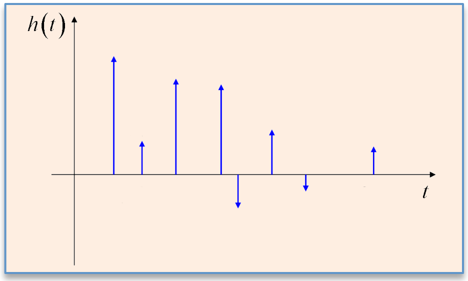
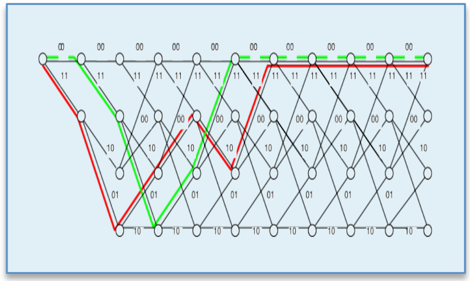
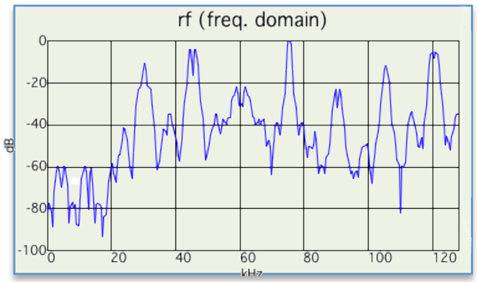
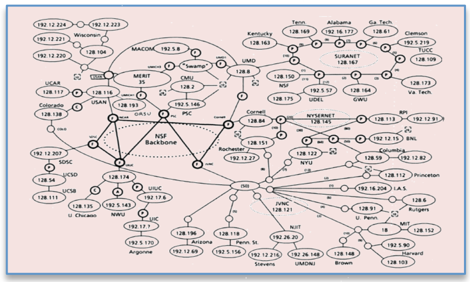








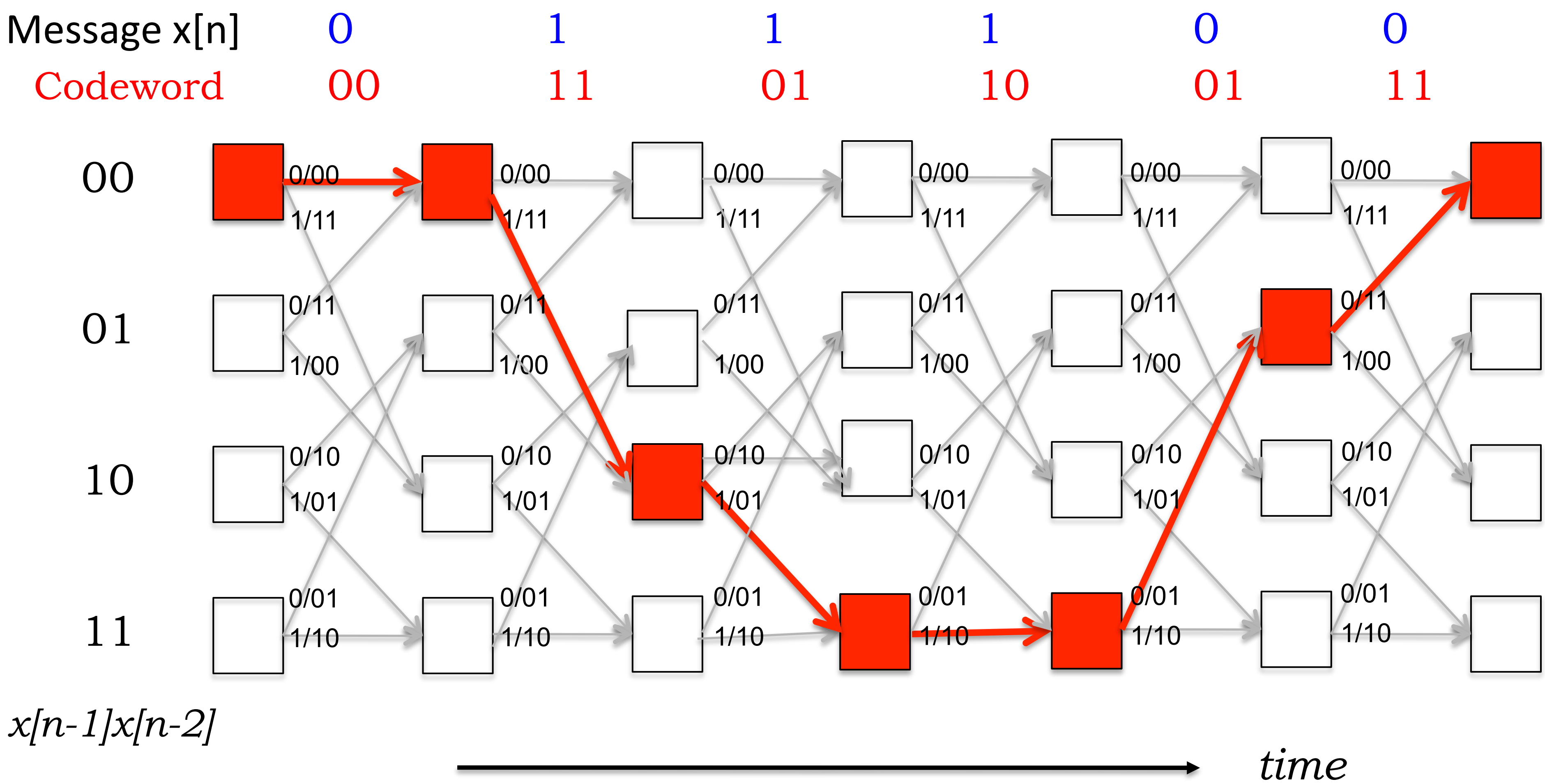
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2013
Lecture #8

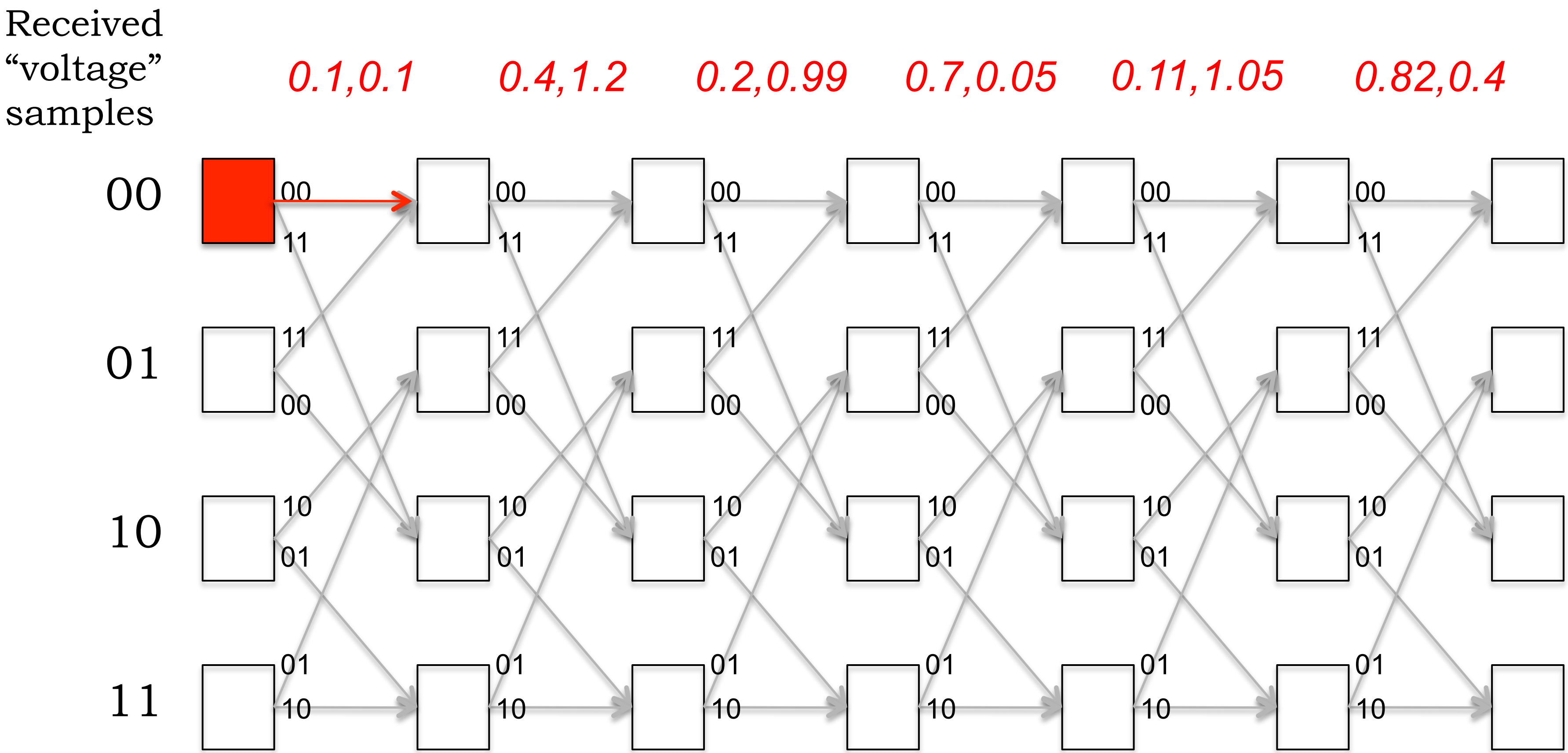
- A bit more on Viterbi
- Noise
- PDF's, means, variances, Gaussian noise
- Bit error rate for bipolar signaling

