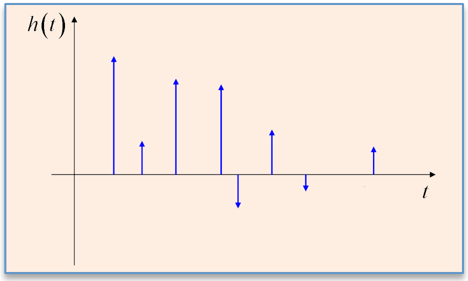
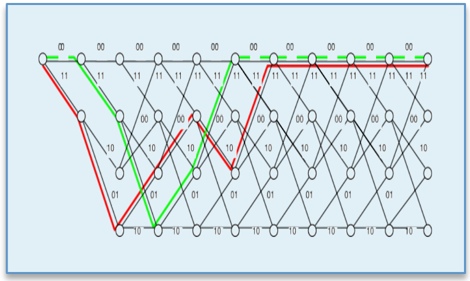
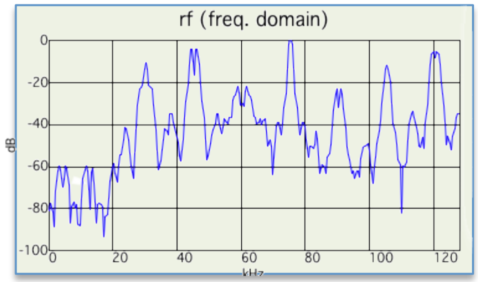
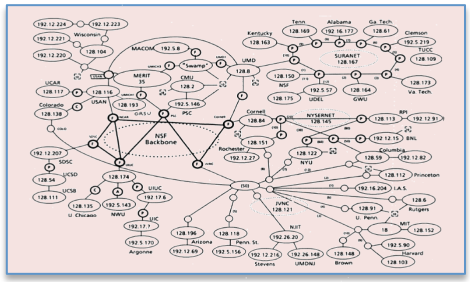








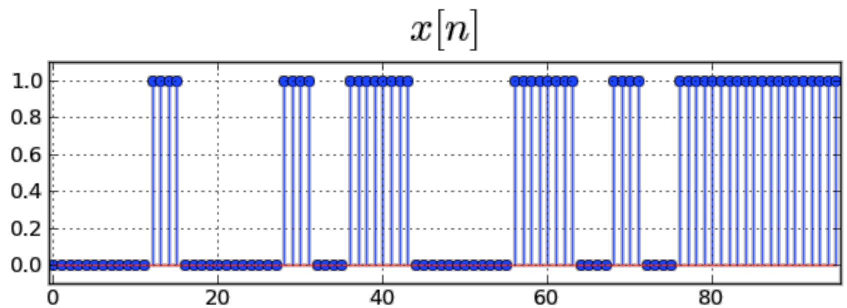
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

