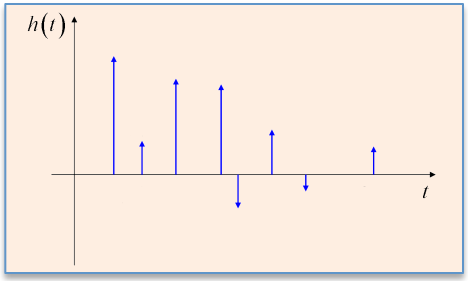
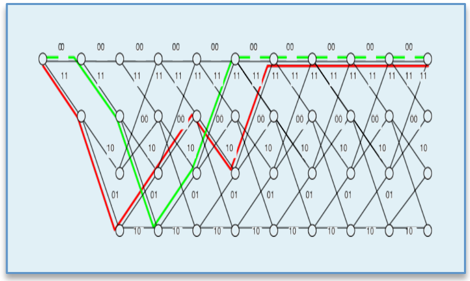
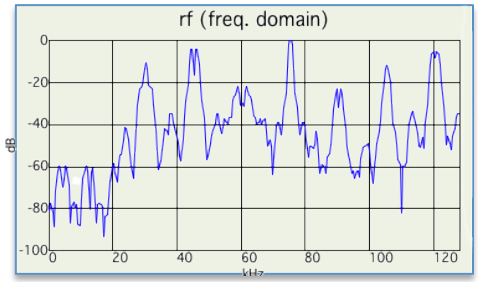
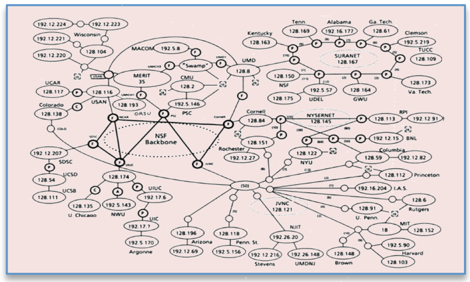








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2013
Lecture #10

- Modulation/demodulation
- Channel models
- Linear time-invariant (LTI) models
- Superposition

Averaging the Received Samples in a Bit Slot

- Average M (≤L) samples with independent noise components:

$$y[n] = \pm V_p + w[n] \quad so \quad \textbf{avg}\ \{y[n]\} = \pm V_p + \textbf{avg}\ \{w[n]\}$$

- **Claim: avg** {w[n]} is still Gaussian, still has mean 0, but its variance is now σ^2/M instead of σ^2 , so

  "SNR" is increased by a factor of M

- Same analysis and formulas as before, but now with

  $\sigma^2 \rightarrow \sigma^2/M$, or equivalently

  sample energy E_s ➔ bit (or symbol) energy $E_b = M.E_s$

Implications for Signaling Rate

- As the noise intensity increases, we need to increase the number of samples per bit (L), to get higher energy in the (M≤L) samples extracted from a bit interval, if we wish to maintain the same error performance. This results in a lower signaling rate.
  - e.g. Voyager 2 was transmitting at 115 **kilobits**/s when it was near Jupiter in 1979. When it was over 9 billion miles away, 13 light hours away from the sun, twice as far away from the sun as Pluto, it was transmitting at only 160 **bits**/s. The received power at the Deep Space Network antennas on earth when Voyager was near Neptune was on the order of 10^(-16) watts!! --- 20 billion times smaller than an ordinary digital watch consumes. The power now is estimated at less than 10^(-19) watts.

Flipped bits can have serious consequences!

- "On **November 30, 2006**, a telemetered command to *Voyager 2* was incorrectly decoded by its on-board computer—in a random error—as a command to turn on the electrical heaters of the spacecraft's magnetometer. These heaters remained turned on until December 4, 2006, and during that time, there was a resulting high temperature above 130 °C (266 °F), significantly higher than the magnetometers were designed to endure, and a sensor rotated away from the correct orientation. It has not been possible to fully diagnose and correct for the damage caused to the *Voyager 2's* magnetometer, although efforts to do so are proceeding."

- "On **April 22, 2010**, *Voyager 2* encountered scientific data format problems as reported by the Associated Press on May 6, 2010. On **May 17, 2010**, JPL engineers revealed that a flipped bit in an on-board computer had caused the issue, and scheduled a bit reset for May 19. On **May 23, 2010**, *Voyager 2* has resumed sending science data from deep space after engineers fixed the flipped bit."

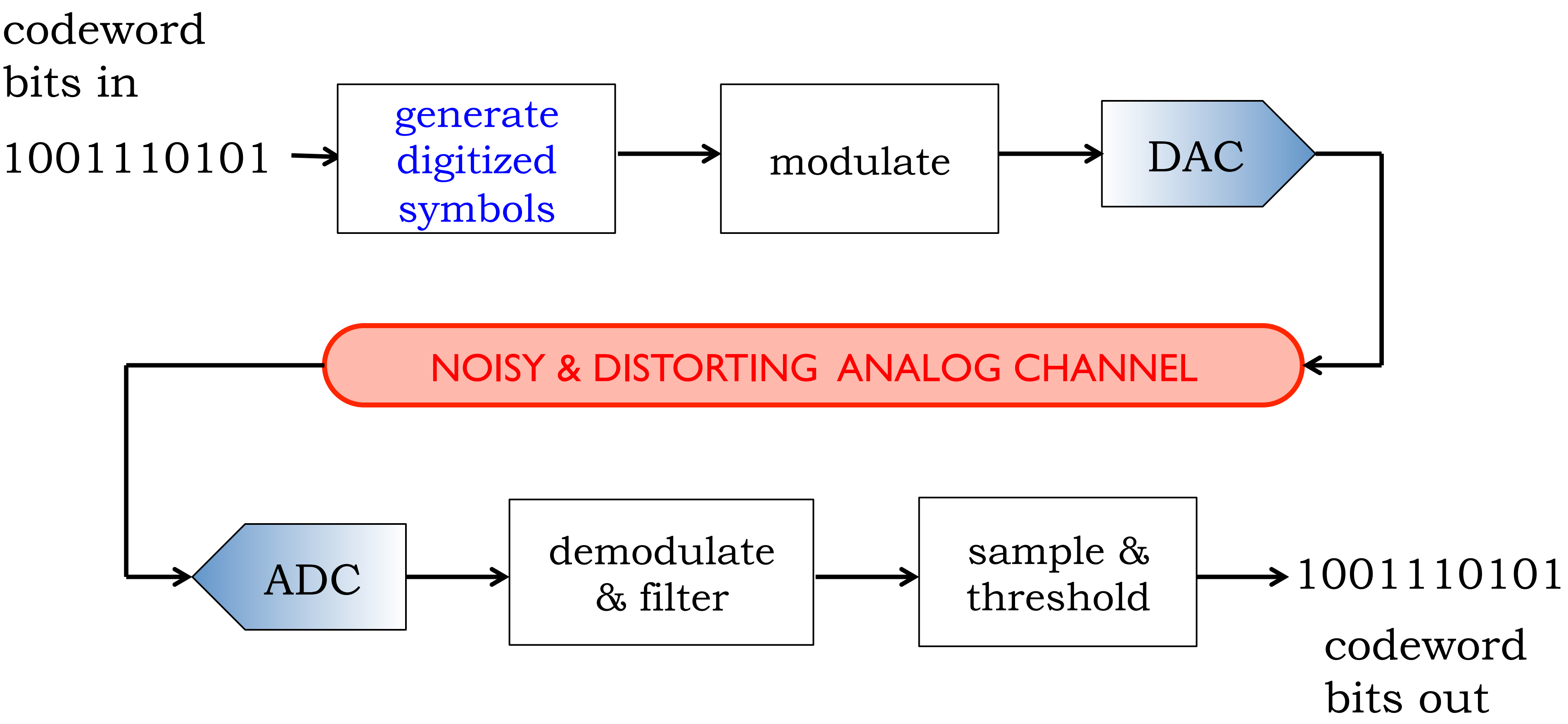
http://en.wikipedia.org/wiki/Voyager_2

The moral of the story is ···

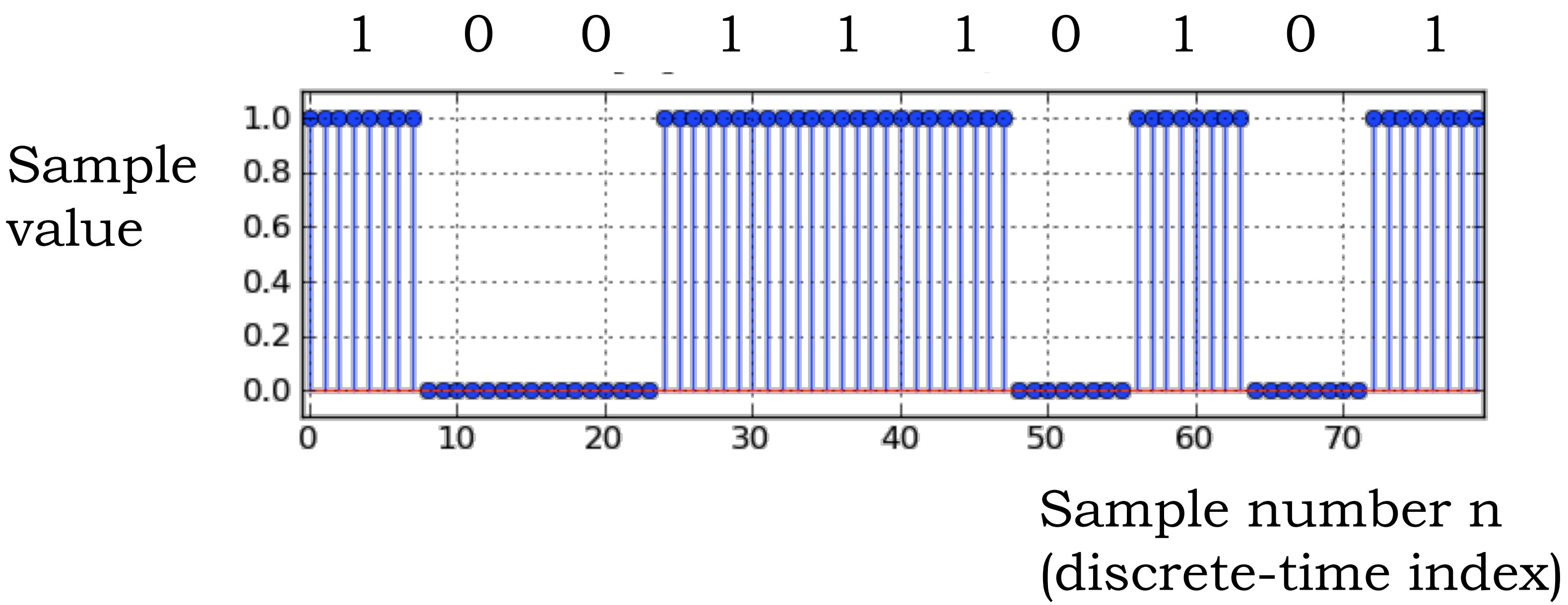
… if you're doing appropriate/optimal processing at the receiver, your effective SNR (and therefore your error performance) in the case of iid Gaussian noise is determined --- through the Q(.) function --- by the ratio of bit (or symbol) energy (not sample energy) to noise variance.

