps until there is only 1 symbol left in S:
  - Choose the two members of S having lowest probabilities. Choose arbitrarily to resolve ties.
  - Remove the selected symbols from S, and create a new node of the decoding tree whose children (sub-nodes) are the symbols you've removed. Label the left branch with a "0", and the right branch with a "1".
  - Add to S a new symbol that represents this new node. Assign this new symbol a probability equal to the sum of the probabilities of the two nodes it replaces.

Huffman Coding Example

- Initially S = { (A, 1/3) (B, 1/2) (C, 1/12) (D, 1/12) }
- First iteration
  - Symbols in S with lowest probabilities: C and D
  - Create new node
  - Add new symbol to S = { (A, 1/3) (B, 1/2) (CD, 1/6) }
- Second iteration
  - Symbols in S with lowest probabilities: A and CD
  - Create new node
  - Add new symbol to S = { (B, 1/2) (ACD, 1/2) }
- Third iteration
  - Symbols in S with lowest probabilities: B and ACD
  - Create new node
  - Add new symbol to S = { (BACD, 1) }
- Done



Huffman Codes – the final word?

- Given static symbol probabilities, the Huffman algorithm creates an optimal encoding when each symbol is encoded separately. (optimal ≡ no other encoding will have a shorter expected message length). It can be proved that

  expected length L satisfies $H \leq L \leq H+1$

- Huffman codes have the biggest impact on average message length when some symbols are substantially more likely than other symbols.

- You can improve the results by adding encodings for symbol pairs, triples, quads, etc. From example code:

- *Pairs: 1.646 bits/sym, Triples: 1.637, Quads 1.633, …*

- This can get somewhat intractable.

- Symbol probabilities change message-to-message, or even within a single message. Can we do adaptive variable-length encoding?

  ➔ next lecture.