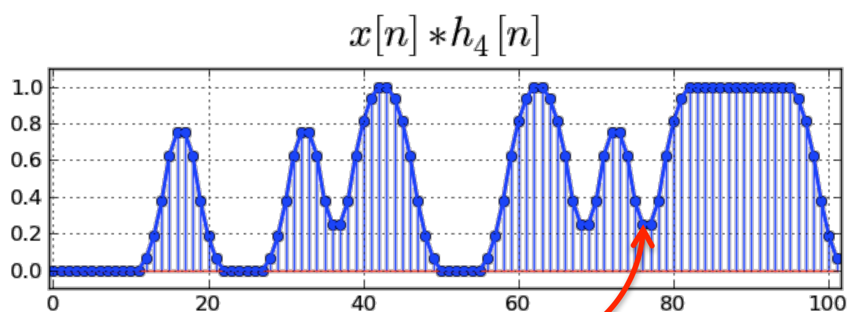
6.02 Fall 2013
Lecture #12

- Intersymbol interference (ISI)
- Eye diagrams
- Frequency response

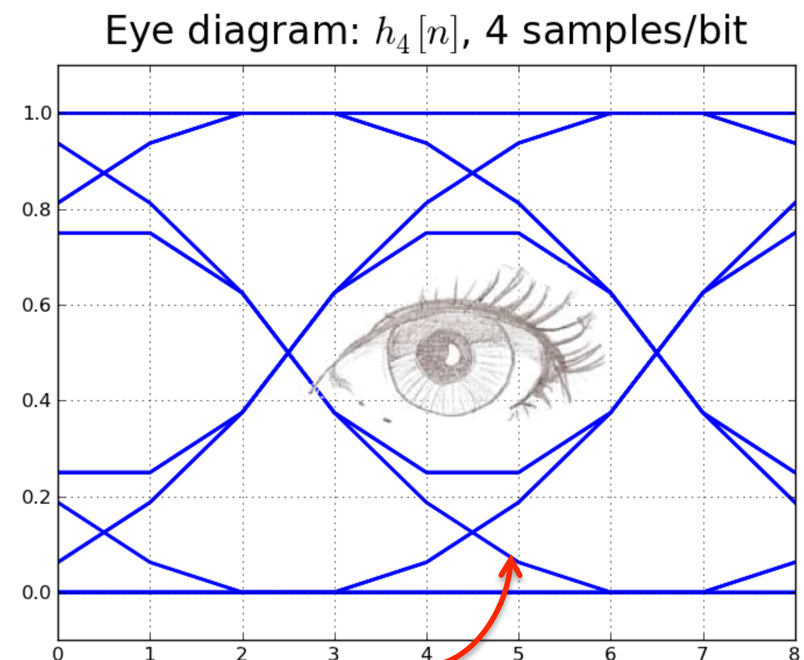
Intersymbol Interference & Eye Diagrams



000 100 010 110 001 101 011 111

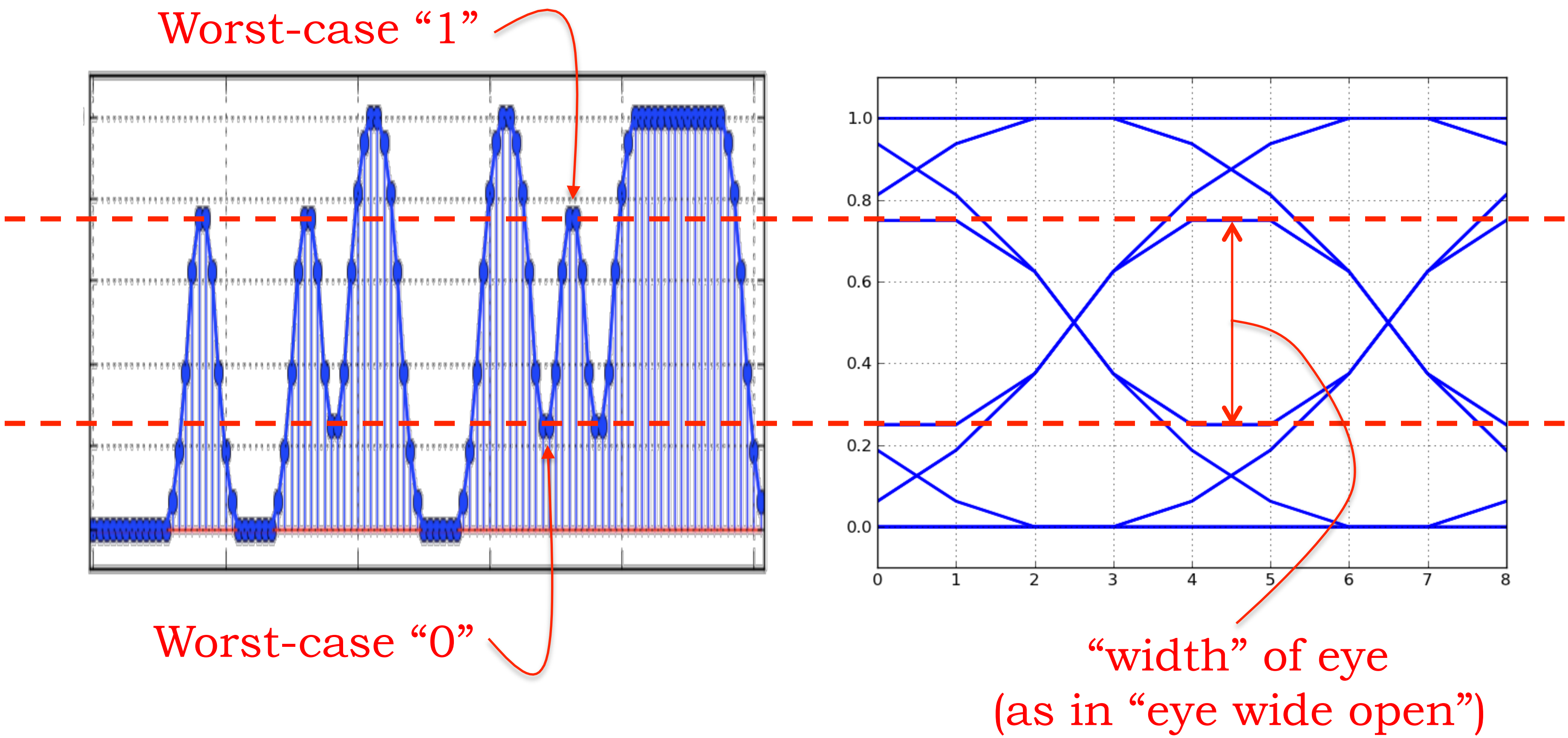


Intersymbol interference



These are overlaid two-bit-slot segments of step responses, plotted without the ‘stems’ of the stem plot on the left