Back to noise-free distortion …

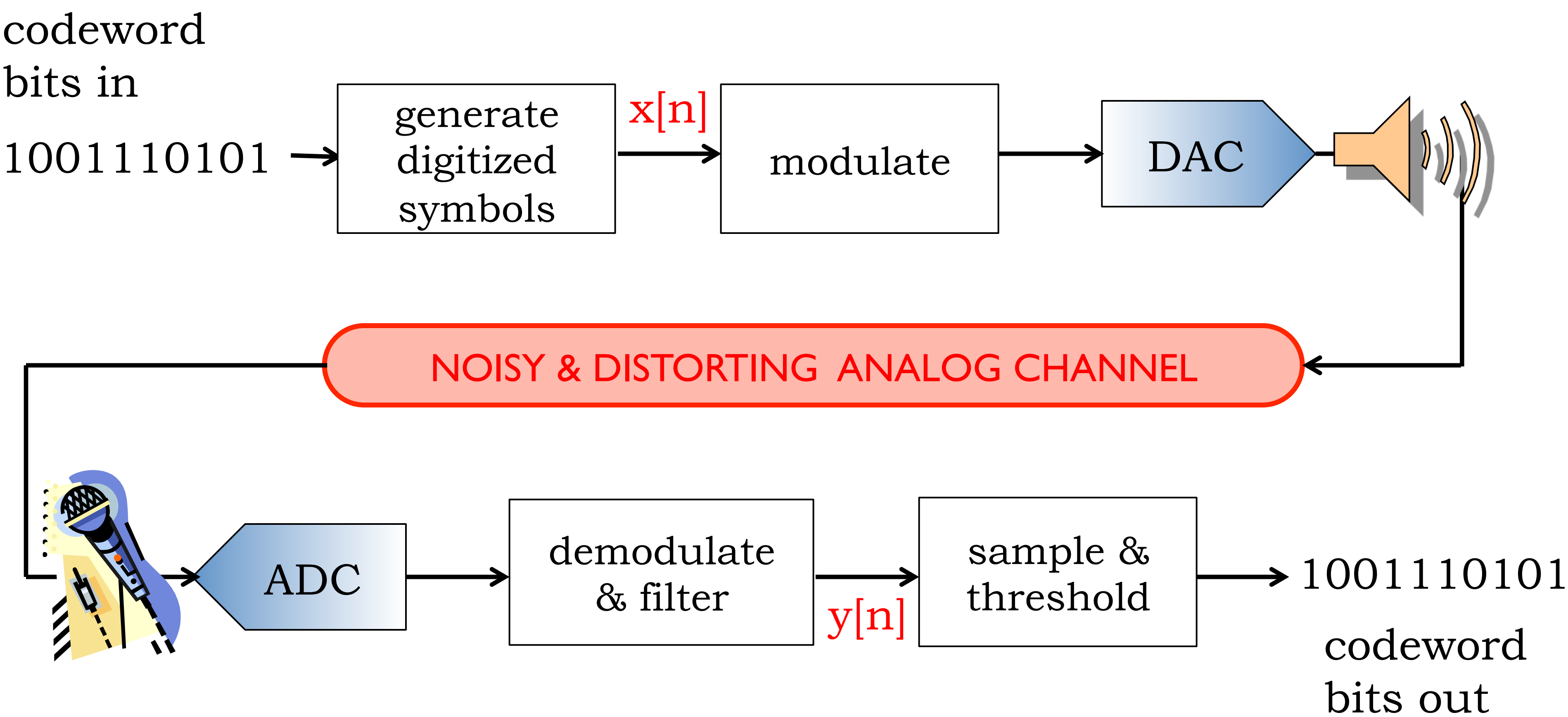
A Single Link



Digitized Symbols



From Baseband to Modulated Signal, and Back



Modulation (at the Transmitter)

Adapts the digitized signal x[n] to the characteristics of the channel.

e.g., Acoustic channel from laptop speaker to microphone is *not* well suited to transmitting *constant* levels v_0 and v_1 to represent 0 and 1. So instead transmit **sinusoidal** pressure-wave signals proportional to speaker voltages

$$v_0 \cos(2\pi f_c t) \quad \text{and} \quad v_1 \cos(2\pi f_c t)$$

where f_c is the *carrier frequency* (e.g., 2kHz; wavelength at 340 m/s = 17cm, comparable with speaker dimensions) and

$$v_0 = 0 \qquad v_1 = V > 0$$

(on-off or *amplitude* keying)

or alternatively

$$v_0 = -V \qquad v_1 = V > 0$$

(bipolar or *phase-shift* keying)

Could also key the *frequency*.

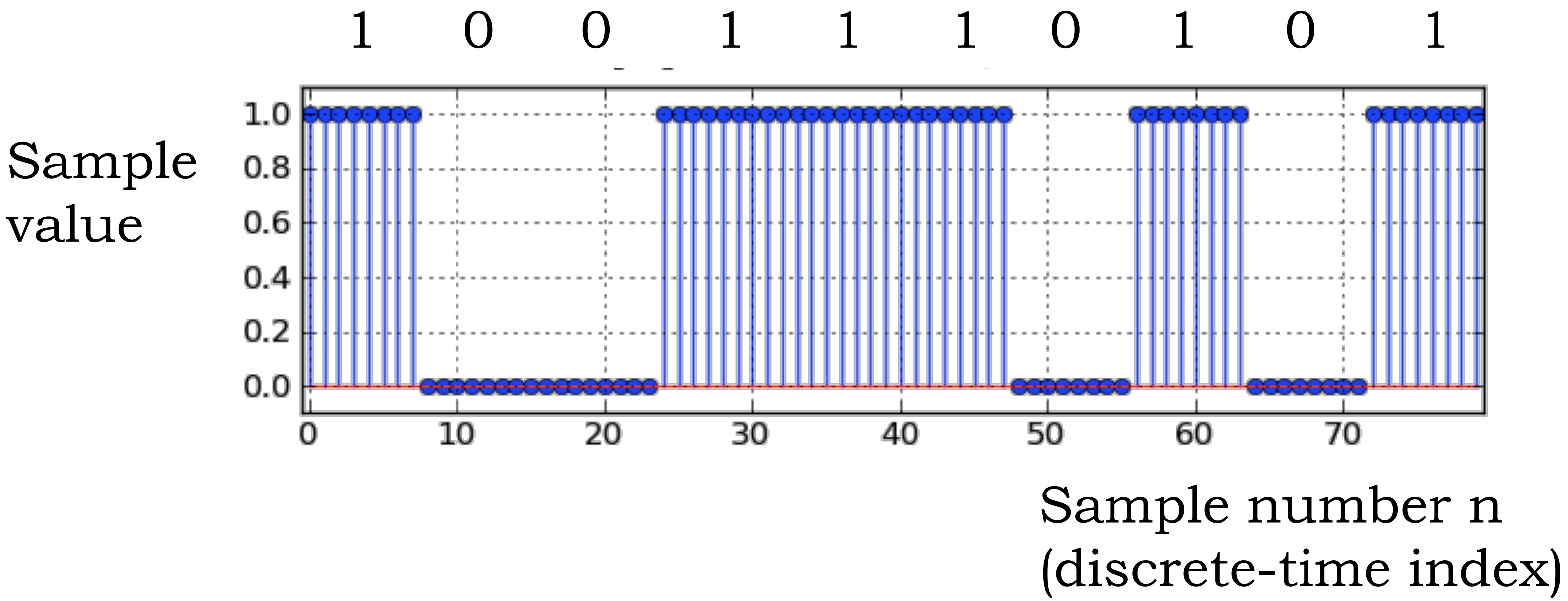
From Brant Rock tower, radio age was sparked
By Carolyn Y. Johnson, Globe Staff | July 30, 2006

MARSHFIELD, MA -- A century ago, radio pioneer Reginald A. Fessenden* used a massive 420-foot radio tower that dwarfed Brant Rock to send voice and music to ships along the Atlantic coast, in what has become known as the world's first voice radio broadcast. This week, Marshfield will lay claim to its little-known radio heritage with a three-day extravaganza to celebrate the feat -- including pilgrimages to the base of the long-dismantled tower, a cocktail to be named the Fessenden Fizz, and a dramatic reenactment of the historic moment, called ``Miracle at Brant Rock."

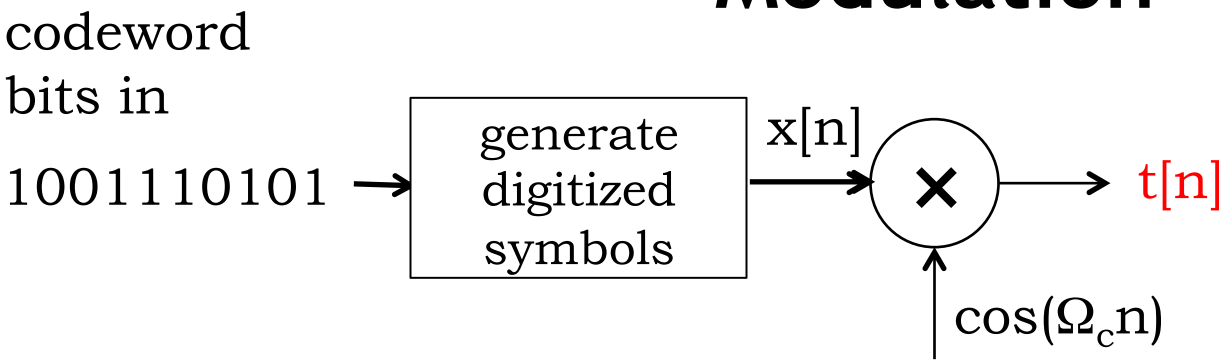
Amplitude Modulation (AM)


Wireless Station, Brant Rock, Mass.

