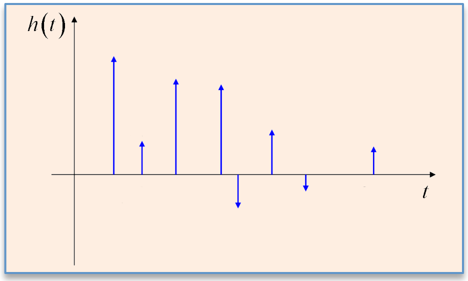
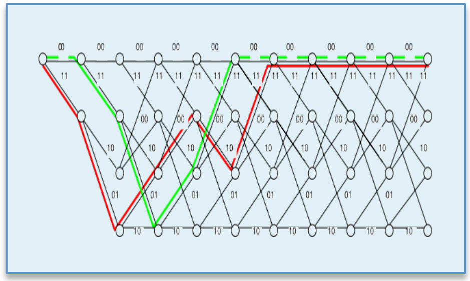
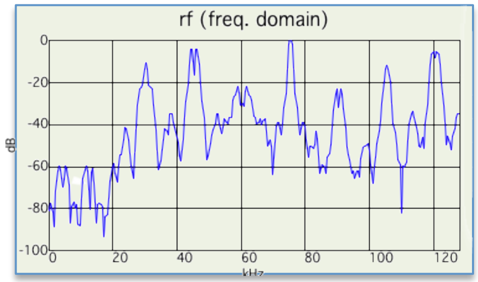
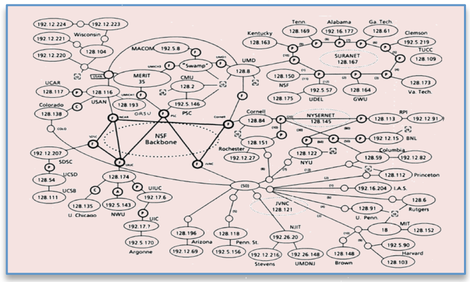








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2013
Lecture #9

- Revisit/expand story on bit detection in noise
- Introduction to modulation and demodulation
- Modeling the channel

Bit Detection in Noise

We've decided it's easier to tell you the full story rather than a partial one! So:

Recall that the receiver samples the value

$$y = x + w$$

in a particular bit slot (one sample per bit slot, for now),

where x is the transmitted value

$= V_0$ if the sender's codeword bit B=0 , probability p_0

$= V_1$ if the sender's codeword bit B=1 , probability p_1

$V_0 = 0$ and $V_1 = V_p$ for on-off signaling

$V_0 = -V_p$ and $V_1 = V_p$ for bipolar signaling

and

w is the value of the additive channel noise in this bit slot.

Conditional PDFs of received sample *Y*

- Think in terms of random variables,

$$Y = X + W$$

with each of these taking specific values *y, x, w* in the particular bit slot.

(*X* and *W* are assumed independent, i.e., knowing one tells us nothing about the other --- if the channel noise is signal-dependent, things get harder)

- How is *Y* distributed if $B=0$? → Described by **conditional PDF** $f_{Y|B}(y|0)$

- How is *Y* distributed if $B=1$? → Described by **conditional PDF** $f_{Y|B}(y|1)$

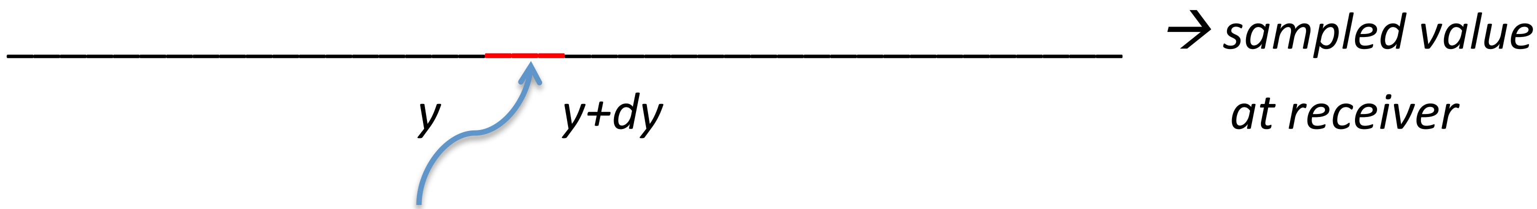
- What is $f_{Y|B}(y|0)$ if *W* is Gaussian, mean *0*, variance σ^2 ?