“Width” of Eye

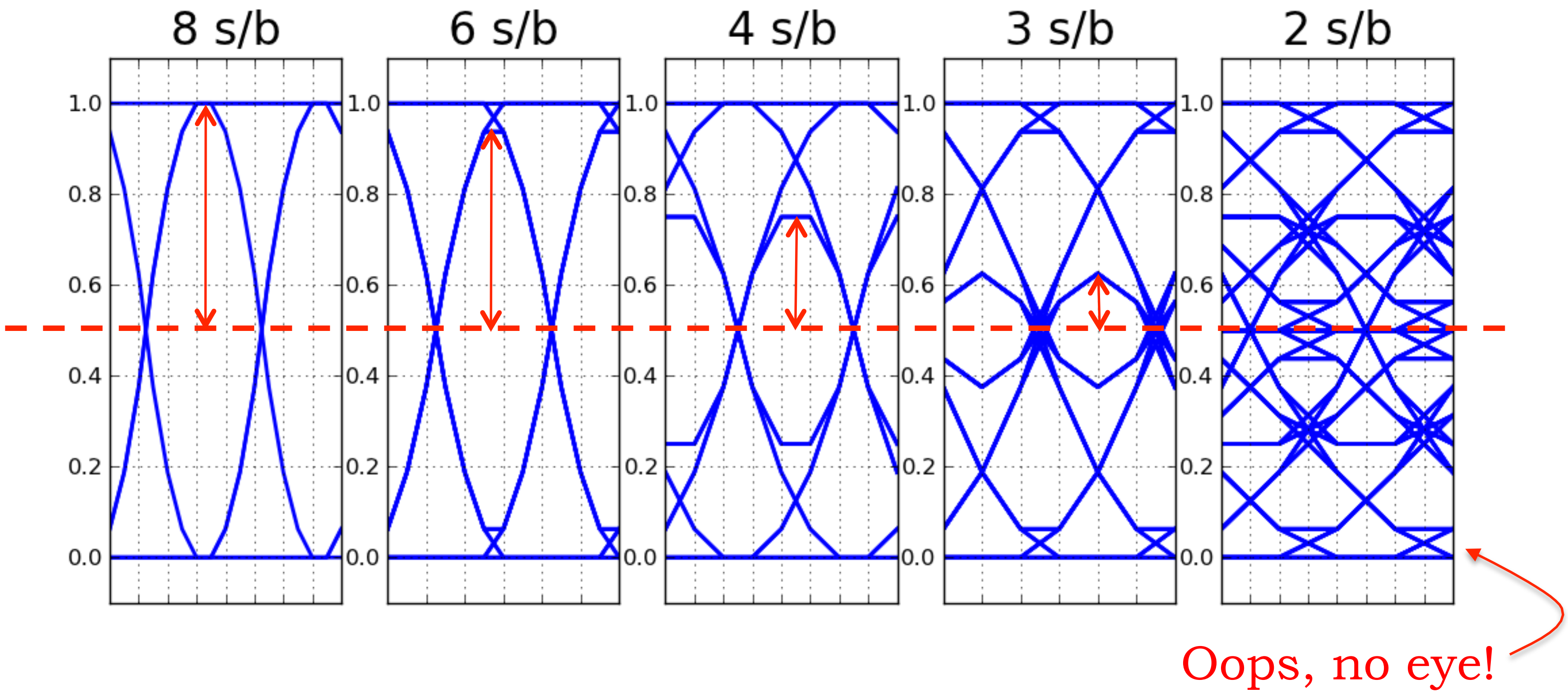


To maximize noise margins:

Pick the best sample point → widest point in the eye

Pick the best digitization threshold → half-way across width

Choosing Samples/Bit



Given h[n], you can use the eye diagram to pick the number of samples transmitted for each bit (N):

Reduce N until you reach the noise margin you feel is the minimum acceptable value.