Trellis View at Transmitter



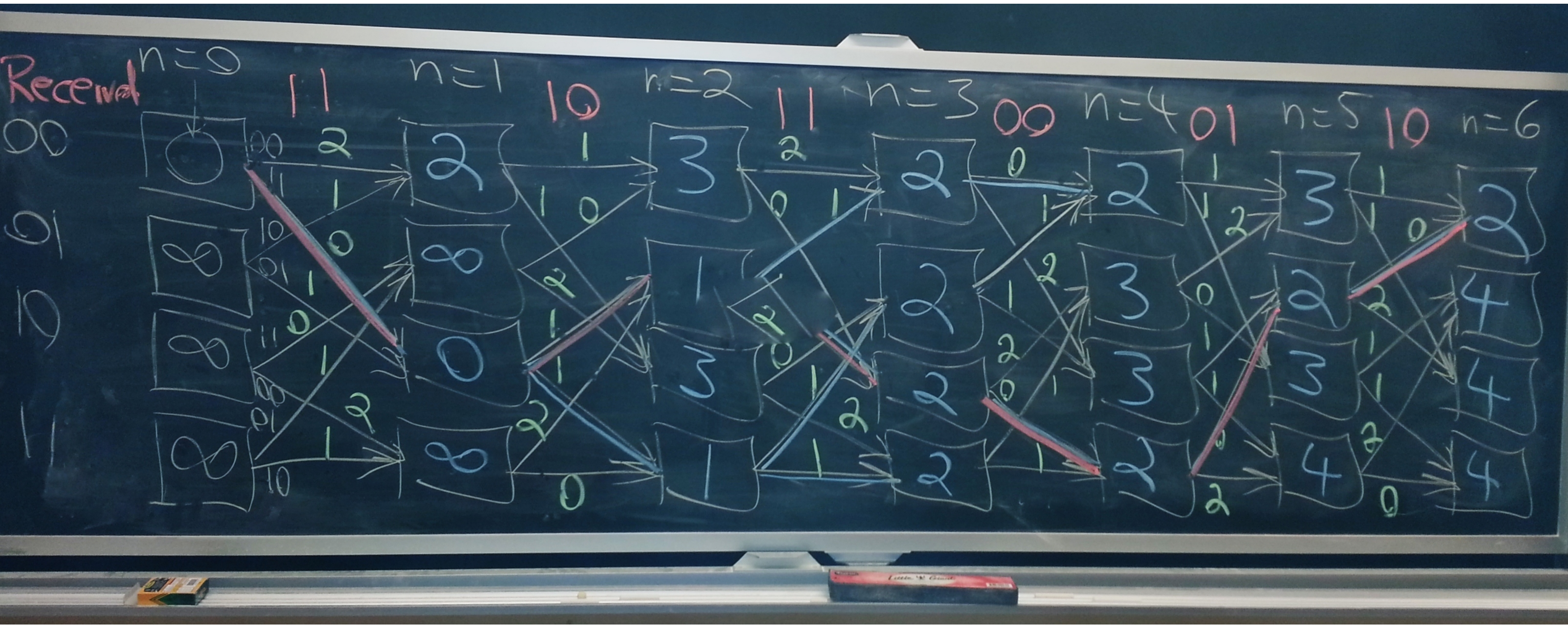
Free distance = Minimum weight of nonzero codeword that returns to zero state = 5

Decoding: Finding Max Likelihood Path



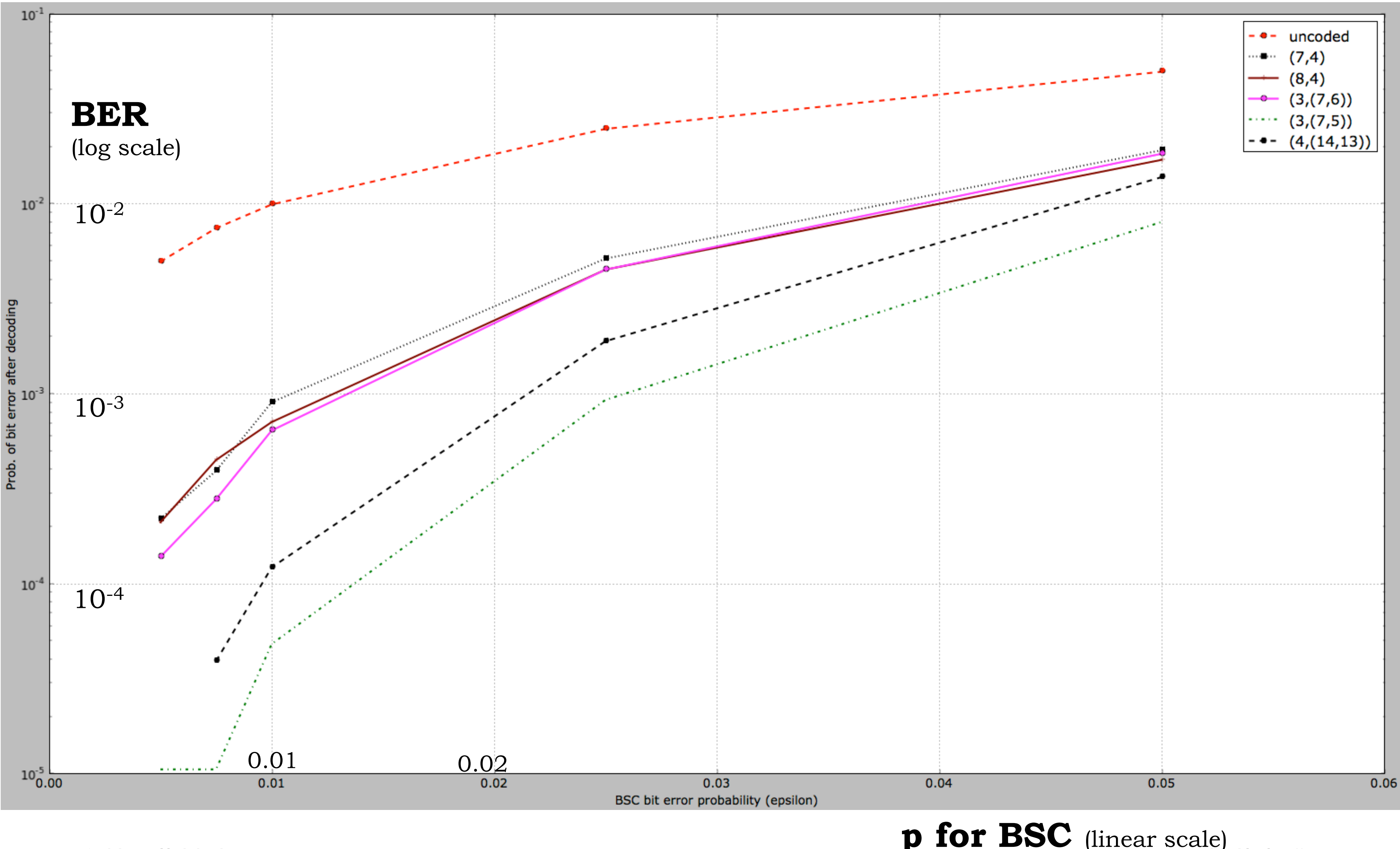
Given received "voltage" samples, find most-likely path through trellis, i.e., minimize "distance" between the received values and those produced along the trellis path.