  Gaussian, mean V_0 , variance σ^2

- What is $f_{Y|B}(y|1)$ if *W* is Gaussian, mean *0*, variance σ^2 ?

  Gaussian, mean V_1 , variance σ^2

Bit Detection with Min Probability of Error



What is the probability that the received sample falls in this interval of length *dy* ?

$$f_{Y|B}(y|0)\,dy \quad \text{if} \quad B=0$$
$$f_{Y|B}(y|1)\,dy \quad \text{if} \quad B=1$$

What is the probability of error if receiver decides "0" when *y* lies here?

$$p_1 \cdot f_{Y|B}(y|1)\,dy$$

What is the probability of error if receiver decides "1" when *y* lies here ?

$$p_0 \cdot f_{Y|B}(y|0)\,dy$$

So, for min P(error) ⋯

- Decide "1" for all *y* where $p_1 \,.\, f_{Y|B}(y|1) \quad > \quad p_0 \,.\, f_{Y|B}(y|0)$
- Decide "0" for all *y* where $p_1 \,.\, f_{Y|B}(y|1) \quad < \quad p_0 \,.\, f_{Y|B}(y|0)$
- And the associated probability of error is

$$\int p_1 \,.\, f_{Y|B}(y|1)\, dy \quad \text{over the "0" region}$$

$$+ \quad \int p_0 \,.\, f_{Y|B}(y|0)\, dy \quad \text{over the "1" region}$$

- This is bit-by-bit detection, i.e., **hard detection** (not soft)

The Q(.) Function ---
Area in the Tail of a Standard Gaussian

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-v^2/2}\, dv$$

$$Q(-t) = 1 - Q(t)$$

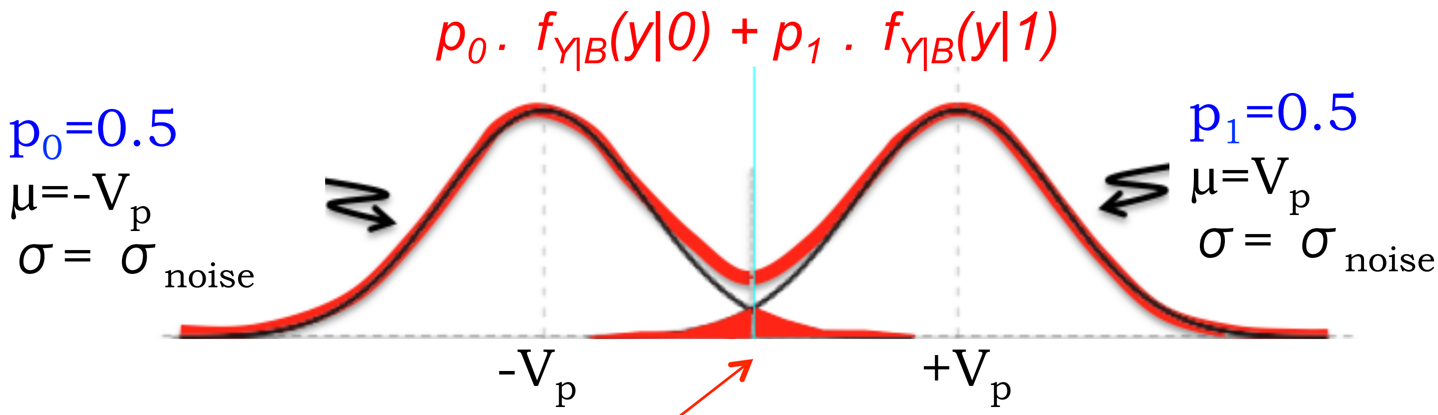
$$\frac{t}{(1+t^2)} \frac{e^{-t^2/2}}{\sqrt{2\pi}} < Q(t) < \frac{1}{t} \frac{e^{-t^2/2}}{\sqrt{2\pi}} \;, \quad t > 0$$