Digitized Symbols

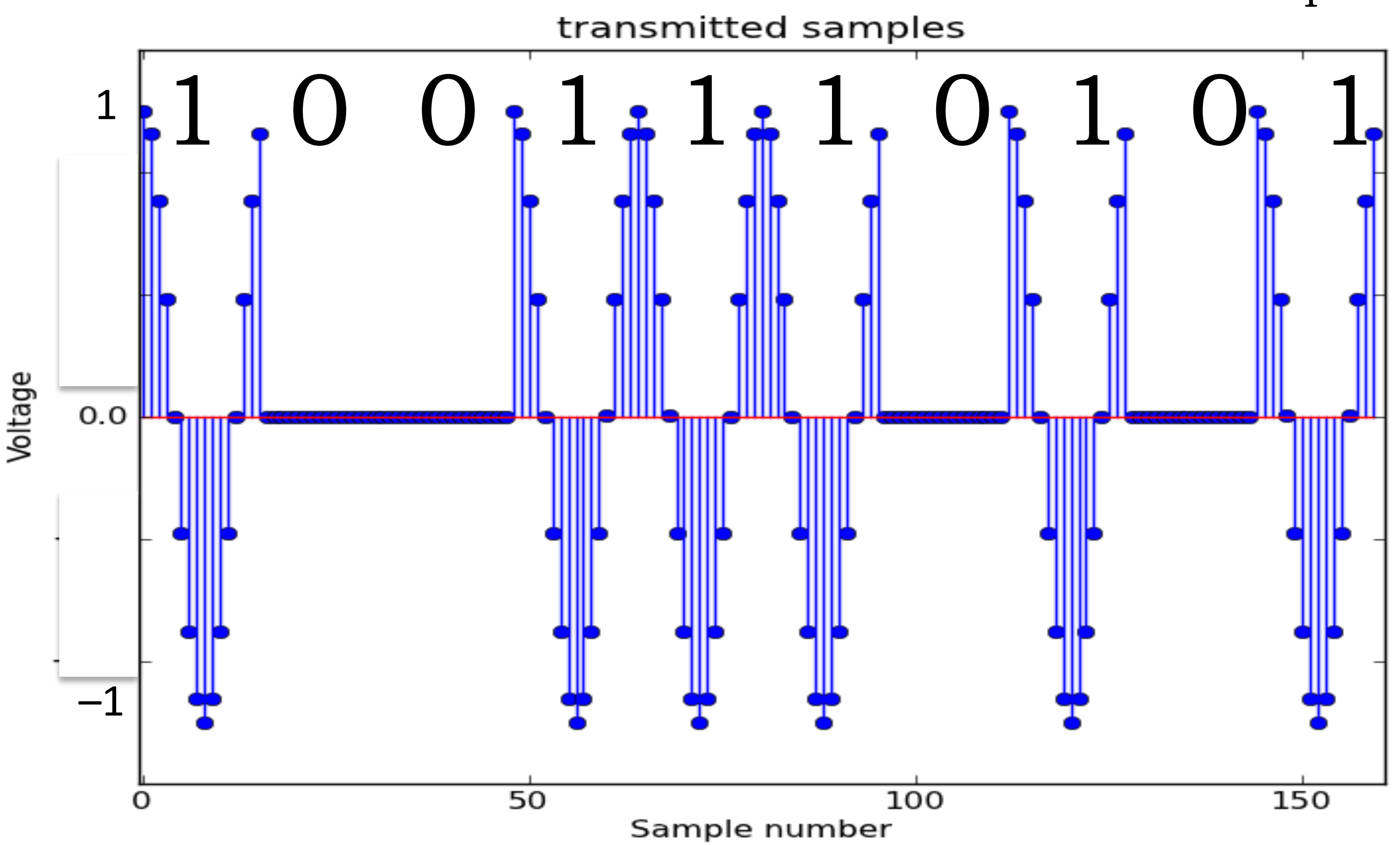


Modulation



$\Omega_c = 2\pi/16$

16 samples per cycle



Ideas for Demodulation

- For on-off keying, it suffices to detect when there's signal and when there isn't, since we're only trying to distinguish

$$v_0 = 0 \qquad v_1 = V > 0$$

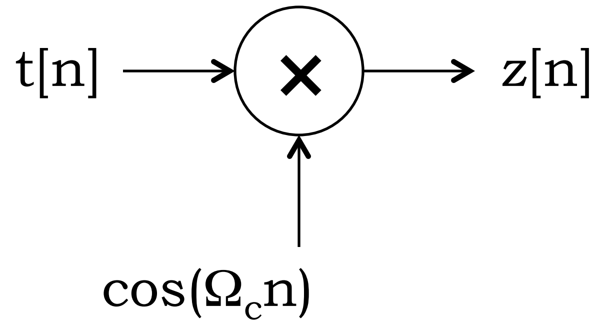
Many ways to do that, e.g., take absolute value and then local average over half-period of carrier

- For bipolar keying, we need the sign:

$$v_0 = -V \qquad v_1 = V > 0$$

Demodulation

Assuming no distortion or noise on channel, so what was transmitted is received



Why not just divide t[n] by $\cos(\Omega_c n)$?

$$z[n] = t[n]\cos(\Omega_c n)$$

$$z[n] = x[n]\cos(\Omega_c n)\cos(\Omega_c n)$$

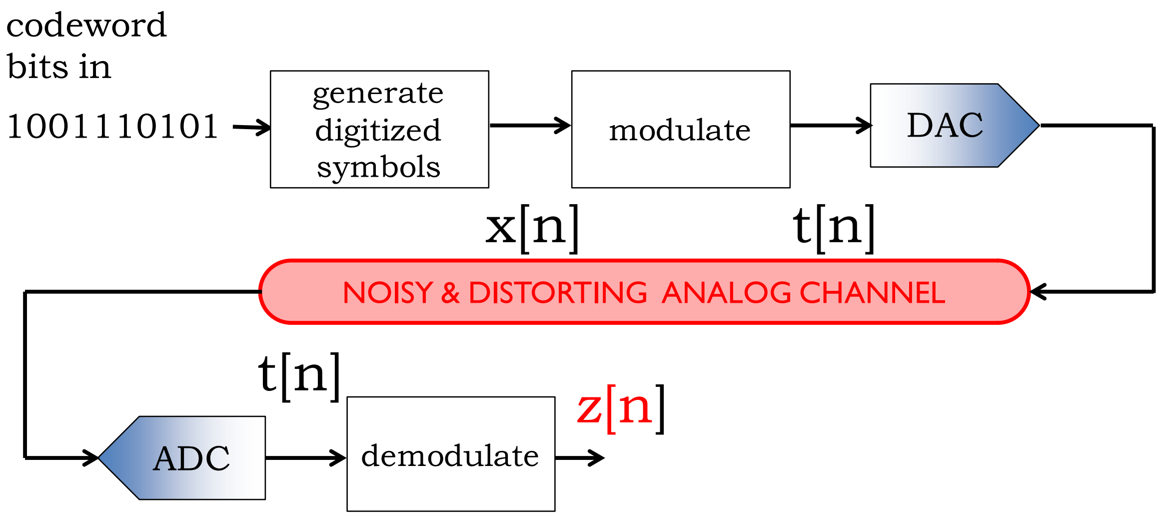
'Heterodyning' - invented by Fessenden

$$z[n] = 0.5x[n](1+\cos(2\Omega_c n))$$

$$z[n] = 0.5x[n] + 0.5x[n]\cos(2\Omega_c n)$$

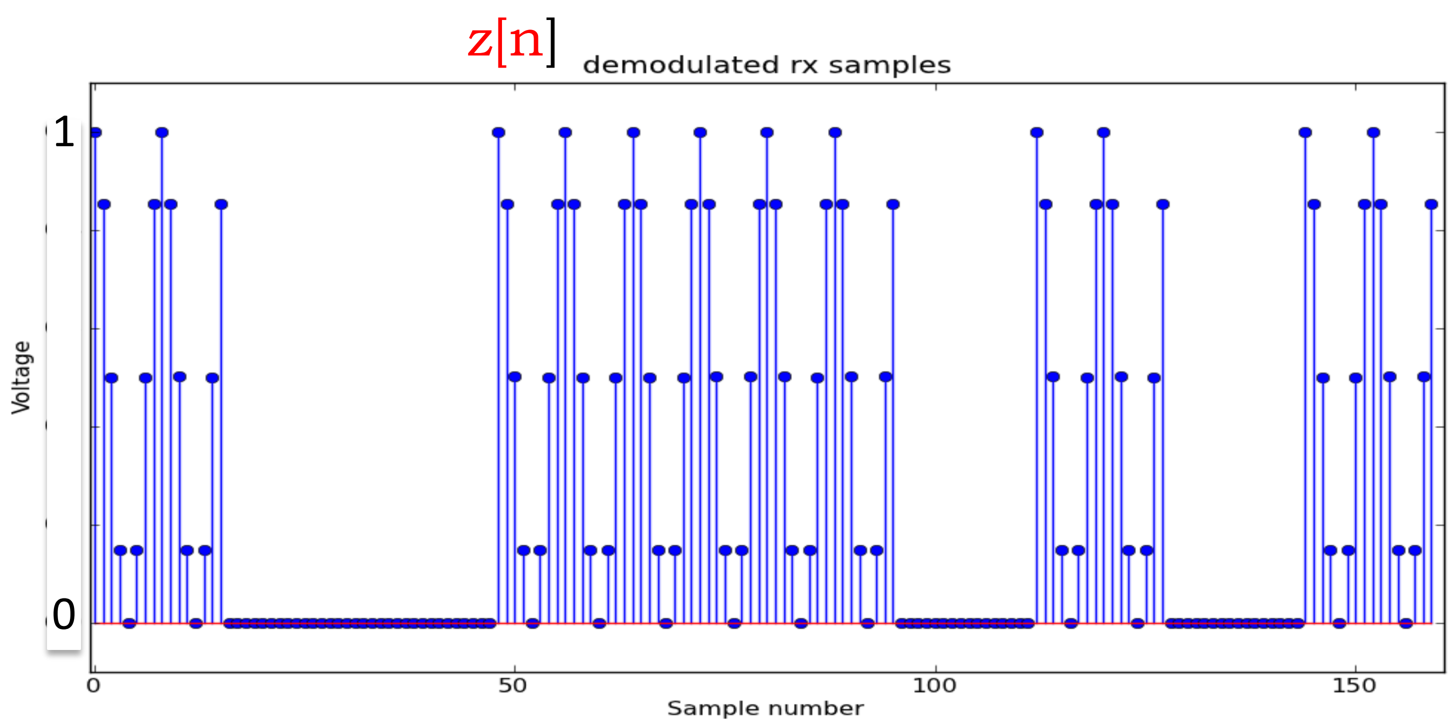
Extract this!