Viterbi at the Board*

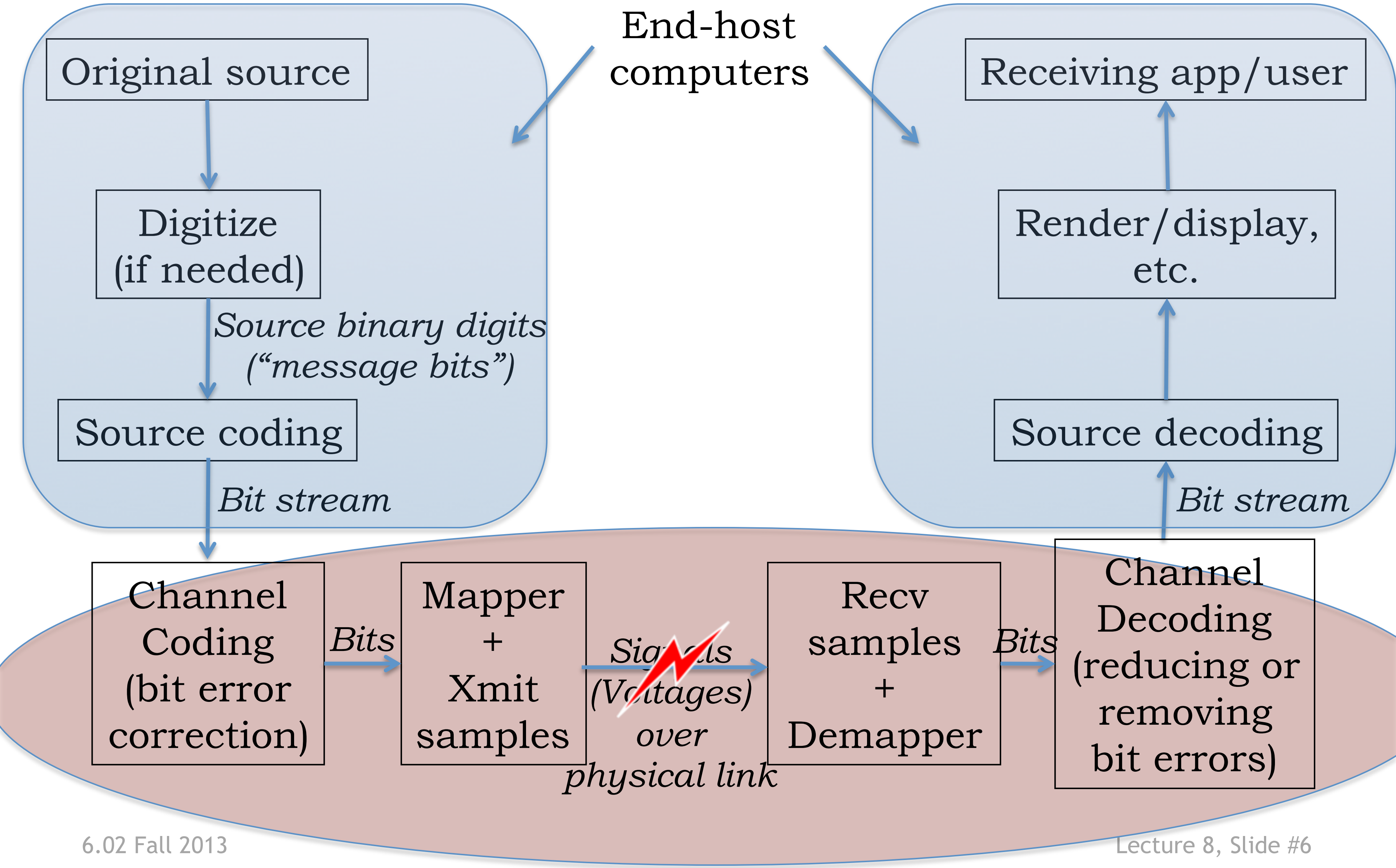


*Thanks to Babak Ayazifar, hard-decision decoding using Viterbi, from recitation

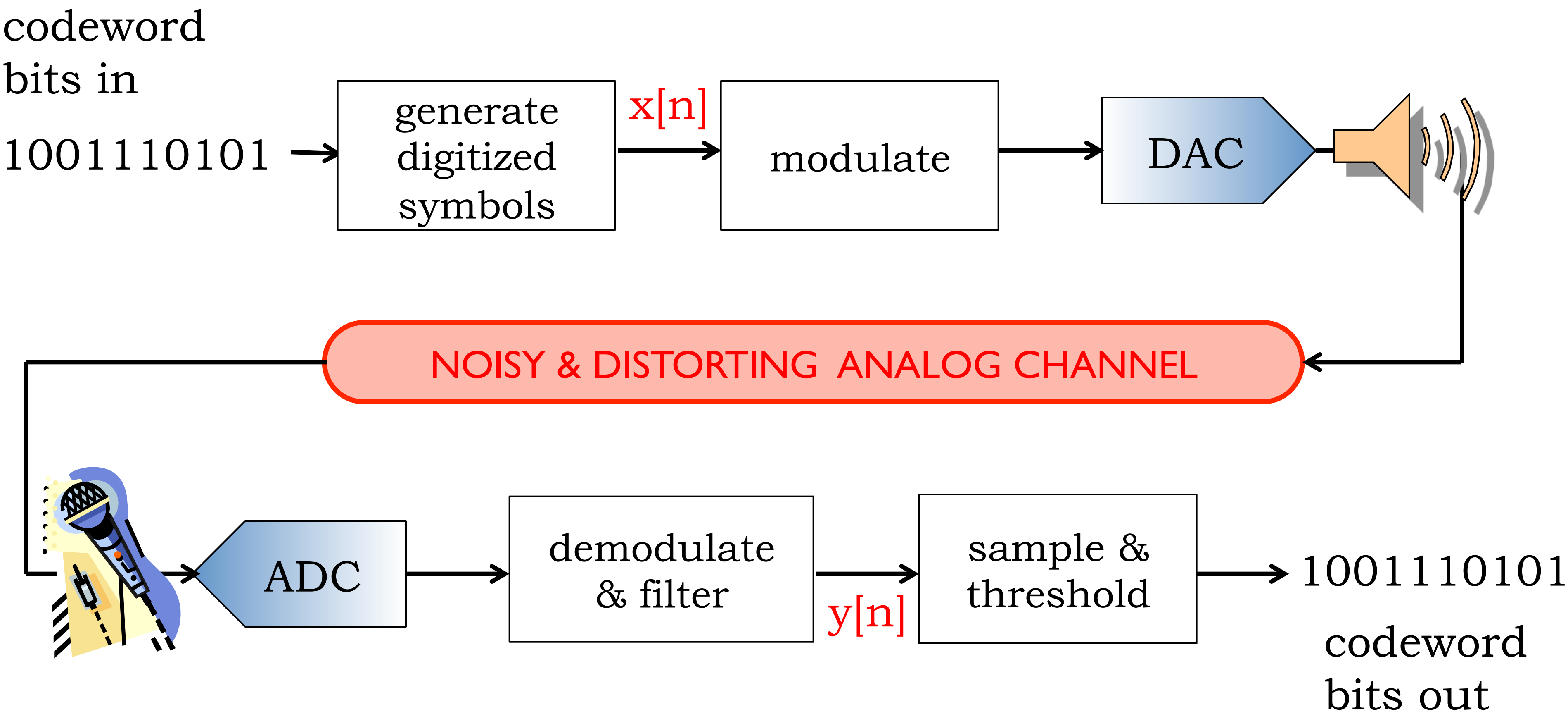
Post-decoding BER v. BSC error probability