Tail probability of a general Gaussian in terms of the Q(.) function

$$\frac{1}{\sqrt{2\pi\sigma^2}}\int_t^{\infty} e^{-(v-\mu)^2/(2\sigma^2)}\,dv$$

$$= Q\left(\frac{t-\mu}{\sigma}\right)$$

Connecting the "SNR" and BER for Bipolar Signaling



$$P(error) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{V_p}^{\infty} e^{-w^2/(2\sigma^2)}\, dw$$

$$= Q\left(\frac{V_p}{\sigma}\right)$$

$$= Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

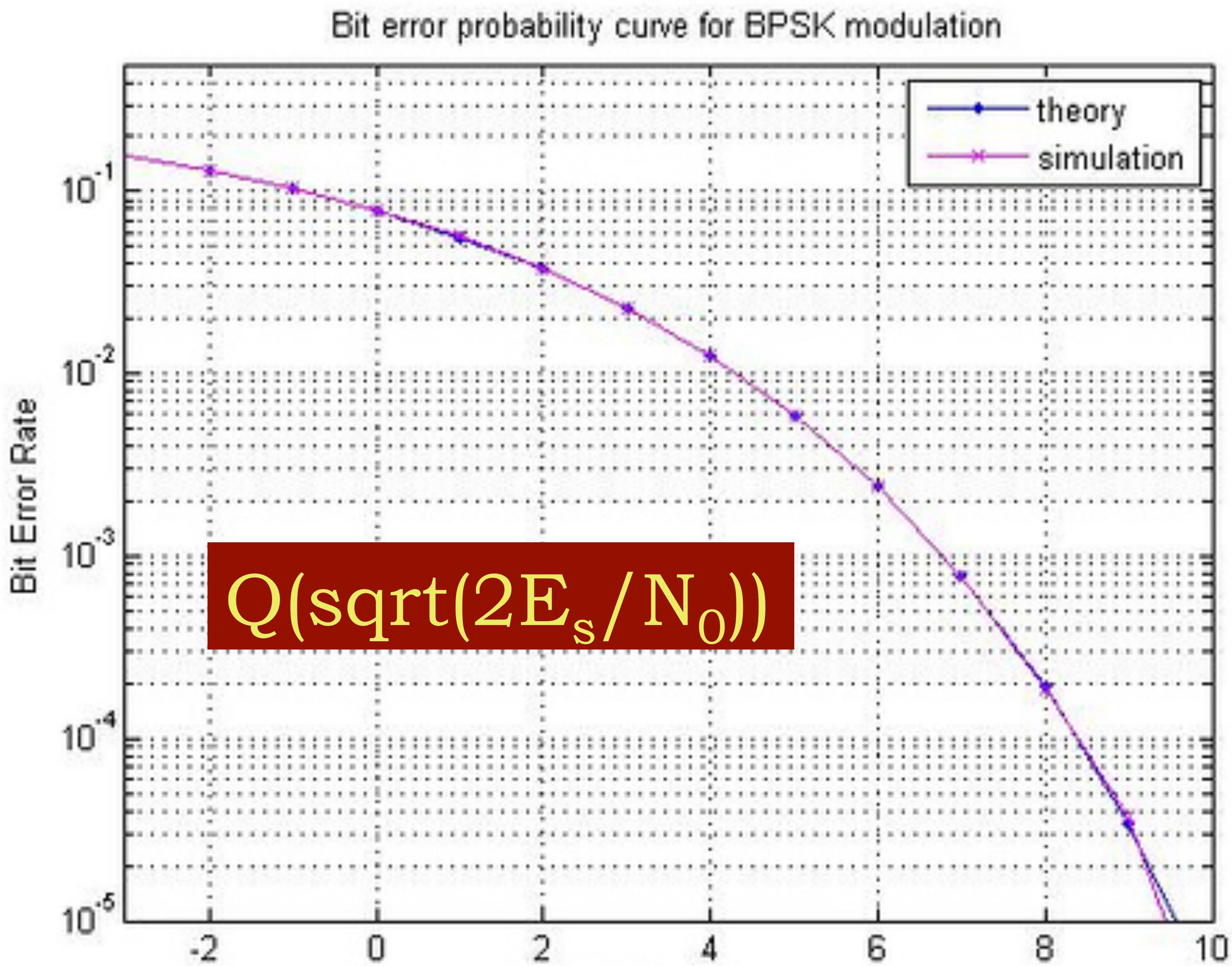
$$V_p = \sqrt{E_S}$$