Demodulation

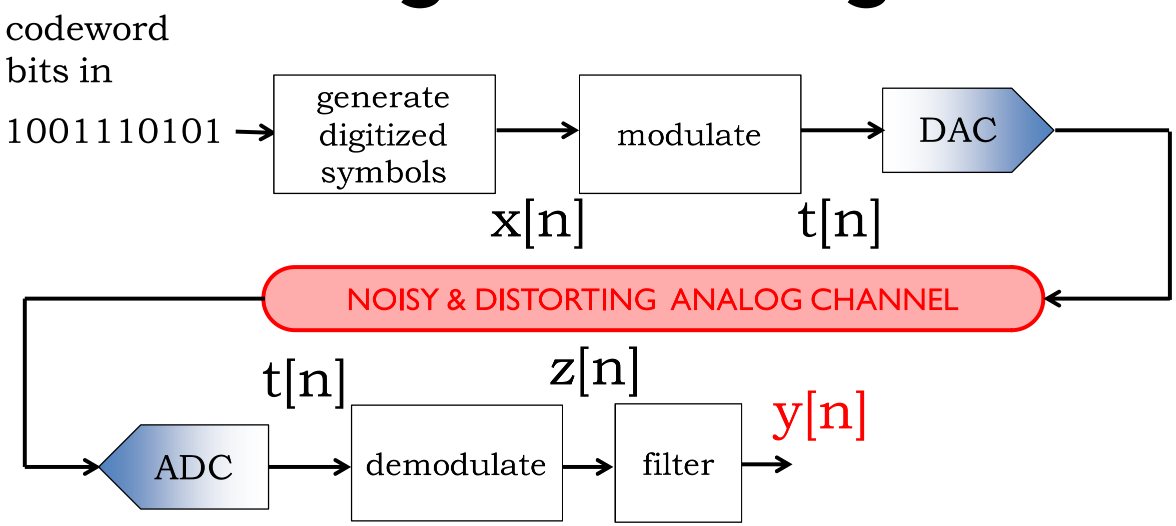


$\Omega_c = 2\pi / 16$

16 samples per cycle

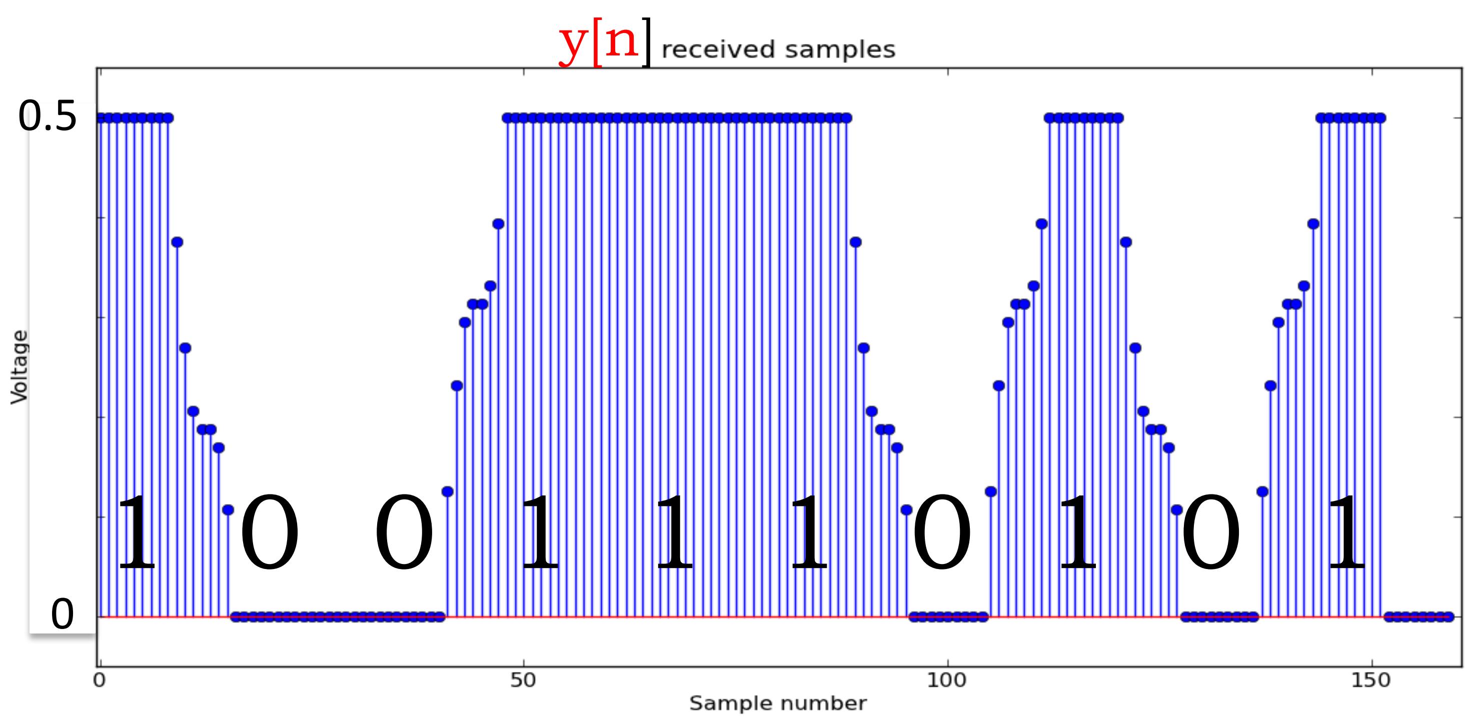


Filtering: Removing the $2\Omega_c$ component

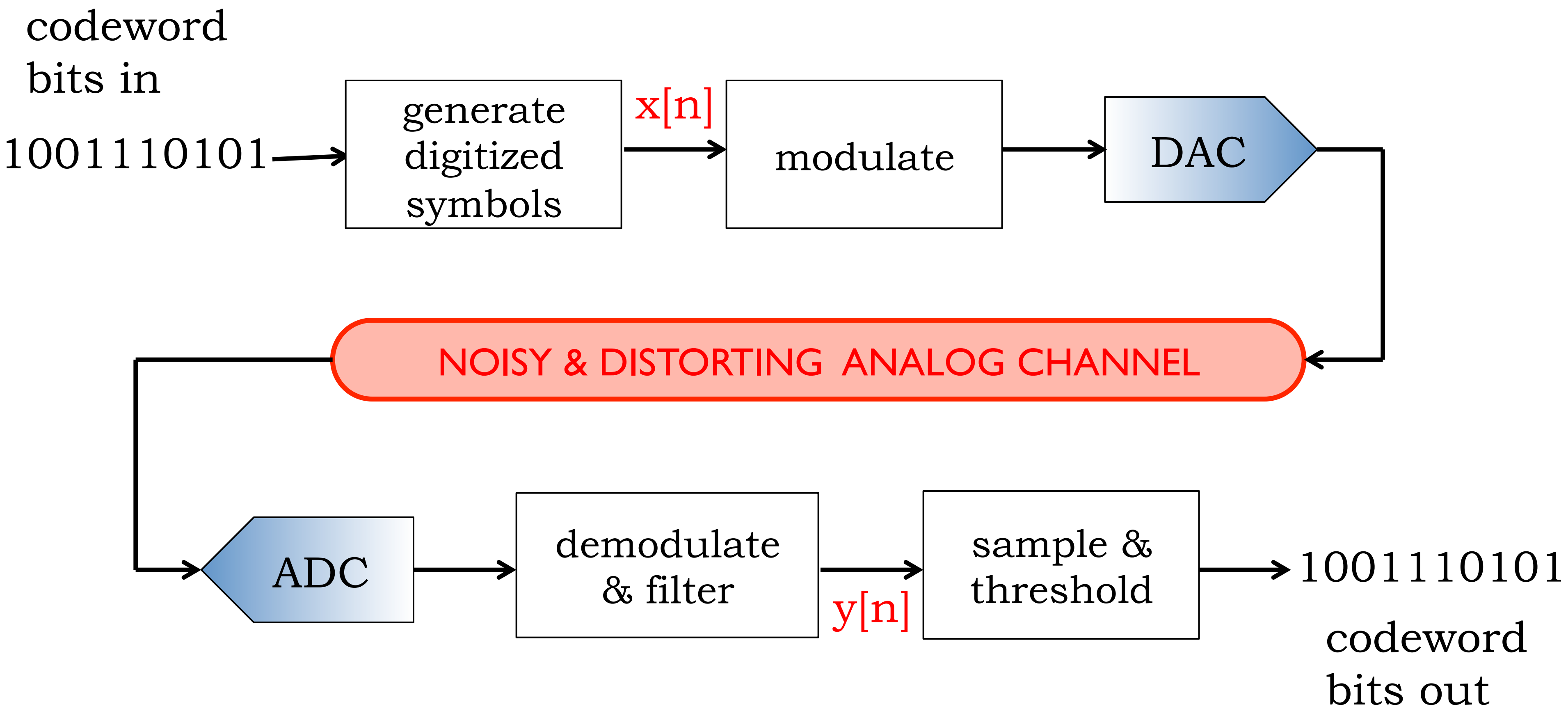


$\Omega_c = 2\pi/16$

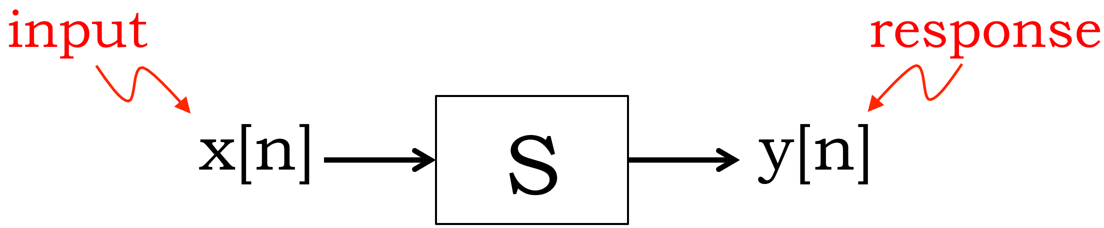
16 samples per cycle



Modeling The Baseband Channel



The Baseband** Channel



The sequence of *output* values y[.] is the *response* of system S to the *input* sequence x[.]. The above picture is a snapshot at a particular time n.

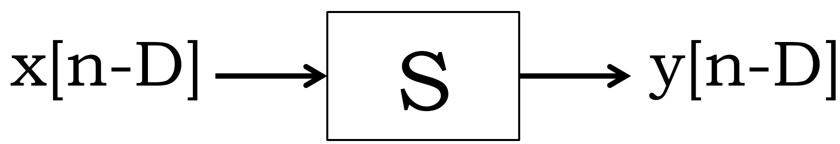
The system is causal if y[k] depends only on x[j] for j≤k

**From before the modulator till after the demodulator & filter

Time Invariant Systems

Let y[.] be the response of S to input x[.].

If for all possible sequences x[n] (with n ranging over all integer values) and an arbitrary integer D:



then system S is said to be *time invariant* (TI).

A time shift D in the input sequence to S results in an identical time shift of the output sequence.

Linear Systems

Let $y_1[.]$ be the response of S to an arbitrary input $x_1[.]$ and $y_2[.]$ be the response to an arbitrary $x_2[.]$.

If, for arbitrary scalar coefficients *a* and *b*, we have:

$$ax_1[n]+bx_2[n] \longrightarrow \boxed{\;S\;} \longrightarrow ay_1[n]+by_2[n]$$

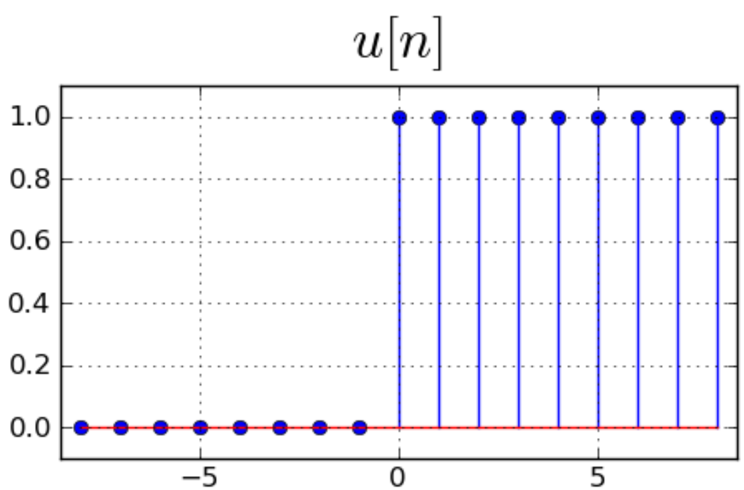
then system S is said to be *linear*. If the input is the weighted sum of several signals, the response is the *superposition* (i.e., same weighted sum) of the response to those signals.

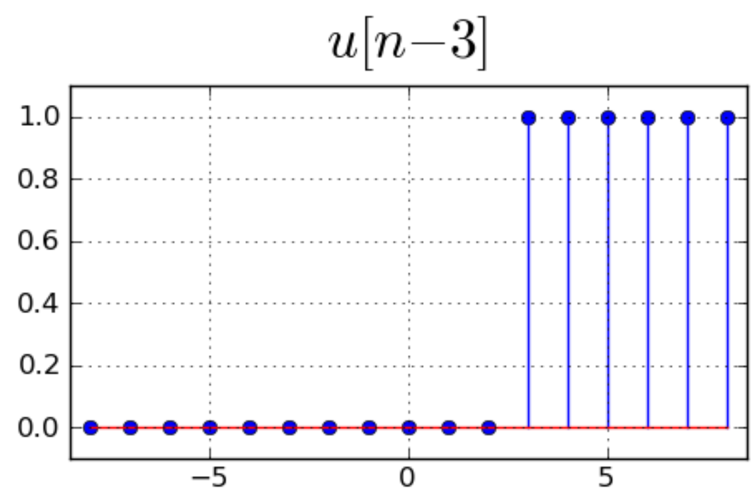
One key consequence: If the input is identically 0 for a linear system, the output must also be identically 0.