Single Link Communication Model



From Baseband to Modulated Signal, and Back

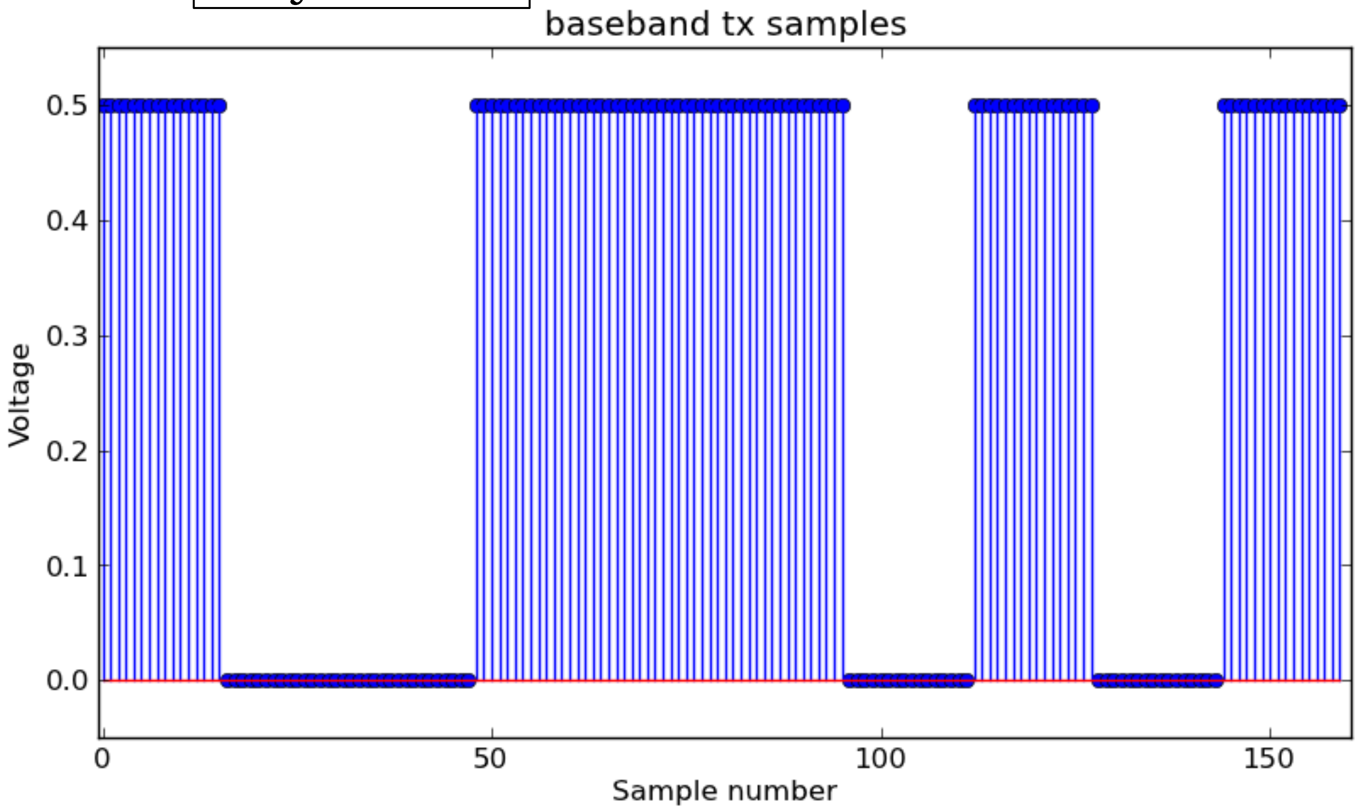


Mapping Bits to Samples at Transmitter

codeword
bits in

1001110101 → generate digitized symbols → x[n]

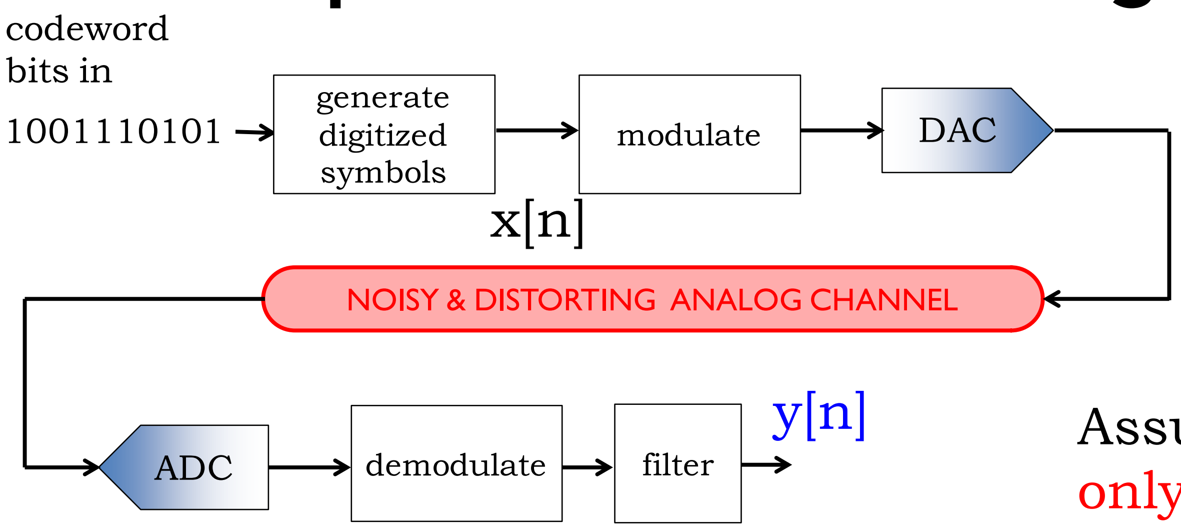
L=16 samples per bit



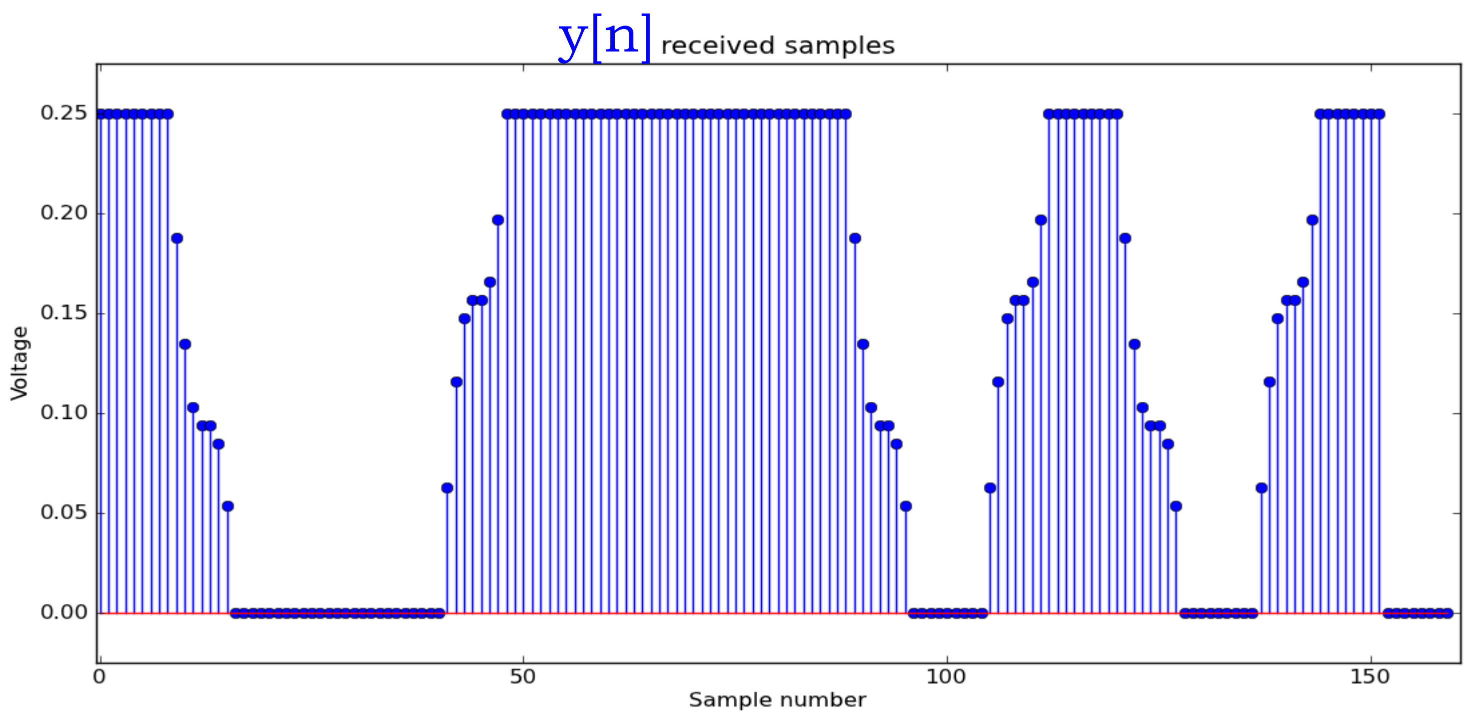
On-Off Signaling
'0'→0 volts
'1'→V volts