Constructing the Eye Diagram (no need to wade through all this unless you really want to!)

1. Generate an input bit sequence pattern that contains all possible combinations of B bits (e.g., B=3 or 4), so a sequence of $2^B B$ bits. (Otherwise, a random sequence of comparable length is fine.)

2. Transmit the corresponding x[n] over the channel ($2^B BN$ samples, if there are N samples/bit)

3. Instead of one long plot of y[n], plot the response as an *eye diagram:*
   a. break the plot up into short segments, each containing KN samples, starting at sample 0, KN, 2KN, 3KN, … (e.g., K=2 or 3)
   b. plot all the short segments on top of each other

Time Now for a **Frequency-Domain** Story

in which
convolution
is transformed to
multiplication,
and other
good things
happen

A First Step

Do periodic inputs to an LTI system, i.e., x[n] such that

$$x[n+P] = x[n] \text{ for all } n, \text{ some fixed } P$$

(with P usually picked to be the smallest positive integer for which this is true) yield periodic outputs? If so, of period P?

Yes! --- Since the system is TI, using input x delayed by P should yield y delayed by P. But x delayed by P is x again, so y delayed by P must be y. (Linearity is not needed.)

Alternate argument: use Flip/Slide/Dot.Product to see this easily: sliding by P gives the same picture back again, hence the same output value.

But much more is true for Sinusoidal Inputs to LTI Systems

Sinusoidal inputs, i.e.,

$$x[n] = \cos(\Omega n + \phi)$$

yield sinusoidal outputs at the same 'frequency' Ω rads/sample.

And observe that such inputs are not even periodic in general!

Periodic if and only if $2\pi/\Omega$ is rational, =P/Q for some integers P(>0), Q. The smallest such P is the period.

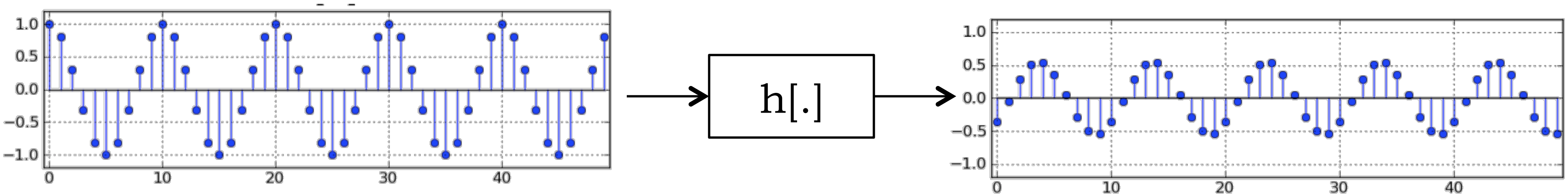
Nevertheless, we often refer to $2\pi/\Omega$ as the 'period' of this sinusoid, whether or not it is a periodic discrete-time sequence. This is the period of an underlying continuous-time signal.

Examples

cos(3πn/4) has frequency 3π/4 rad/sample, and period 8; shifting by integer multiples of 8 yields the same sequence back again, and no integer smaller than 8 accomplishes this.

cos(3n/4) has frequency ¾ rad/sample, and is not periodic as a DT sequence because 8π/3 is irrational, but we could still refer to 8π/3 as its 'period', because we can think of the sequence as arising from sampling the periodic continuous-time signal cos(3t/4) at integer t.

Sinusoidal Inputs and LTI Systems



A very important property of LTI systems or channels:

> If the input x[n] is a sinusoid of a given amplitude, frequency and phase, the response will be a *sinusoid at the same frequency*, although the amplitude and phase may be altered. The change in amplitude and phase will, in general, depend on the frequency of the input.

Let's prove this to be true … but use complex exponentials instead, for clean derivations that take care of sines and cosines (or sinusoids of arbitrary phase) simultaneously.