Unit Step

A simple but useful discrete-time signal is the *unit step* signal or function, u[n], defined as

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

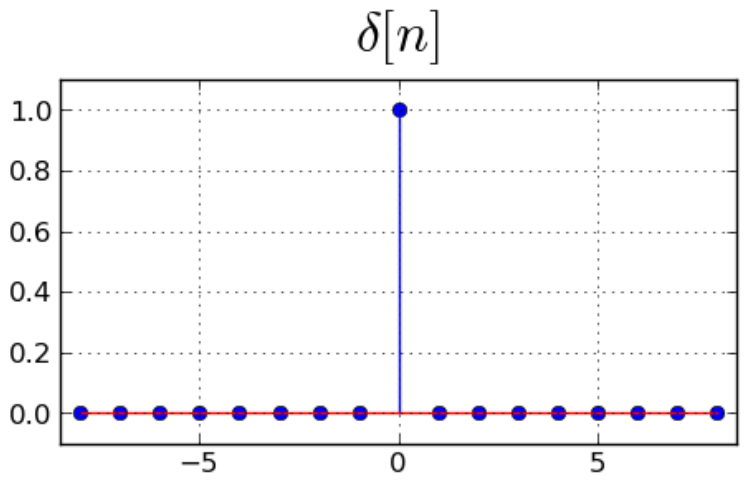


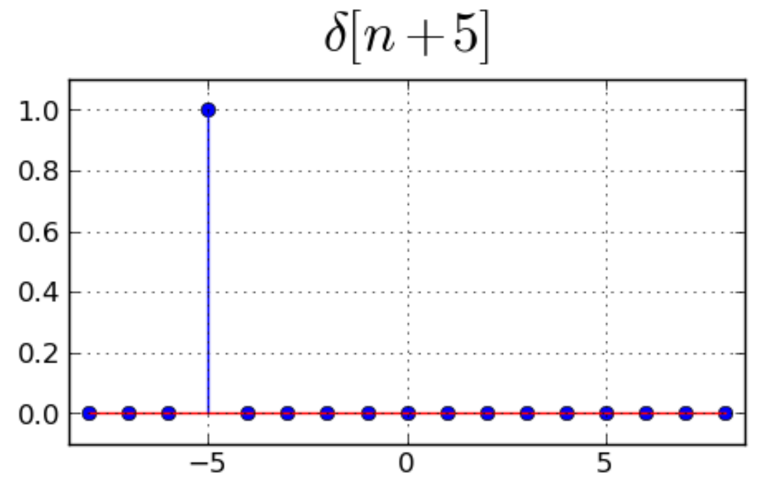


Unit Sample

Another simple but useful discrete-time signal is the *unit sample* signal or function, δ[n], defined as

$$\delta[n] = u[n] - u[n-1] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$





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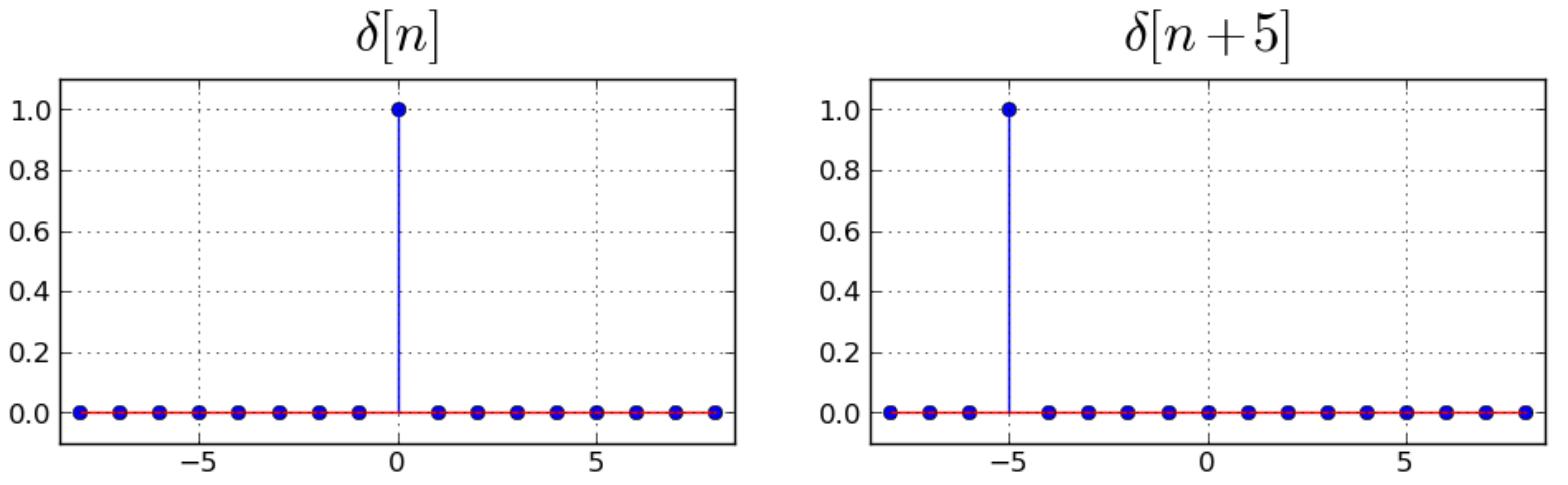
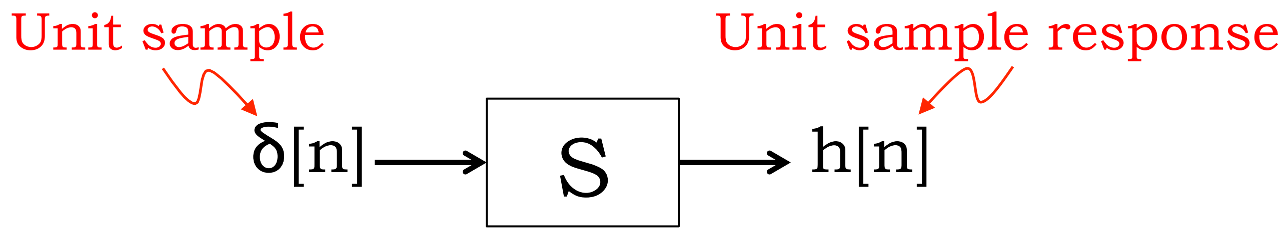


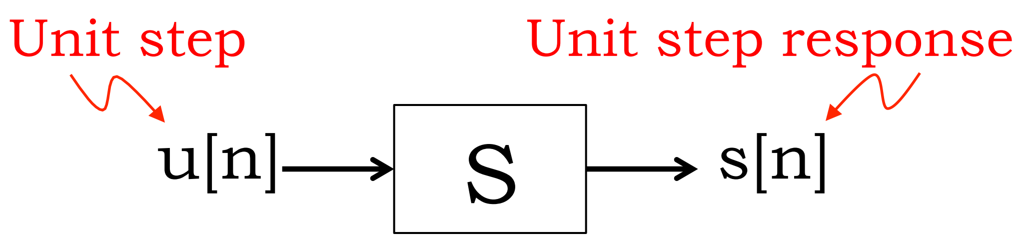
Note that standard algebraic operations on signals (e.g. subtraction, addition, scaling by a constant) are defined in the obvious way, instant by instant.

Unit Sample and Unit Step Responses

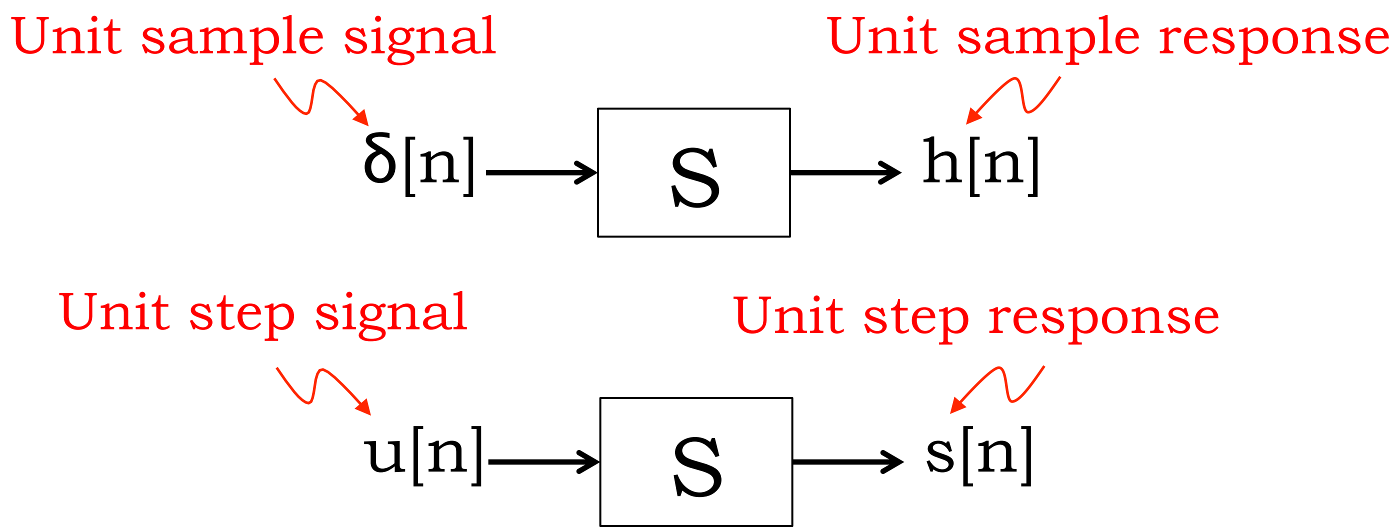


The *unit sample response* of a system S is the response of the system to the unit sample input. We will always denote the unit sample response as h[n].