1 0 0 1 1 1 0 1 0 1

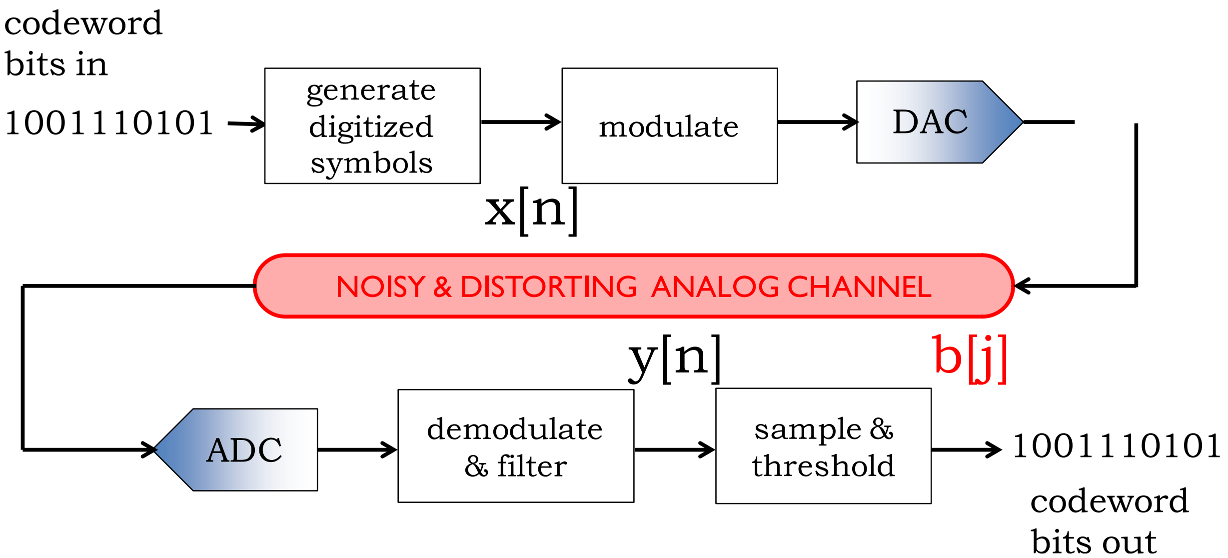
Samples after Processing at Receiver



Assuming no noise,
only end-to-end distortion



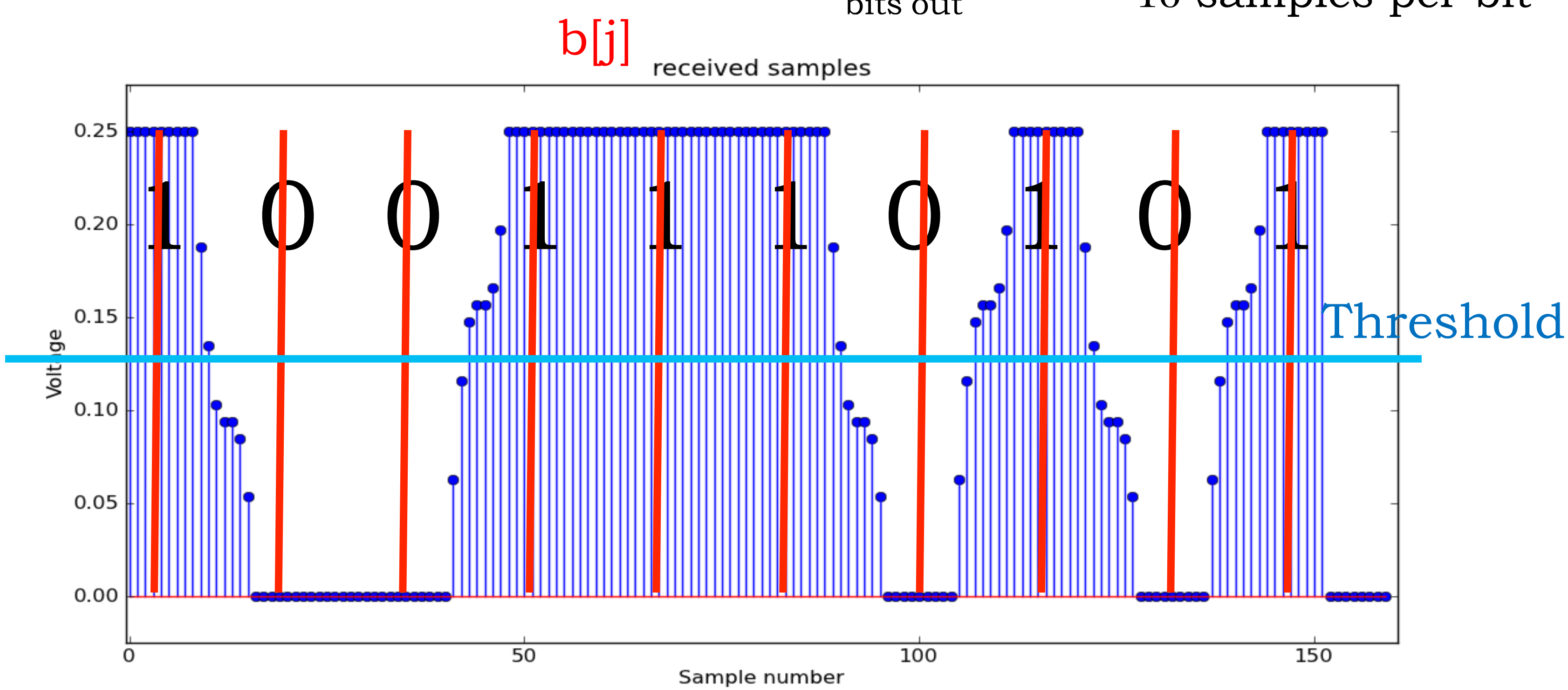
Mapping Samples to Bits at Receiver



n = sample index

j = bit index

16 samples per bit



For now, assume no distortion, only Additive Zero-Mean Noise

- Received signal

  $$b[j] = x[n_j] + w[n_j]$$

  i.e., received samples b[j] = the transmitted sample $x[n_j]$ + zero-mean noise $w[n_j]$ on that sample

  Assume iid (independent and identically distributed at each n_j)

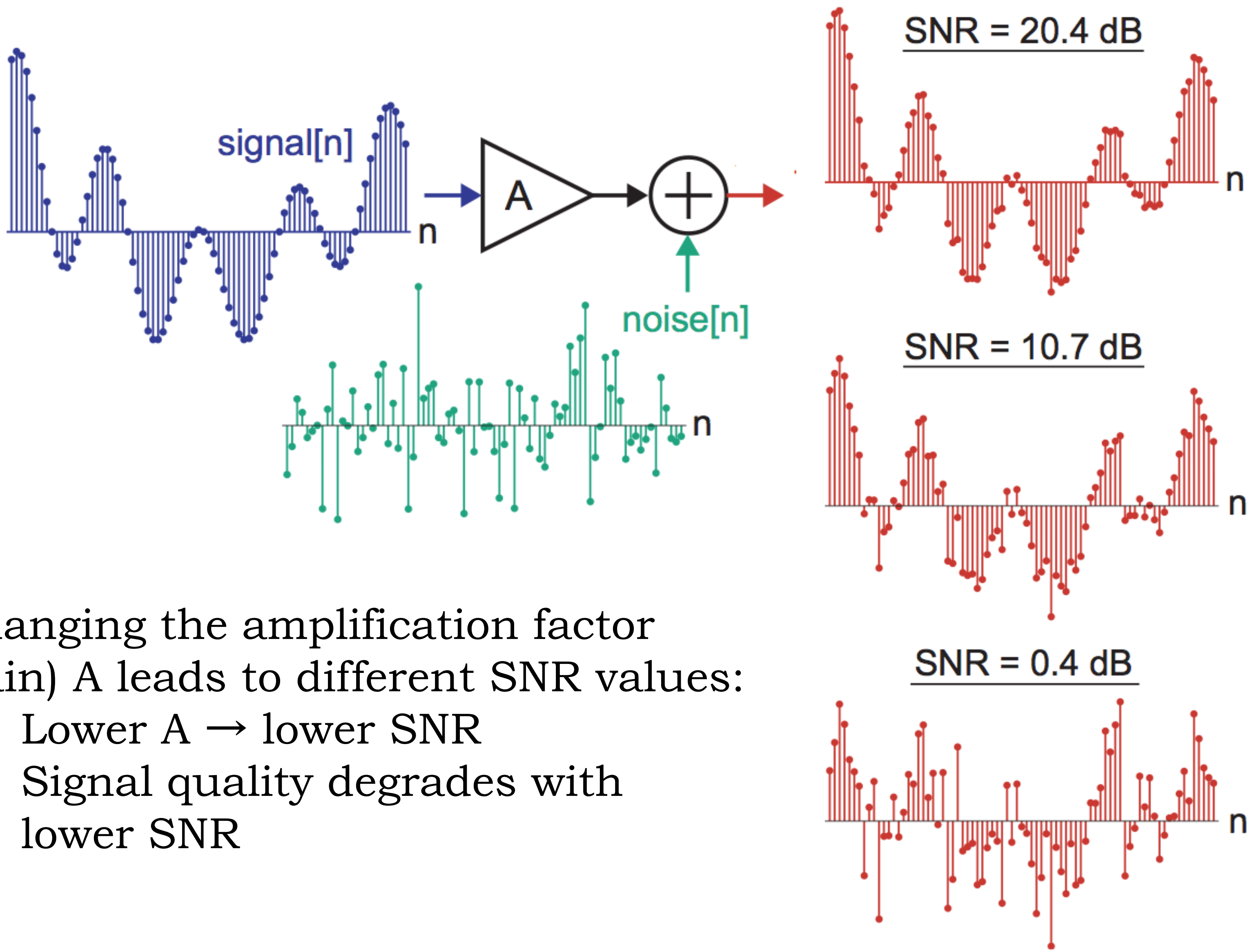
- Signal-to-Noise Ratio (SNR)
  - usually denotes the ratio of

  (time-averaged or peak) signal power, i.e., *squared amplitude* of x[n]

  to

  noise variance, i.e., *expected squared amplitude* of w[n]

SNR Example



Changing the amplification factor (gain) A leads to different SNR values:

- Lower A → lower SNR
- Signal quality degrades with lower SNR

Signal-to-Noise Ratio (SNR)