A related simple case: real discrete-time (DT) exponential inputs also produce exponential outputs of the same type

- Suppose $x[n] = r^n$ for some real number r
- $$y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$
  $$= \sum_{m=-\infty}^{\infty} h[m]r^{n-m}$$
  $$= \left(\sum_{m=-\infty}^{\infty} h[m]r^{-m} \right) r^n$$
- i.e., just a scaled version of the exponential input

Complex Exponentials

Euler's formula shows the relation between complex exponentials and our usual trig functions:

$$e^{j\varphi} = \cos(\varphi) + j\sin(\varphi)$$

$$\cos(\varphi) = \frac{1}{2}e^{j\varphi} + \frac{1}{2}e^{-j\varphi} \qquad \sin(\varphi) = \frac{1}{2j}e^{j\varphi} - \frac{1}{2j}e^{-j\varphi}$$

In the complex plane, $e^{j\varphi} = \cos(\varphi) + j\sin(\varphi)$ is a point on the unit circle, at an angle of ø with respect to the positive real axis.
cos and sin are projections on real and imaginary axes, respectively.

Increasing ø by 2π brings you back to the same point!
So any function of $e^{j\varphi}$ only needs to be studied for ø in [-π, π] .

Useful Properties of $e^{j\phi}$

When φ = 0:

$$e^{j0} = 1$$

When φ = ±π:

$$e^{j\pi} = e^{-j\pi} = -1$$

$$e^{j\pi n} = e^{-j\pi n} = (-1)^n$$

(More properties later)

Frequency Response

$$A(\cos\Omega n + j\sin\Omega n) = Ae^{j\Omega n} \longrightarrow \boxed{h[.]} \longrightarrow y[n]$$

Using the convolution sum we can compute the system's response to a complex exponential (of frequency Ω) as input:

$$\begin{aligned} y[n] &= \sum_m h[m]x[n-m] \\ &= \sum_m h[m]Ae^{j\Omega(n-m)} \\ &= \left(\sum_m h[m]e^{-j\Omega m}\right)Ae^{j\Omega n} \\ &= H(\Omega)\cdot x[n] \end{aligned}$$

where we've defined the *frequency response* of the system as

$$\boxed{H(\Omega) \equiv \sum_m h[m]e^{-j\Omega m}}$$

From Complex Exponentials to Sinusoids

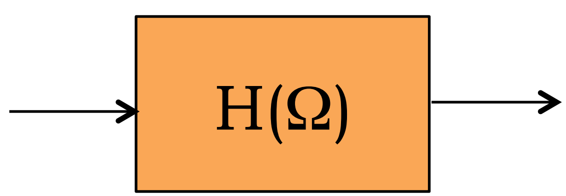
$$\cos(\Omega n) = (e^{j\Omega n} + e^{-j\Omega n)})/2$$

So response to this cosine input is

$$(H(\Omega)e^{j\Omega n} + H(-\Omega)e^{-j\Omega n)})/2 = \text{Real part of } H(\Omega)e^{j\Omega n}$$