Similarly, the *unit step response* s[n]:



Relating h[n] and s[n] of an LTI System



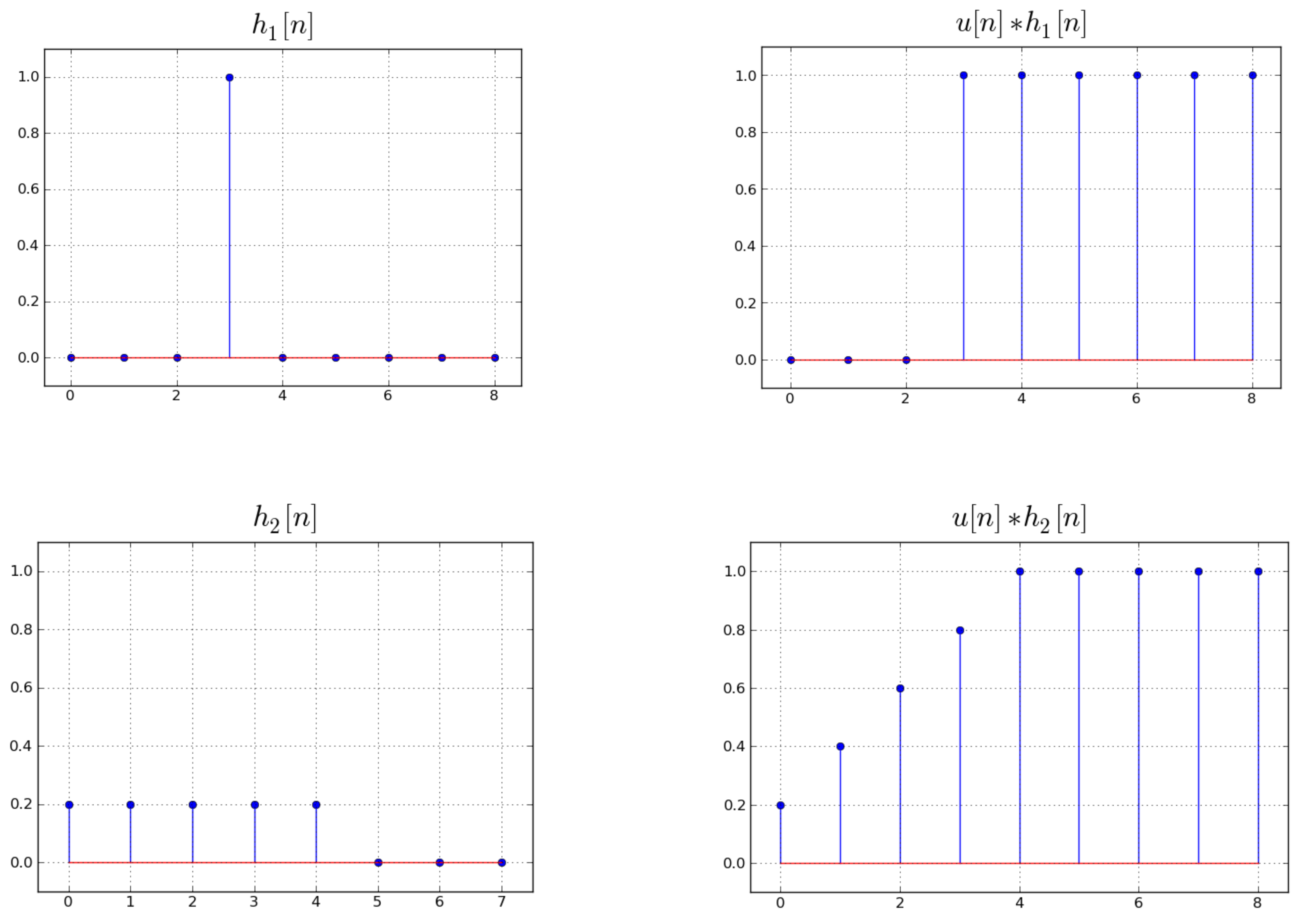
$$\delta[n] = u[n] - u[n-1] \quad \Rightarrow \quad h[n] = s[n] - s[n-1]$$

from which it follows that

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

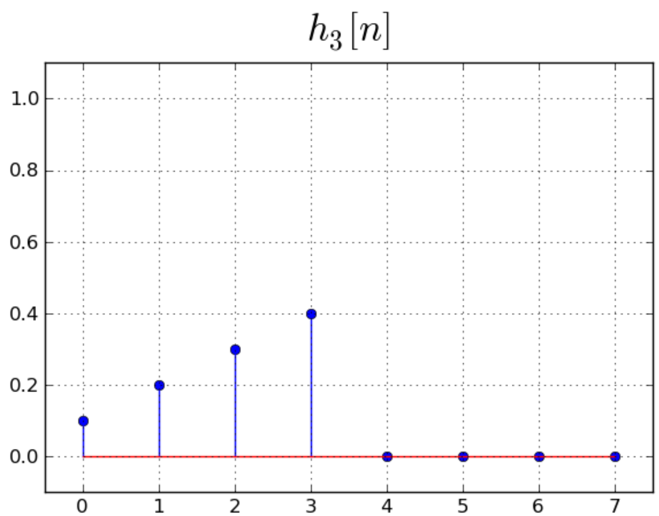
(assuming $s[-\infty] = 0$, e.g., a causal LTI system; more generally, a "right-sided" unit sample response)

h[n] s[n]

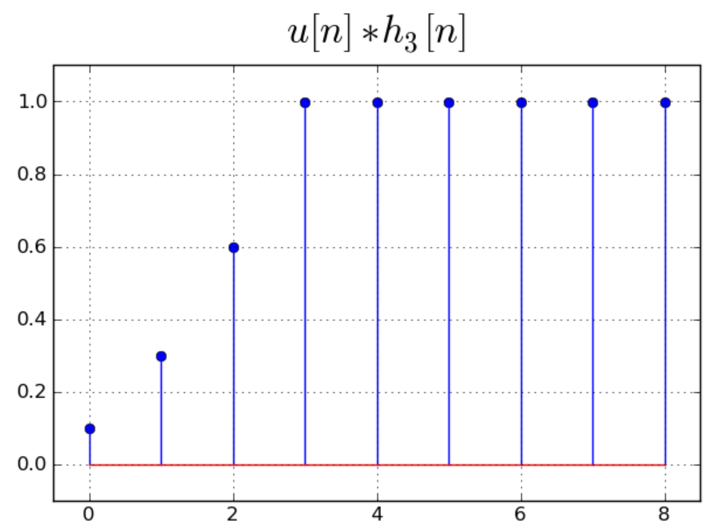


h[n]

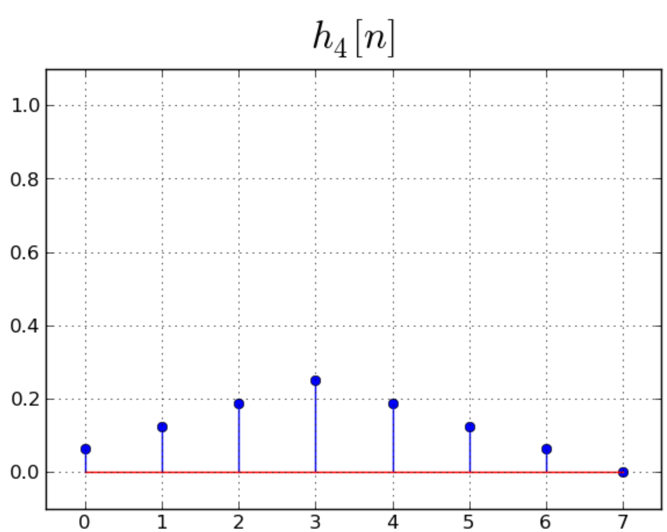
s[n]



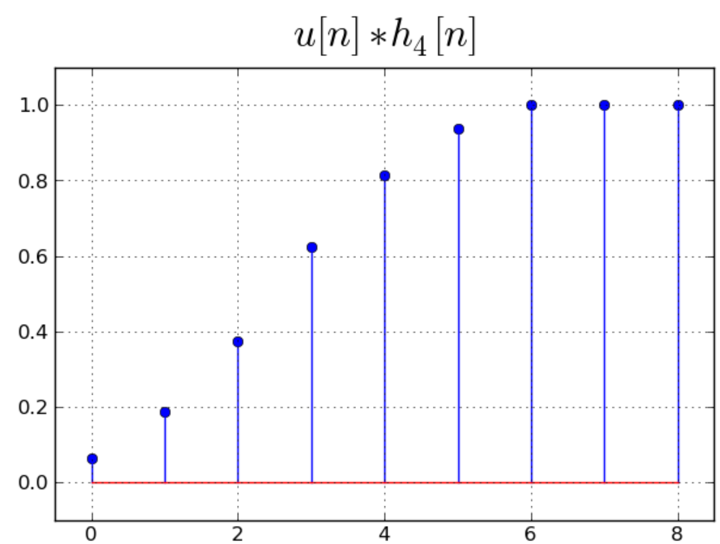
$h_3[n]$
1.0
0.8
0.6
0.4
0.2
0.0
0 1 2 3 4 5 6 7



$u[n] * h_3[n]$
1.0
0.8
0.6
0.4
0.2
0.0
0 2 4 6 8

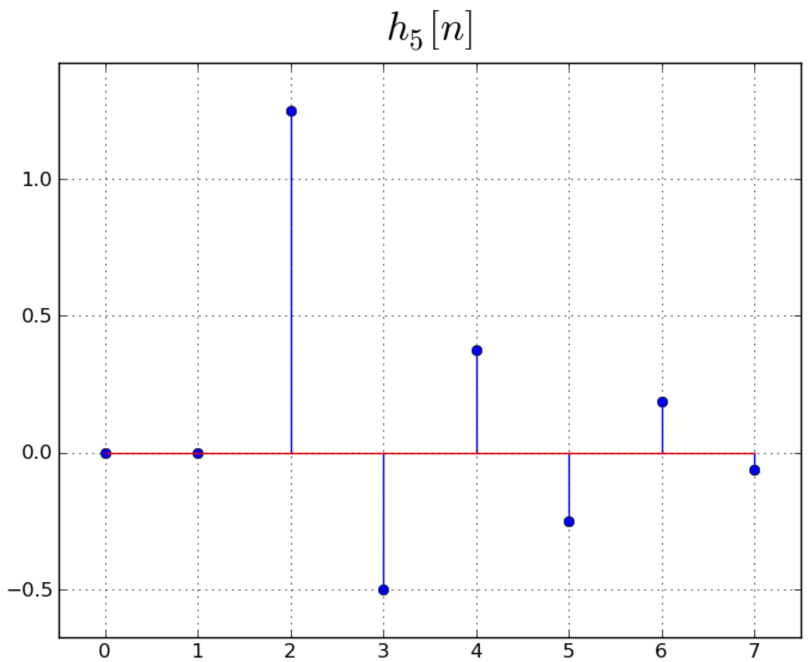


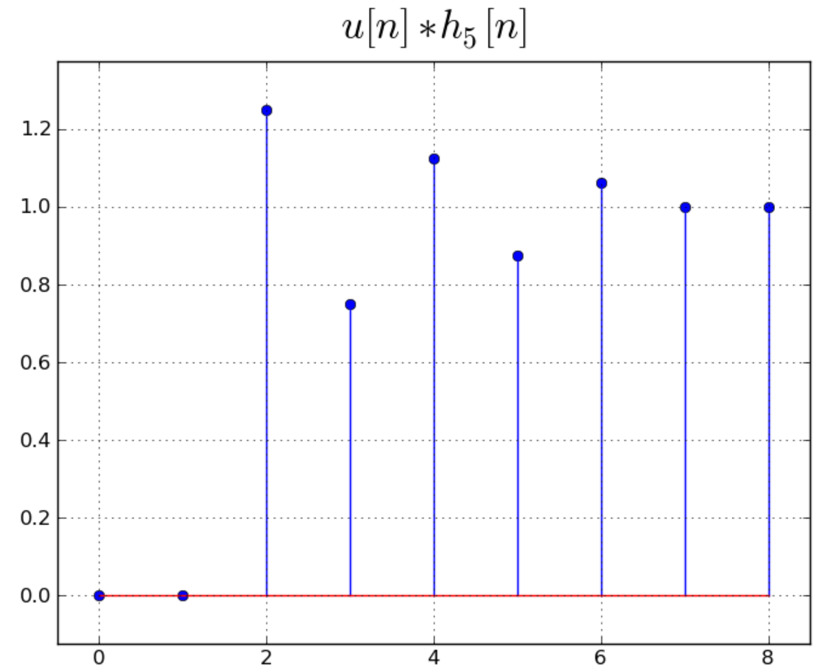
$h_4[n]$
1.0
0.8
0.6
0.4
0.2
0.0
0 1 2 3 4 5 6 7