The Signal-to-Noise ratio (SNR) is useful in judging the impact of noise on system performance:

$$\text{SNR} = \frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}$$

SNR for power is often measured in decibels (dB):

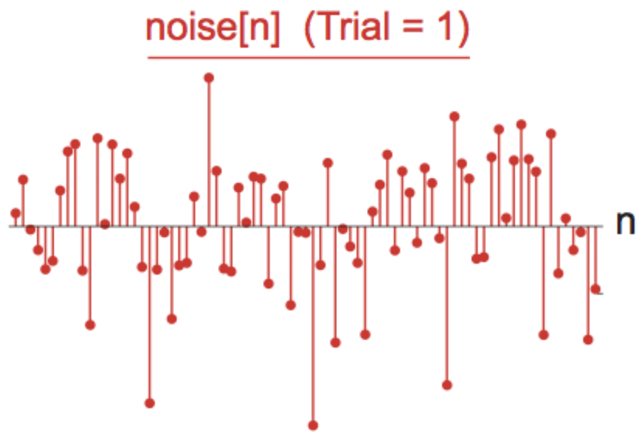
$$\text{SNR (dB)} = 10\log_{10}\left(\frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}\right)$$

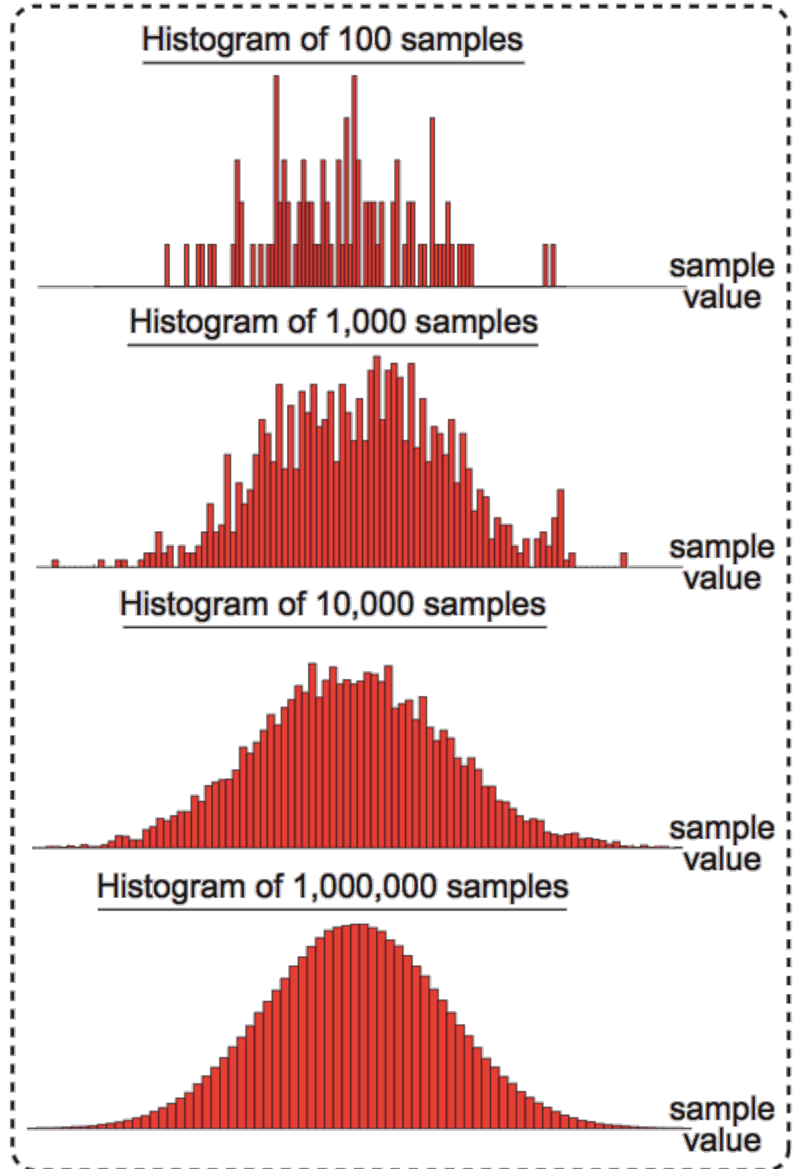
Caution: For measuring ratios of *amplitudes* rather than powers, take 20 $\log_{10}$ (ratio).

10logX	X
100	10000000000
90	1000000000
80	100000000
70	10000000
60	1000000
50	100000
40	10000
30	1000
20	100
10	10
0	1
-10	0.1
-20	0.01
-30	0.001
-40	0.0001
-50	0.000001
-60	0.0000001
-70	0.00000001
-80	0.000000001
-90	0.0000000001
-100	0.00000000001

3.01db is a factor of 2 in power ratio

Noise Characterization: From Histogram to PDF





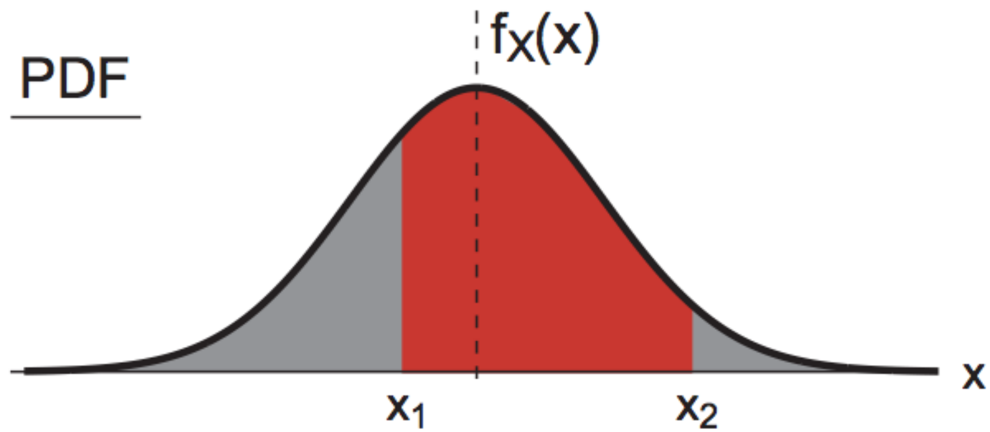
Experiment: create histograms of sample values from independent trials of increasing lengths.

Histogram typically "converges" to a shape that is known – after *normalization to unit area* – as a probability density function (PDF)

Using the PDF in Probability Calculations

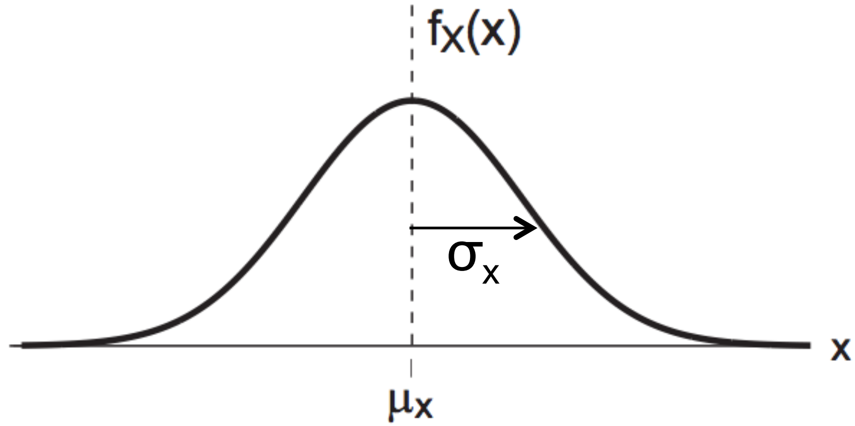
We say that X is a continuous random variable governed by the PDF $f_X(x) \geq 0$ if X takes on a numerical value in the range of x_1 to x_2 with a probability calculated from the PDF of X as:

$$p(x_1 < X < x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$$



A PDF is **not** a probability – its associated *integrals* are.
Note that the PDF is nonnegative, and the area under it is 1.