$$2\sigma^2 = N_0$$

$$"SNR" = E_S / N_0$$

How does this relate to *p* of BSC?

Bit Error Rate for Bipolar Signaling Scheme with Single-Sample Decision



E_s / N_0 , dB

Noise Model for iid Process *w[n_j]*

- Assume each *w[n_j]* is distributed as a Gaussian random variable *W*, with mean 0 and variance σ^2, and independently of *w[.]* at sample times in all other bit slots.
- Under the iid assumption

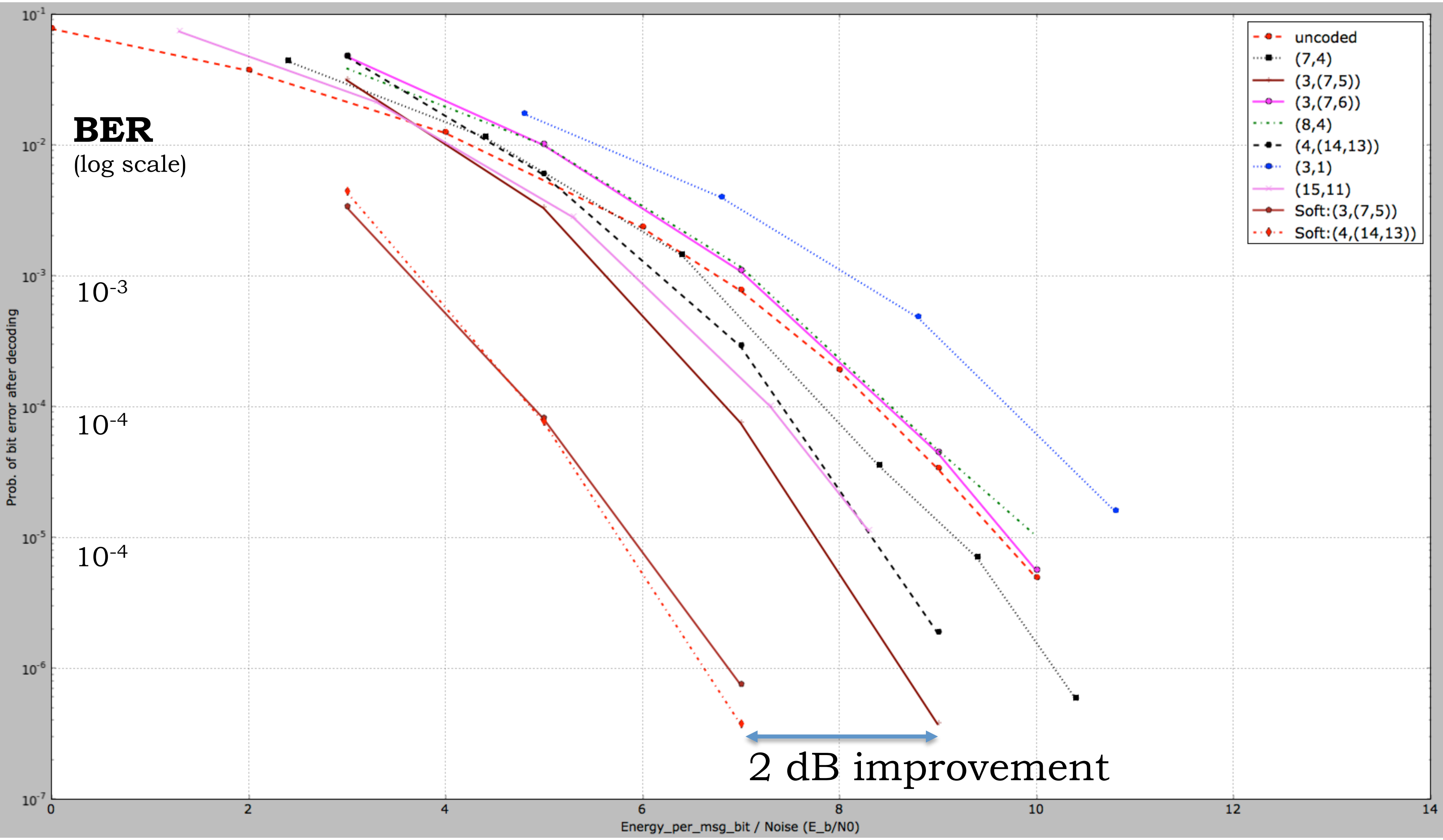
$$P(w_a < w[n_1] < w_b \text{ and } w_c < w[n_2] < w_d)$$