$$= \text{Real part of } |H(\Omega)|e^{j(\Omega n + <H(\Omega))}$$

$\cos(\Omega_0 n)$

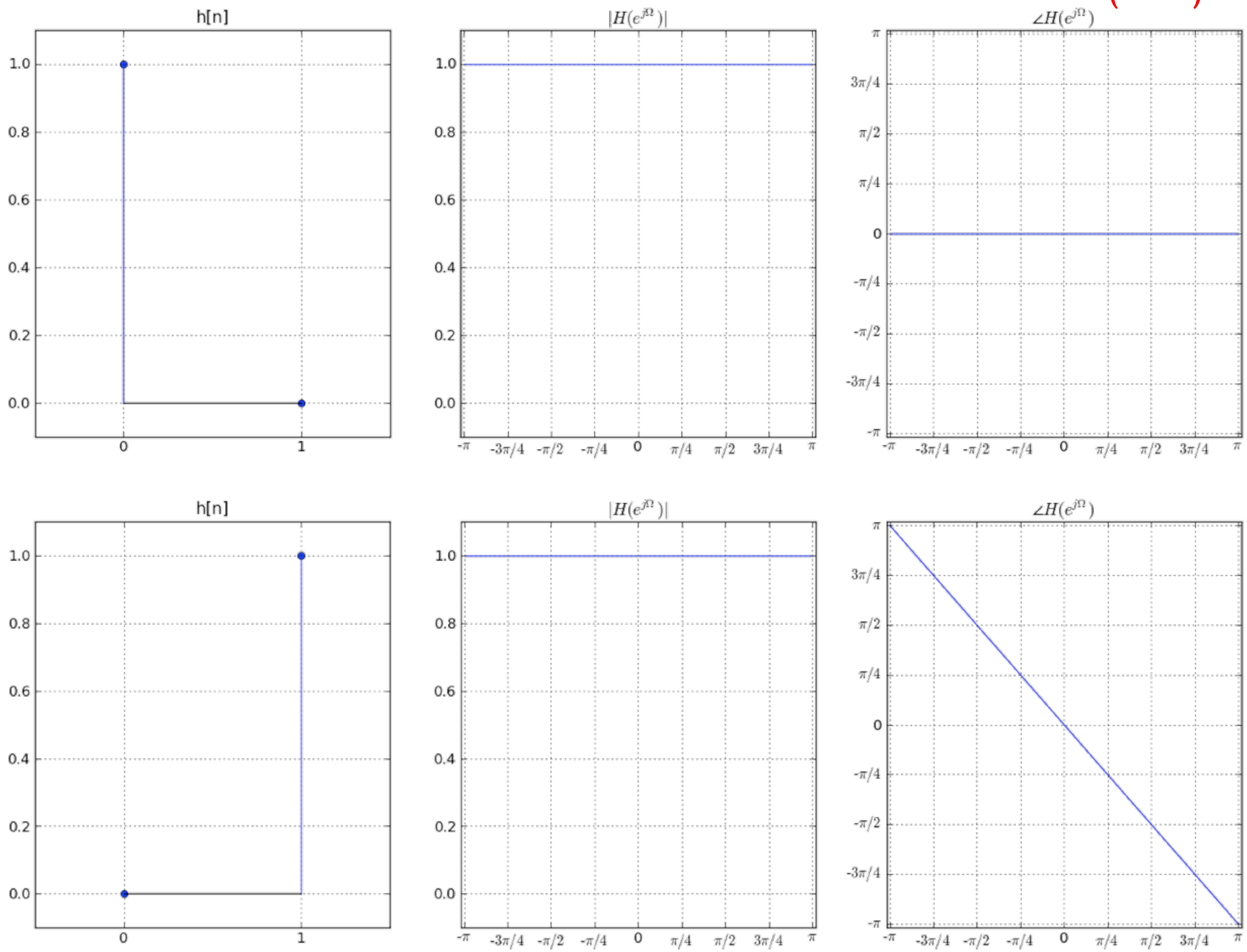


$|H(\Omega_0)|\cos(\Omega_0 n + <H(\Omega_0))$

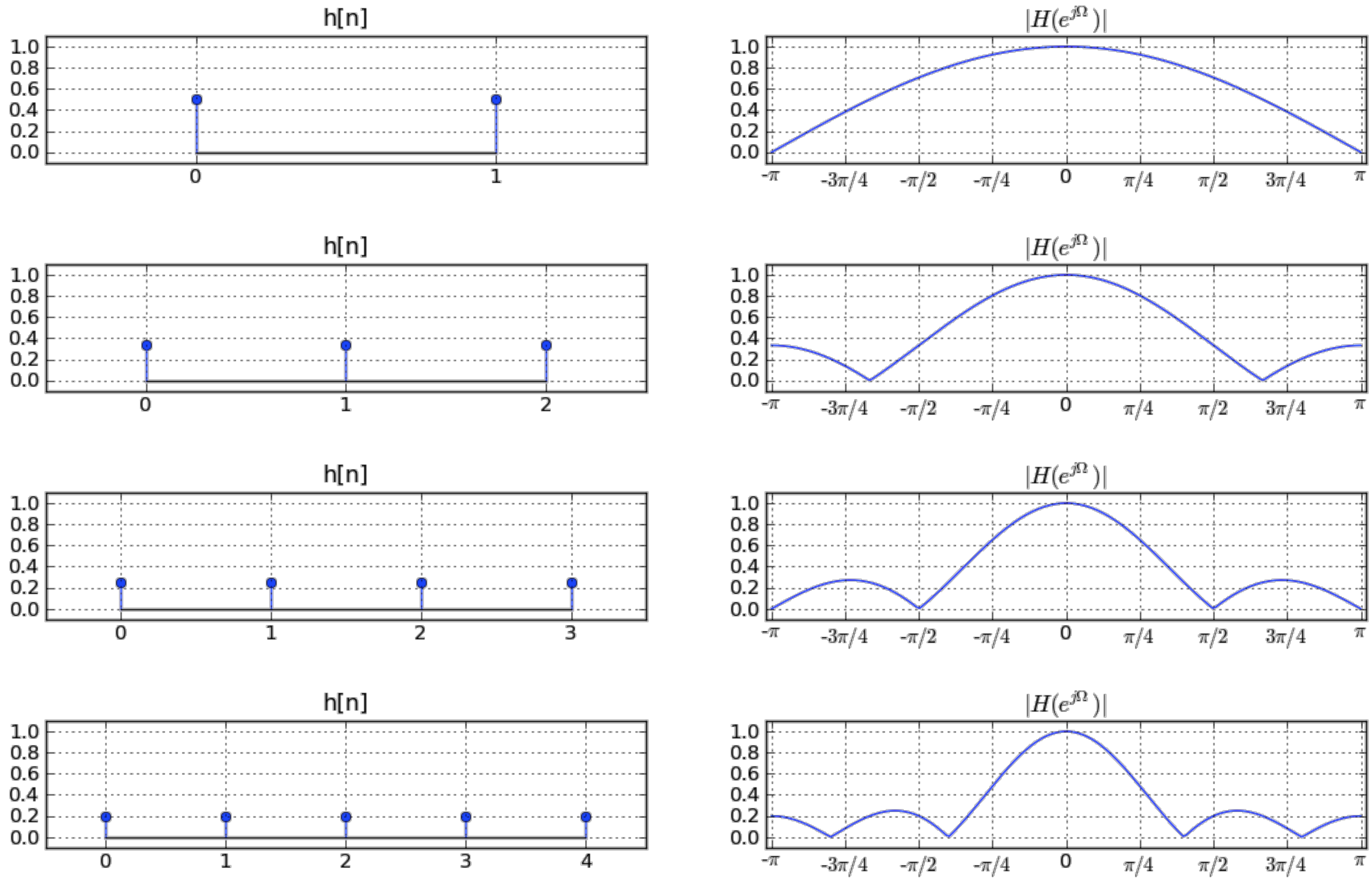
This is IMPORTANT

Example h[n] and H(Ω)

Sometimes written as $H(e^{j\Omega n})$