Mean & Variance of Continuous r.v. *X*



The *mean* or *expected value* μ_X is defined and computed as:

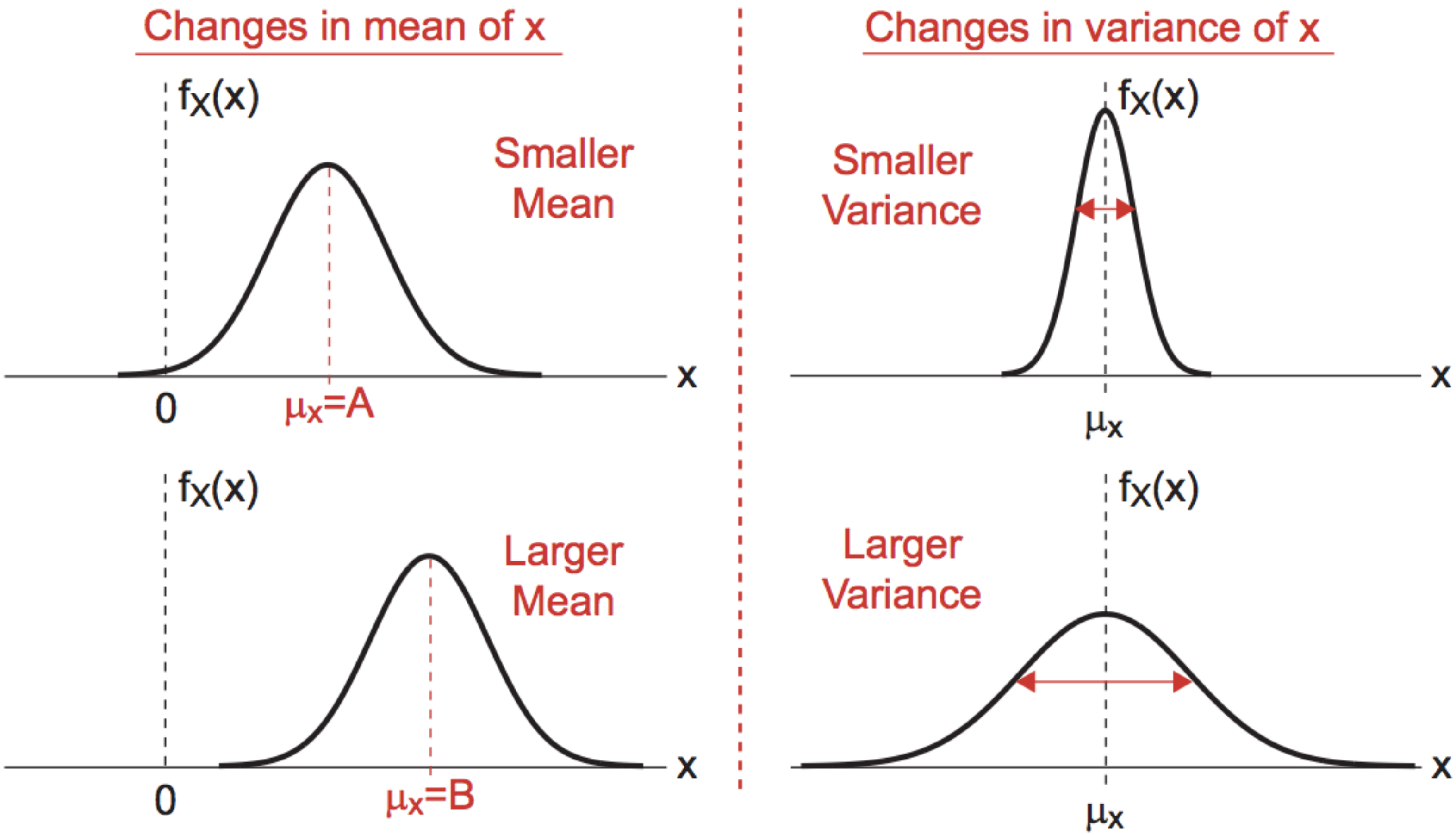
$$\mu_X = \int_{-\infty}^{\infty} x\, f_X(x)dx$$

The *variance* σ_X^2 is the expected squared variation or deviation of the random variable around the mean, and is thus computed as:

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x)dx$$

The square root of the variance is the *standard deviation*, σ_X

Visualizing Mean and Variance



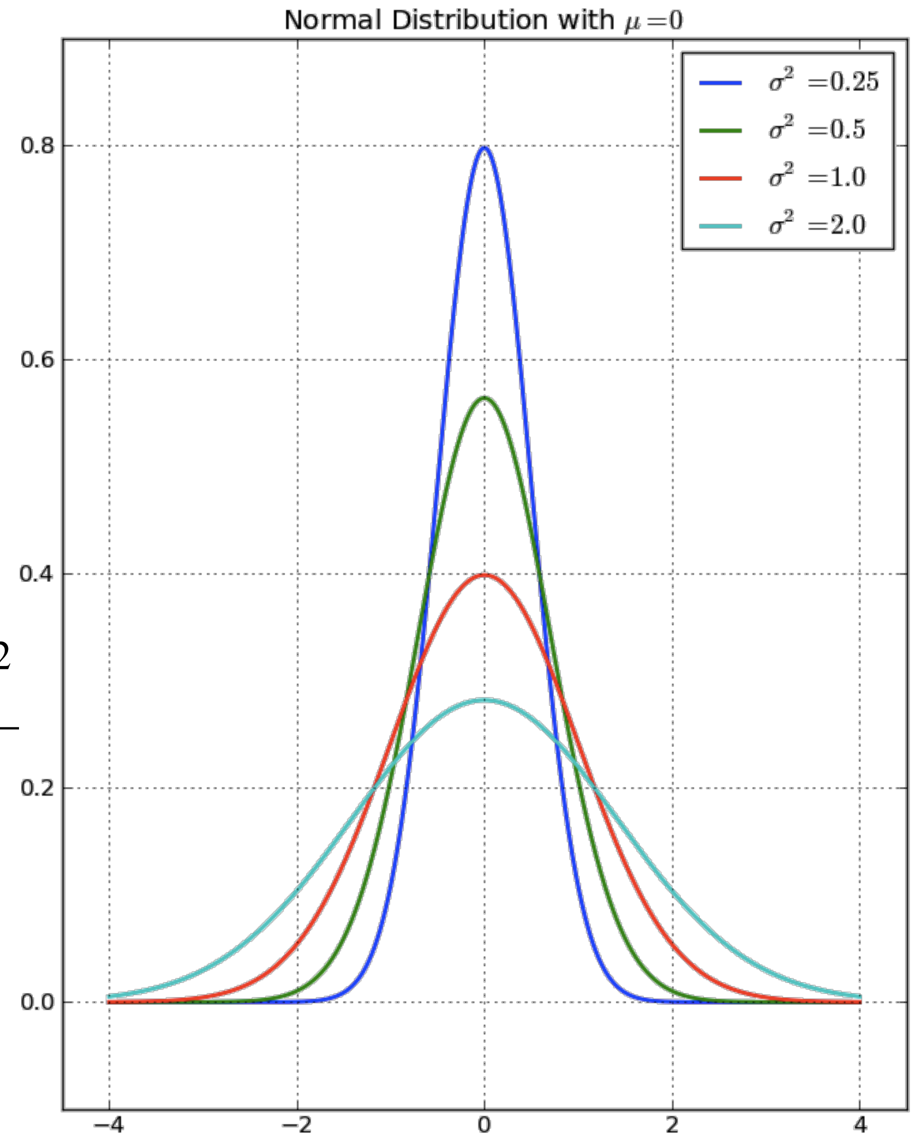
Changes in mean shift the center of mass of PDF

Changes in variance narrow or broaden the PDF (but area is always equal to 1)

The Gaussian Distribution

A Gaussian random variable W with mean μ and variance σ^2 has a PDF described by

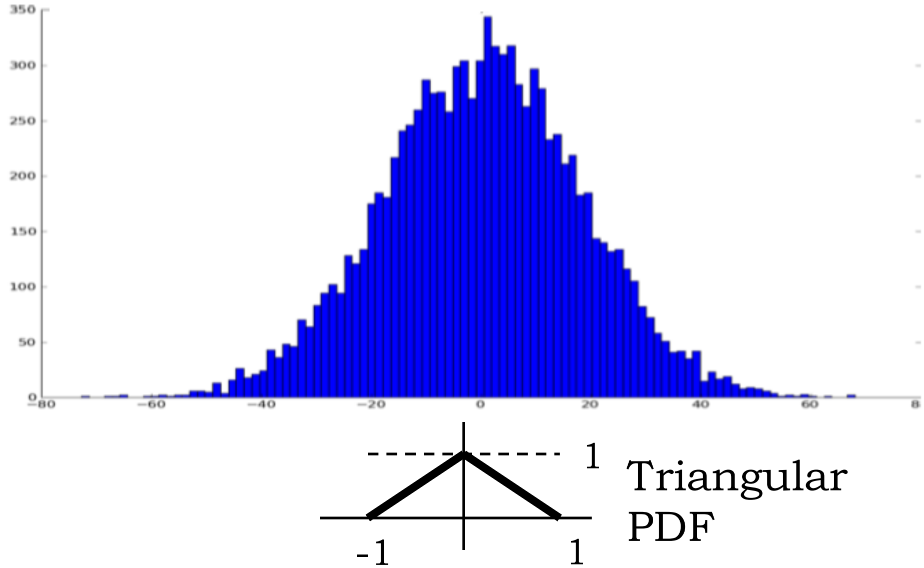
$$f_W(w) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(w-\mu)^2}{2\sigma^2}}$$