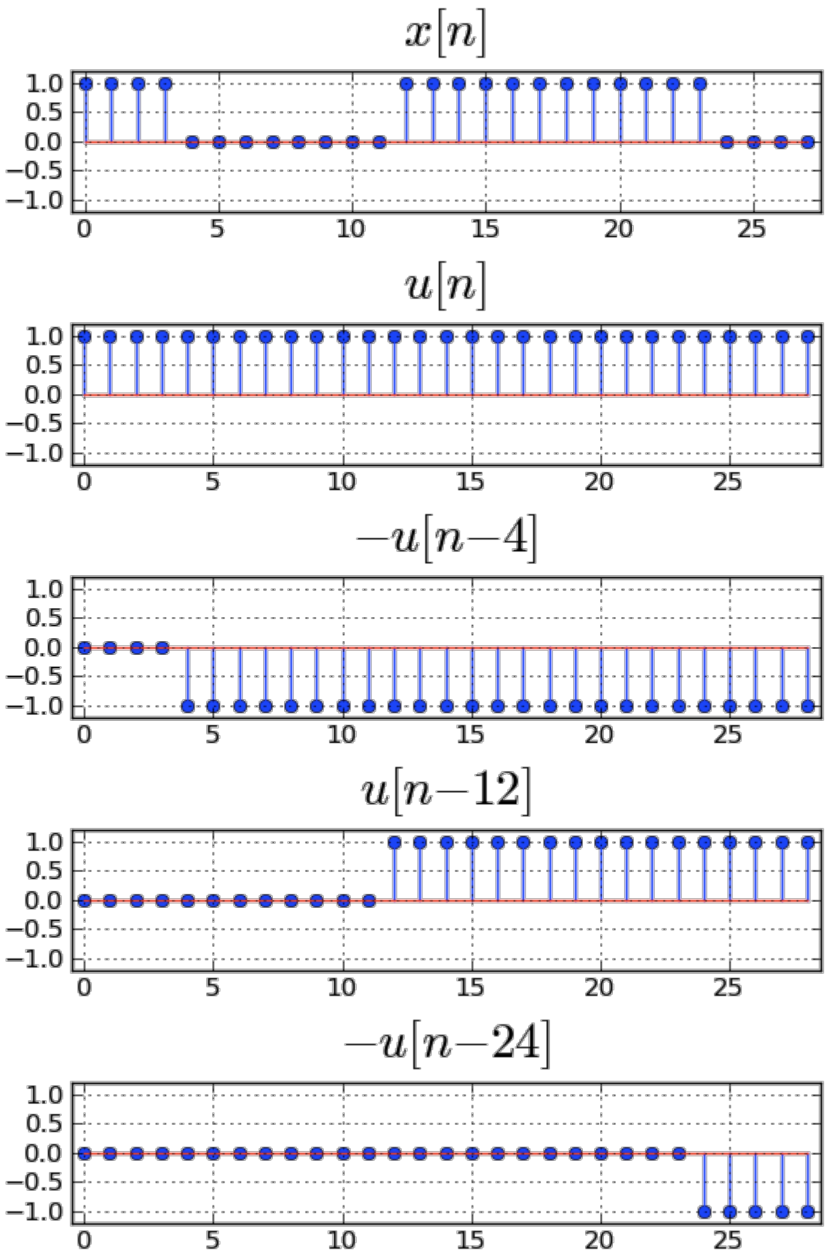


$u[n] * h_4[n]$
1.0
0.8
0.6
0.4
0.2
0.0
0 2 4 6 8

h[n] s[n]







Unit Step Decomposition

"Rectangular-wave" digital signaling waveforms, of the sort we have been considering, are easily decomposed into time-shifted, scaled unit steps --- each transition corresponds to another shifted, scaled unit step.

e.g., if x[n] is the transmission of 1001110 using 4 samples/bit:

$$
\begin{aligned}
x[n] \\
&= u[n] \\
&\quad - u[n-4] \\
&\quad + u[n-12] \\
&\quad - u[n-24]
\end{aligned}
$$

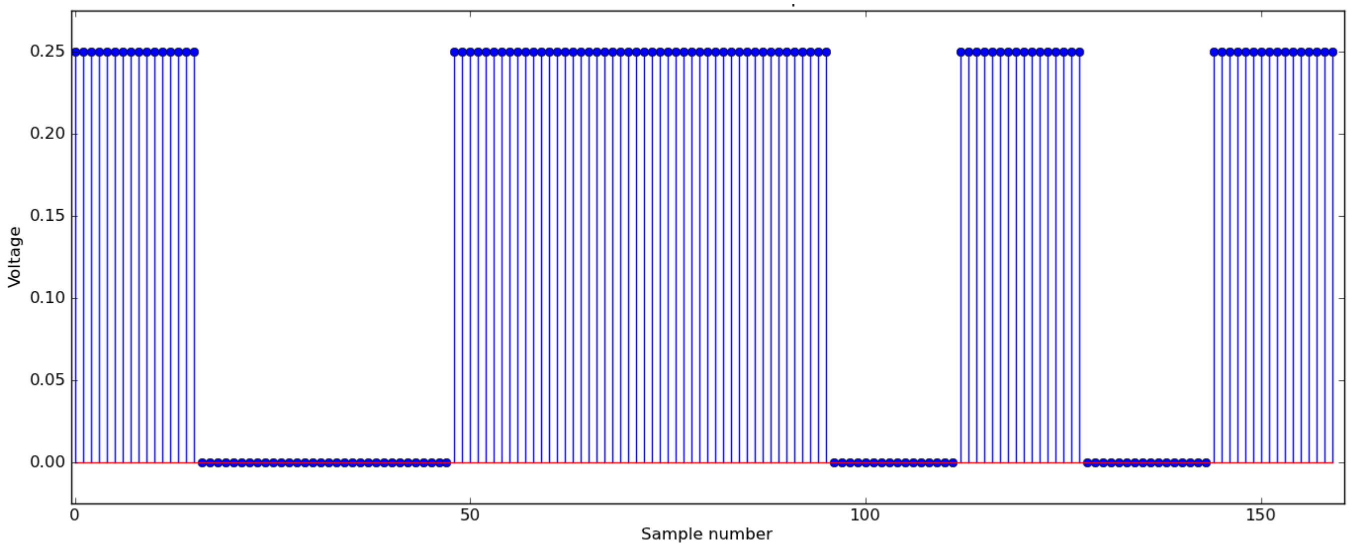
… so the corresponding response is

$$x[n] = u[n] - u[n-4] + u[n-12] - u[n-24]$$

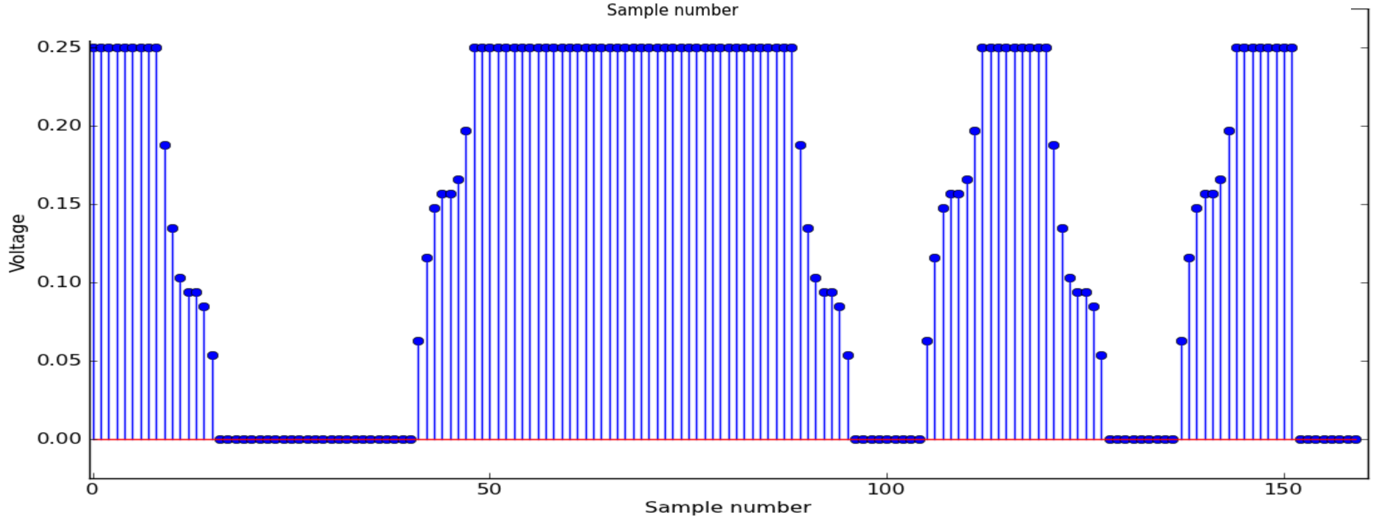
$$y[n] = s[n] - s[n-4] + s[n-12] - s[n-24]$$

Note how we have invoked linearity and time invariance!

Example



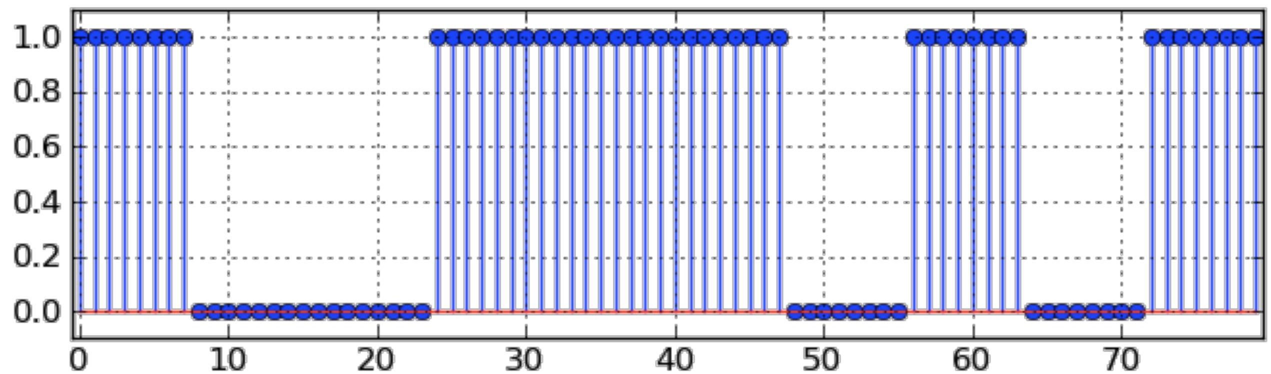
0.25
0.20
0.15
0.10
0.05
0.00
Voltage
0
50
100
150
Sample number