Frequency Response of “Moving Average” Filters

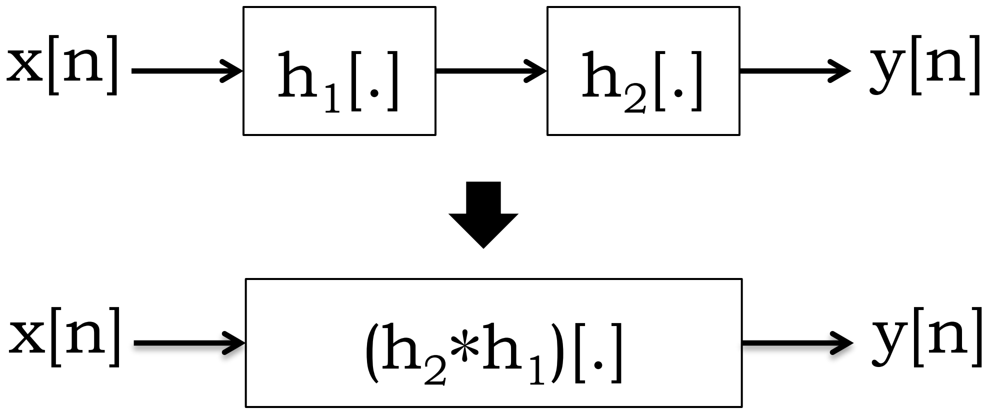


Relating Time- and Frequency-Domain Properties

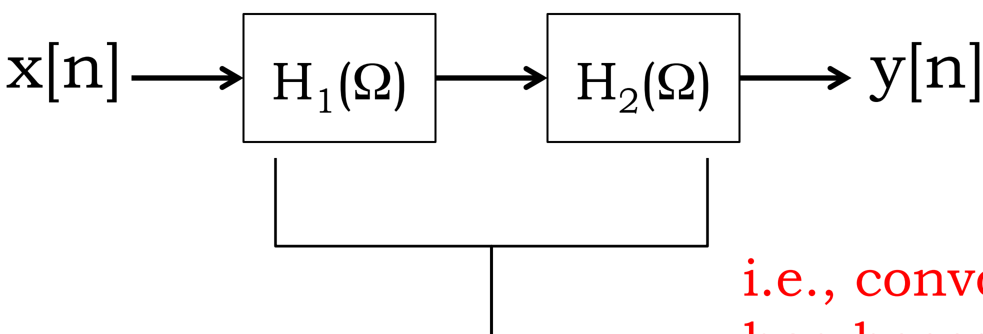
- Why does the 5-point moving average filter on the preceding page have nulls in its frequency response magnitude at $\pm 0.4\pi$ and $\pm 0.8\pi$?