$$= \int_{w_a}^{w_b} f_W(\alpha)\, d\alpha \,.\, \int_{w_c}^{w_d} f_W(\beta)\, d\beta$$

etc. --- i.e., individual PDFs multiply for independent continuous random variables (just as probabilities multiply for independent discrete events) --- so in Gaussian case exponents add.

→ Justifies soft-decision decoding with sum-of-squares metric

Soft Decoding Beats Hard Decoding



E_s / N_0 dB

But we can do better!

- Why just take a single sample from a bit interval?
- Instead, average M (≤L) samples with independent noise components:

$$y[n] = \pm V_p + w[n] \quad so \quad \textbf{avg}\ \{y[n]\} = \pm V_p + \textbf{avg}\ \{w[n]\}$$

- **Claim: avg** {w[n]} is still Gaussian, still has mean 0, but its variance is now σ^2/M instead of σ^2 , so

  "SNR" is increased by a factor of M

- Same analysis and formulas as before, but now with

  $\sigma^2 \rightarrow \sigma^2/M$, or equivalently

  sample energy E_s ➔ bit (or symbol) energy $E_b = M.E_s$

Implications for Signaling Rate

- As the noise intensity increases, we need to slow down the signaling rate, i.e., increase the number of samples per bit (L), to get higher energy in the (M≤L) samples extracted from a bit interval, if we wish to maintain the same error performance.

  – e.g. Voyager 2 was transmitting at 115 **kilobits**/s when it was near Jupiter in 1979. When it was over 9 billion miles away, 13 light hours away from the sun, twice as far away from the sun as Pluto, it was transmitting at only 160 **bits**/s. The received power at the Deep Space Network antennas on earth when Voyager was near Neptune was on the order of 10^(-16) watts!! --- 20 billion times smaller than an ordinary digital watch consumes. The power now is estimated at less than 10^(-19) watts.

Flipped bits can have serious consequences!

- "On **November 30, 2006**, a telemetered command to *Voyager 2* was incorrectly decoded by its on-board computer—in a random error—as a command to turn on the electrical heaters of the spacecraft's magnetometer. These heaters remained turned on until December 4, 2006, and during that time, there was a resulting high temperature above 130 °C (266 °F), significantly higher than the magnetometers were designed to endure, and a sensor rotated away from the correct orientation. It has not been possible to fully diagnose and correct for the damage caused to the *Voyager 2's* magnetometer, although efforts to do so are proceeding."

- "On **April 22, 2010**, *Voyager 2* encountered scientific data format problems as reported by the Associated Press on May 6, 2010. On **May 17, 2010**, JPL engineers revealed that a flipped bit in an on-board computer had caused the issue, and scheduled a bit reset for May 19. On **May 23, 2010**, *Voyager 2* has resumed sending science data from deep space after engineers fixed the flipped bit."

http://en.wikipedia.org/wiki/Voyager_2

The moral of the story is ···

… if you're doing appropriate/optimal processing at the receiver, your effective SNR (and therefore your error performance) in the case of iid Gaussian noise is determined --- through the Q(.) function --- by the ratio of bit (or symbol) energy (not sample energy) to noise variance.