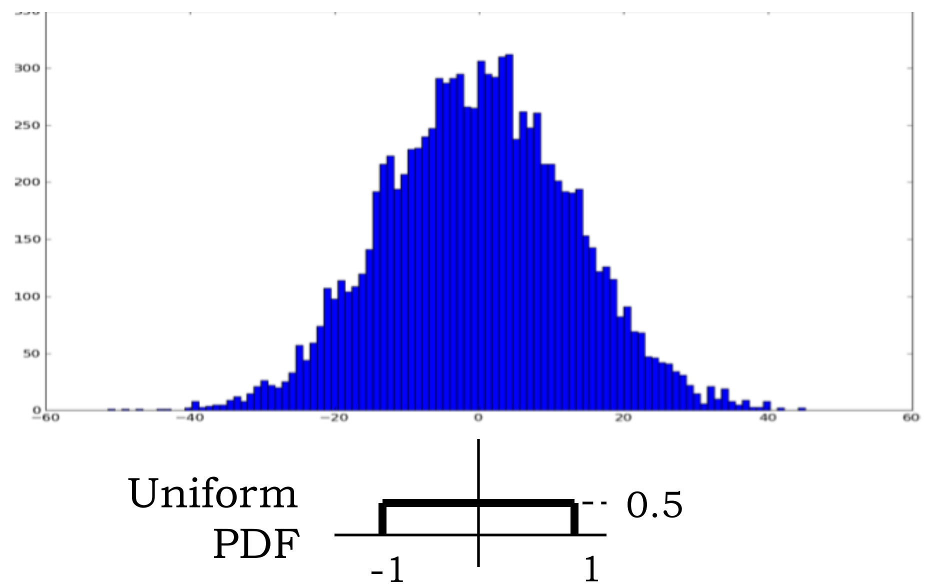


The Ubiquity of Gaussian Noise

The net noise observed at the receiver is often the sum of many small, independent random contributions from many factors. Under fairly mild conditions, the Central Limit Theorem says their sum will be a *Gaussian*.

The figure below shows the histograms of the results of 10,000 trials of summing 100 random samples drawn from [-1,1] using two different distributions.





Distinguishing "1" from "0"

- Assume bipolar signaling:

  Transmit L samples x[.] at $+V_p$ ($=V_1$) to signal a "1"

  Transmit L samples x[.] at $-V_p$ ($=V_0$) to signal a "0"

- Simple-minded receiver: take a single sample value $y[n_j]$ at an appropriately chosen instant n_j in the j-th bit interval. Decide between the following two hypotheses:

$$y[n_j] = +V_p + w[n_j] \quad (\text{==> "1"})$$

or

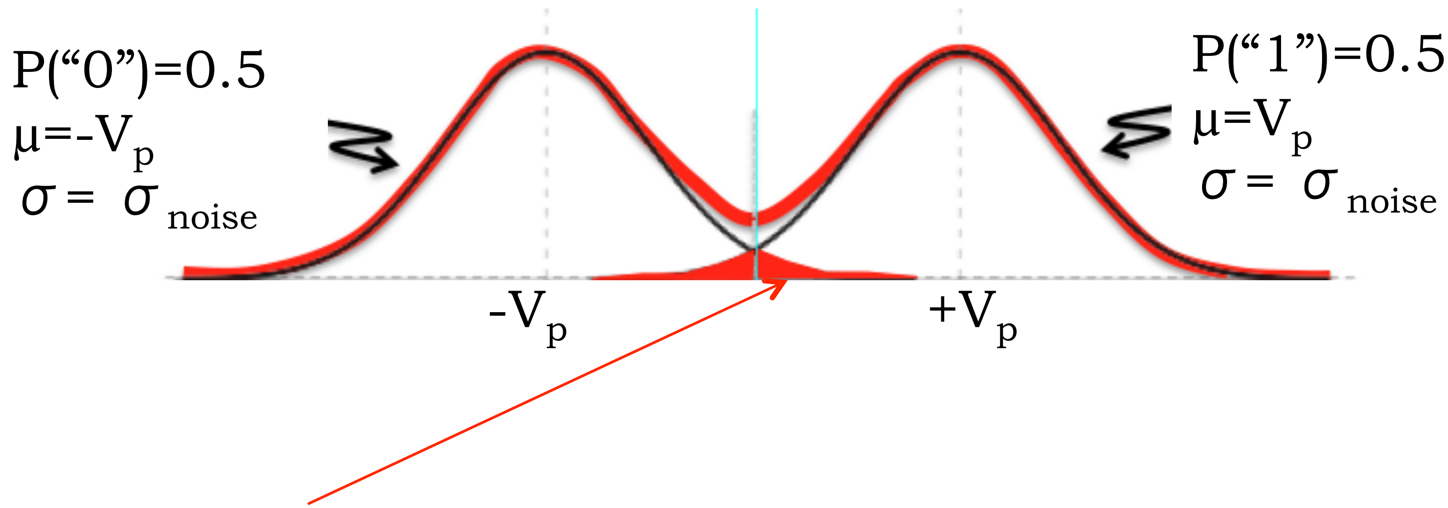
$$y[n_j] = -V_p + w[n_j] \quad (\text{==> "0"})$$

where $w[n_j]$ is Gaussian, zero-mean, variance σ^2

Connecting the SNR and BER

$$V_p = \sqrt{E_S}$$

$$2\sigma^2 = N_0$$



$$\text{BER} = \mathbb{P}(\text{error}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\sqrt{E_s}}^{\infty} e^{-w^2/(2\sigma^2)}\, dw$$

$$BER = P(error) = Q\left(\sqrt{\frac{2E_S}{N_0}}\right)$$

The Q(.) Function --- Area in the Tail of a Standard Gaussian

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-v^2/2}\, dv$$

$$Q(-t) = 1 - Q(t)$$

$$\frac{t}{(1+t^2)} \frac{e^{-t^2/2}}{\sqrt{2\pi}} < Q(t) < \frac{1}{t} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\ , \quad t > 0$$

Tail probability of a general Gaussian in terms of the Q(.) function

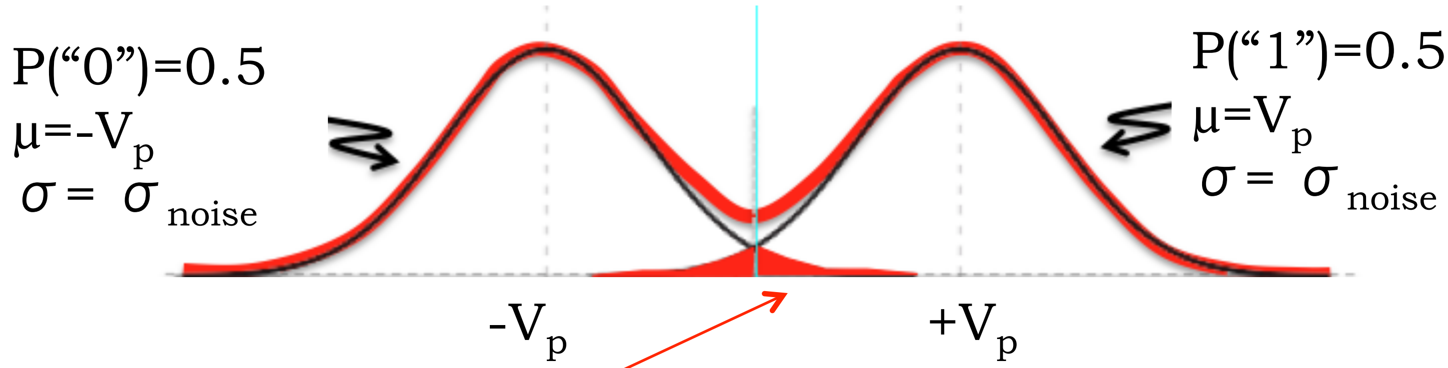
$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_t^{\infty} e^{-(v-\mu)^2/(2\sigma^2)}\, dv$$

$$= Q\left(\frac{t-\mu}{\sigma}\right)$$

Expressed in terms of *erfc* (as in notes)

$$V_p = \sqrt{E_S}$$

$$2\sigma^2 = N_0$$



$$\text{BER} = \mathbb{P}(\text{error}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\sqrt{E_s}}^{\infty} e^{-w^2/(2\sigma^2)}\, dw$$

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \cdot \int_{-\infty}^{z} e^{-v^2}\, dv\ ,$$

$$\text{BER} = \mathbb{P}(\text{error}) = \frac{1}{\sqrt{\pi}} \cdot \int_{\sqrt{E_s/N_0}}^{\infty} e^{-v^2}\, dv$$

$$\text{erfc}(z) = 1 - \text{erf}(z) = \frac{2}{\sqrt{\pi}} \cdot \int_{z}^{\infty} e^{-v^2}\, dv$$

$$BER = P(error) = \frac{1}{2} erfc(\sqrt{\frac{E_S}{N_0}}) = \frac{1}{2} erfc(\frac{V_p}{\sigma\sqrt{2}})$$