(Think of convolving the unit sample response of this filter with sinusoids at these frequencies.)

Convolution in Time <---> Multiplication in Frequency



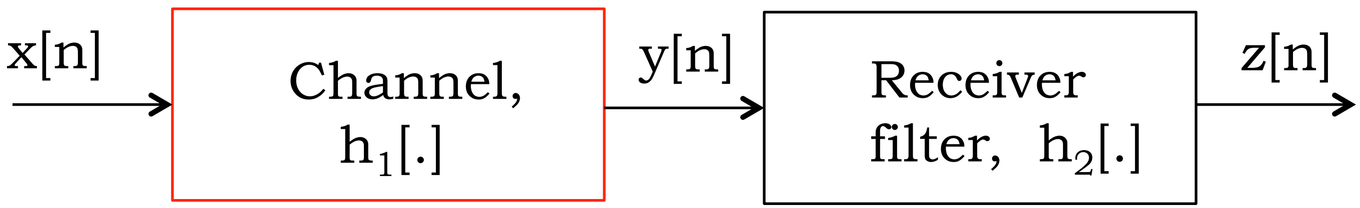
In the frequency domain (i.e., thinking about input-to-output frequency response):



$$H(\Omega)=H_2(\Omega)H_1(\Omega)$$

i.e., convolution in time has become multiplication in frequency!

Example: "Deconvolving" Output of Channel with Echo



Suppose channel is LTI with

$h_1[n]=\delta[n]+0.8\delta[n-1]$

$$H_1(\Omega) = ?? = \boxed{\sum_m h_1[m]e^{-j\Omega m}}$$

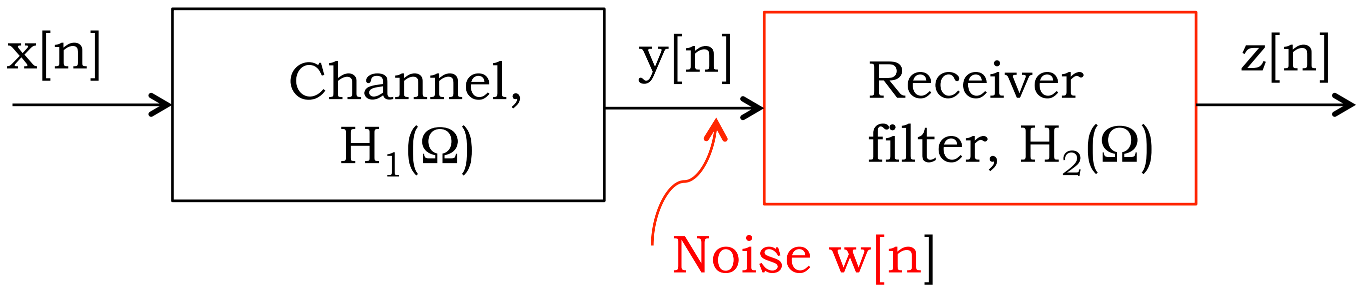
$$= 1 + 0.8e^{-j\Omega} = 1 + 0.8\cos(\Omega) - j0.8\sin(\Omega)$$

So:

$$|H_1(\Omega)| = [1.64 + 1.6\cos(\Omega)]^{1/2}$$ *EVEN function of* Ω;

$$\angle H_1(\Omega) = \arctan\left[-(0.8\sin(\Omega)/[1 + 0.8\cos(\Omega)]\right]$$ *ODD*.

A Frequency-Domain View of Deconvolution



Given $H_1(\Omega)$, what should $H_2(\Omega)$ be, to get $z[n]=x[n]$?

$$H_2(\Omega) = 1/H_1(\Omega) \qquad \text{"Inverse filter"}$$

$$= (1/|H_1(\Omega)|).\ \exp\{-j\angle H_1(\Omega)\}$$

Inverse filter at receiver does very badly in the presence of noise that adds to y[n]:

filter has high gain for noise precisely at frequencies where channel gain $|H_1(\Omega)|$ is low (and channel output is weak)!

A Deeper Reason for Interest in Sinusoidal Inputs

- General inputs x[.] can we written as “sums” of sinusoids
- Each input sinusoidal component is mapped via the frequency response $H(\Omega)$ to its corresponding sinusoidal output component
- Superposition of these output components yields the general response y[.]

We’ll develop this story over the next couple of lectures.