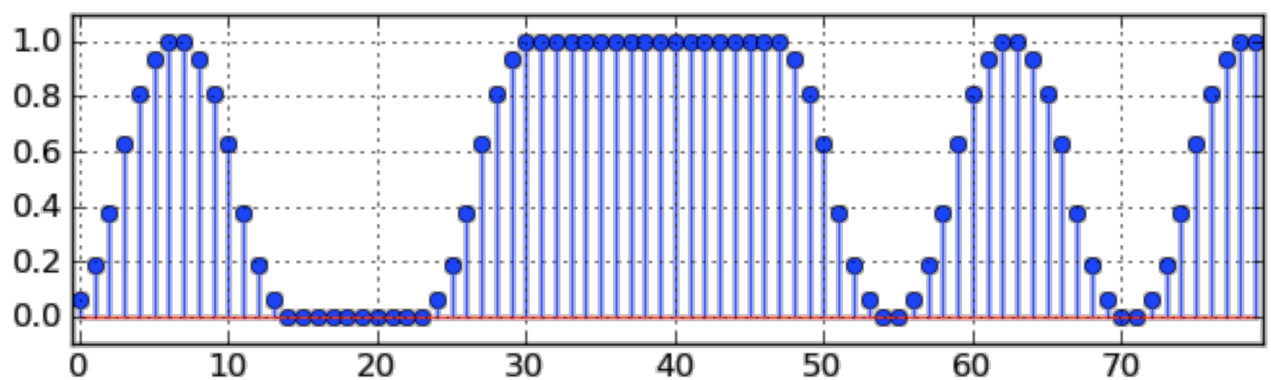
0.25
0.20
0.15
0.10
0.05
0.00
Voltage
0
50
100
150
Sample number

Transmission Over a Channel

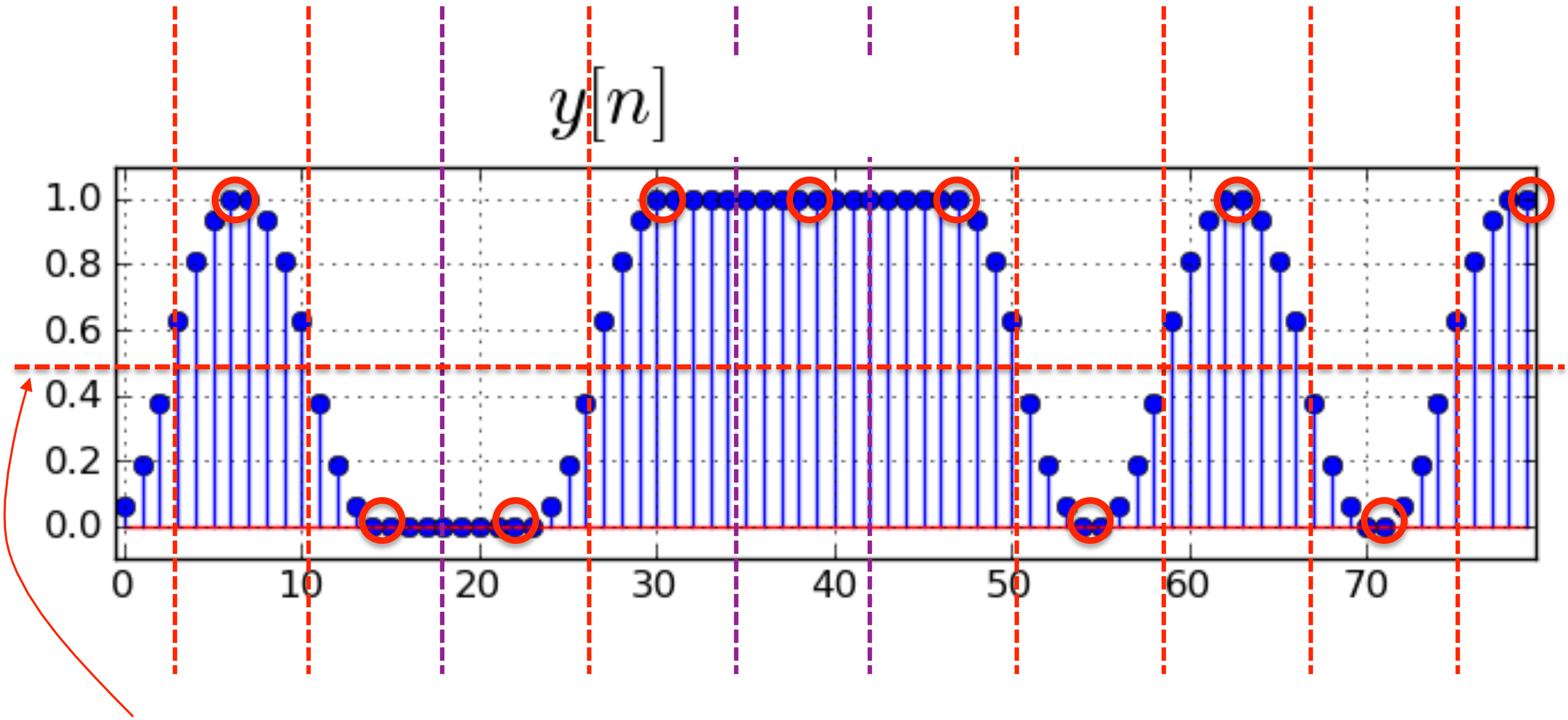
$x[n]$ at 8 samples/bit



$y[n]$

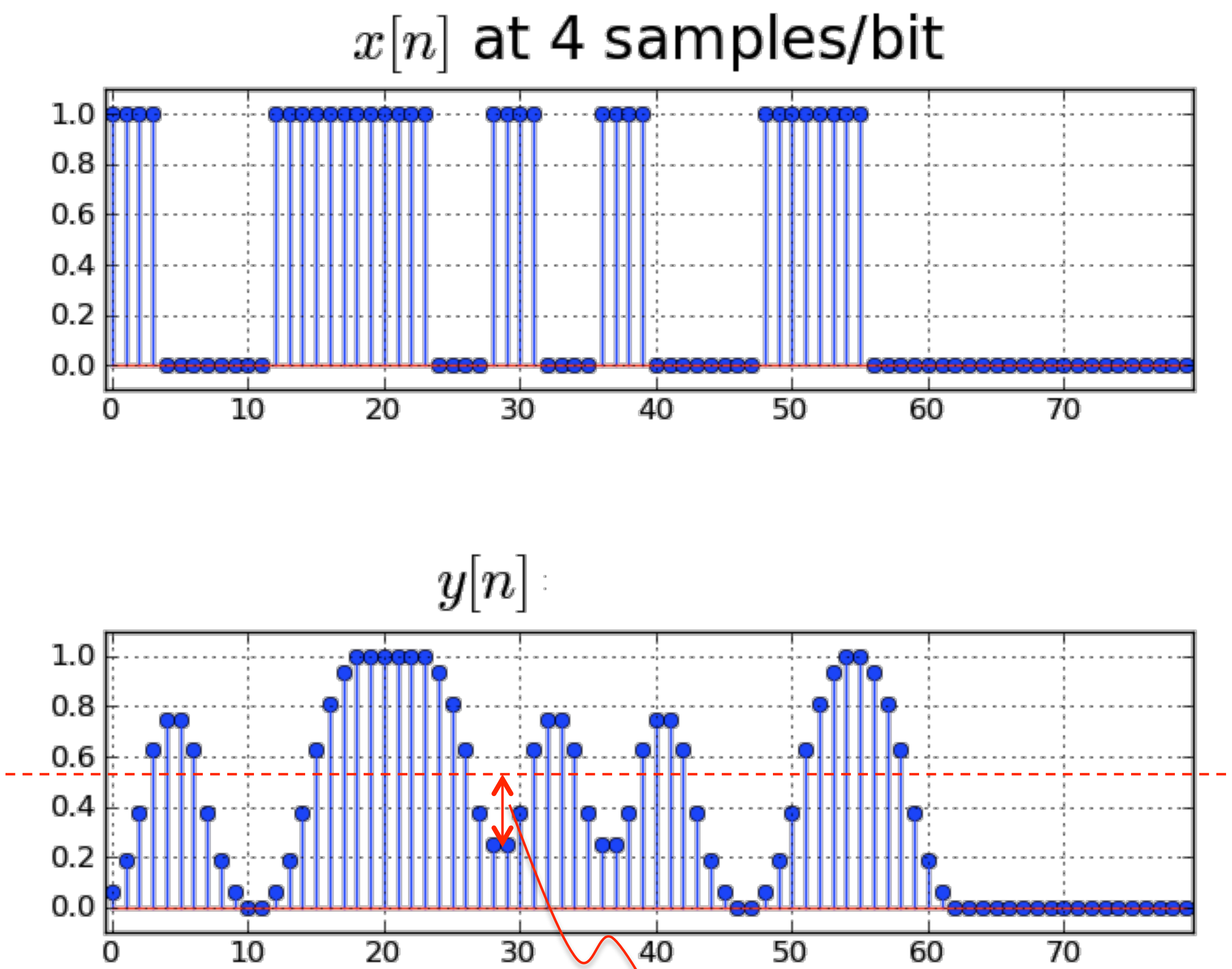


Receiving the Response



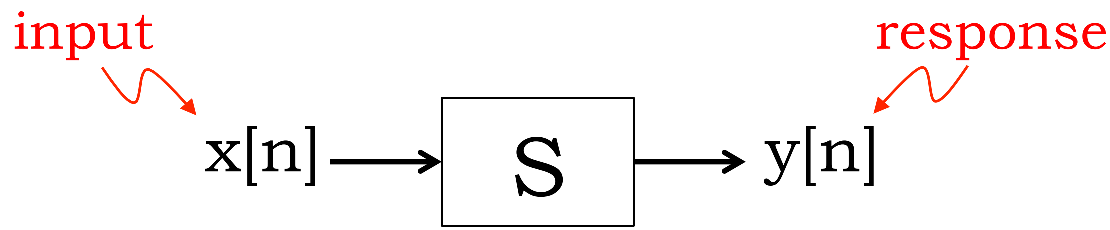
Digitization threshold = 0.5V

Faster Transmission

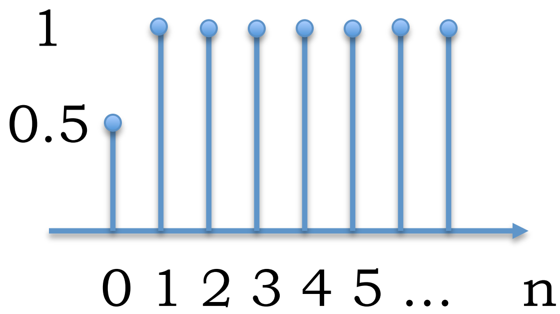


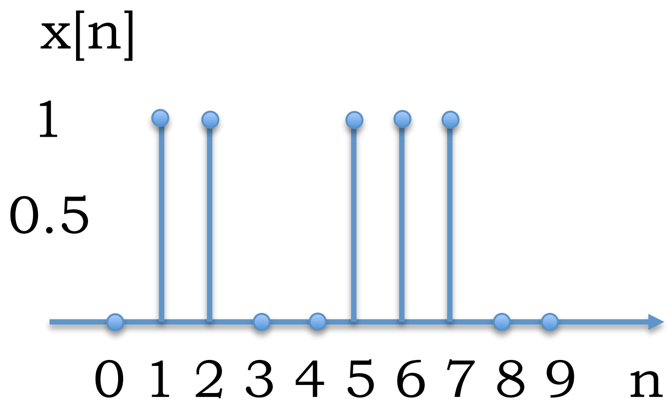
Noise margin? $0.5 - y[28]$

Spot Quiz



Unit step response: s[n]





Find y[n]:

1. Write x[n] as a function of unit steps

2. Write y[n] as a function of unit step responses

3. Draw y[n]