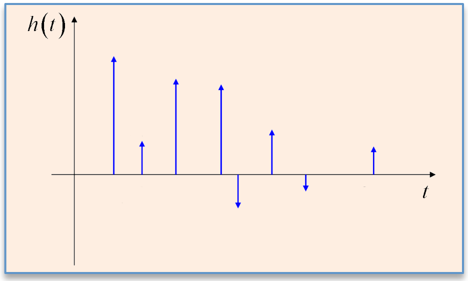
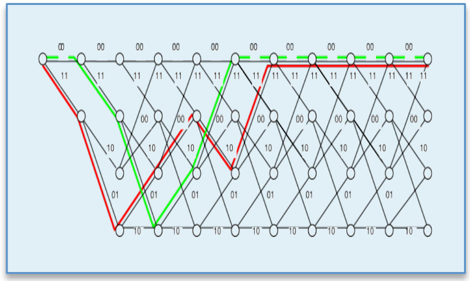
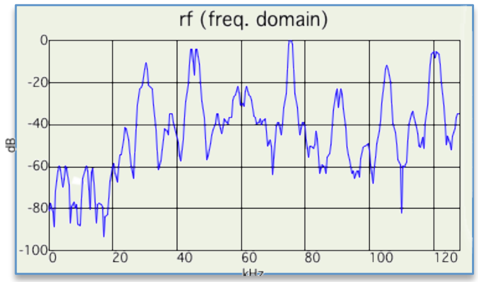
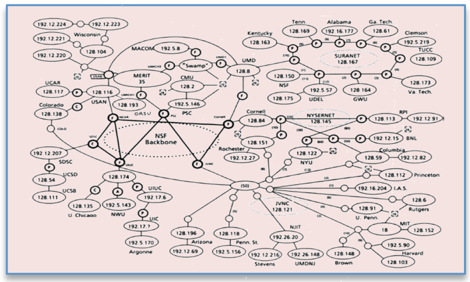








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2013
Lecture #14

- Spectral content of signals via the DTFT

Determining h[n] from H(Ω)

$$H(\Omega) = \sum_{m} h[m] e^{-j\Omega m}$$

Multiply both sides by $e^{j\Omega n}$ and integrate over a (contiguous) 2π interval. Only one term survives!

$$\int_{<2\pi>} H(\Omega) e^{j\Omega n} d\Omega = \int_{<2\pi>} \sum_{m} h[m] e^{-j\Omega(m-n)} d\Omega$$

$$= 2\pi \cdot h[n]$$

$$\Rightarrow \quad h[n] = \frac{1}{2\pi} \int_{<2\pi>} H(\Omega) e^{j\Omega n} d\Omega$$

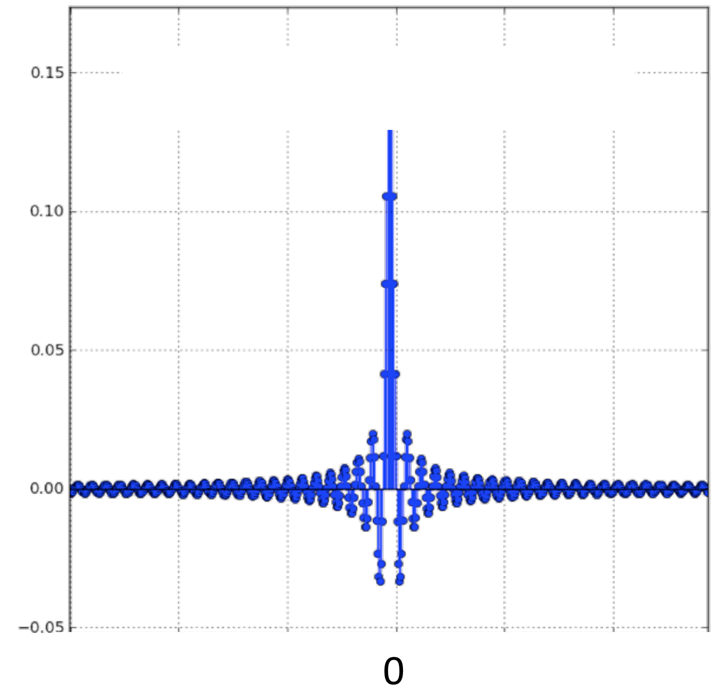
Design **ideal lowpass filter** with cutoff frequency Ω_C and $H(\Omega)=1$ in passband

$$h[n] = \frac{1}{2\pi} \int_{<2\pi>} H(\Omega) e^{j\Omega n} d\Omega$$

$$= \frac{1}{2\pi} \int_{-\Omega_C}^{\Omega_C} 1 \cdot e^{j\Omega n} d\Omega$$

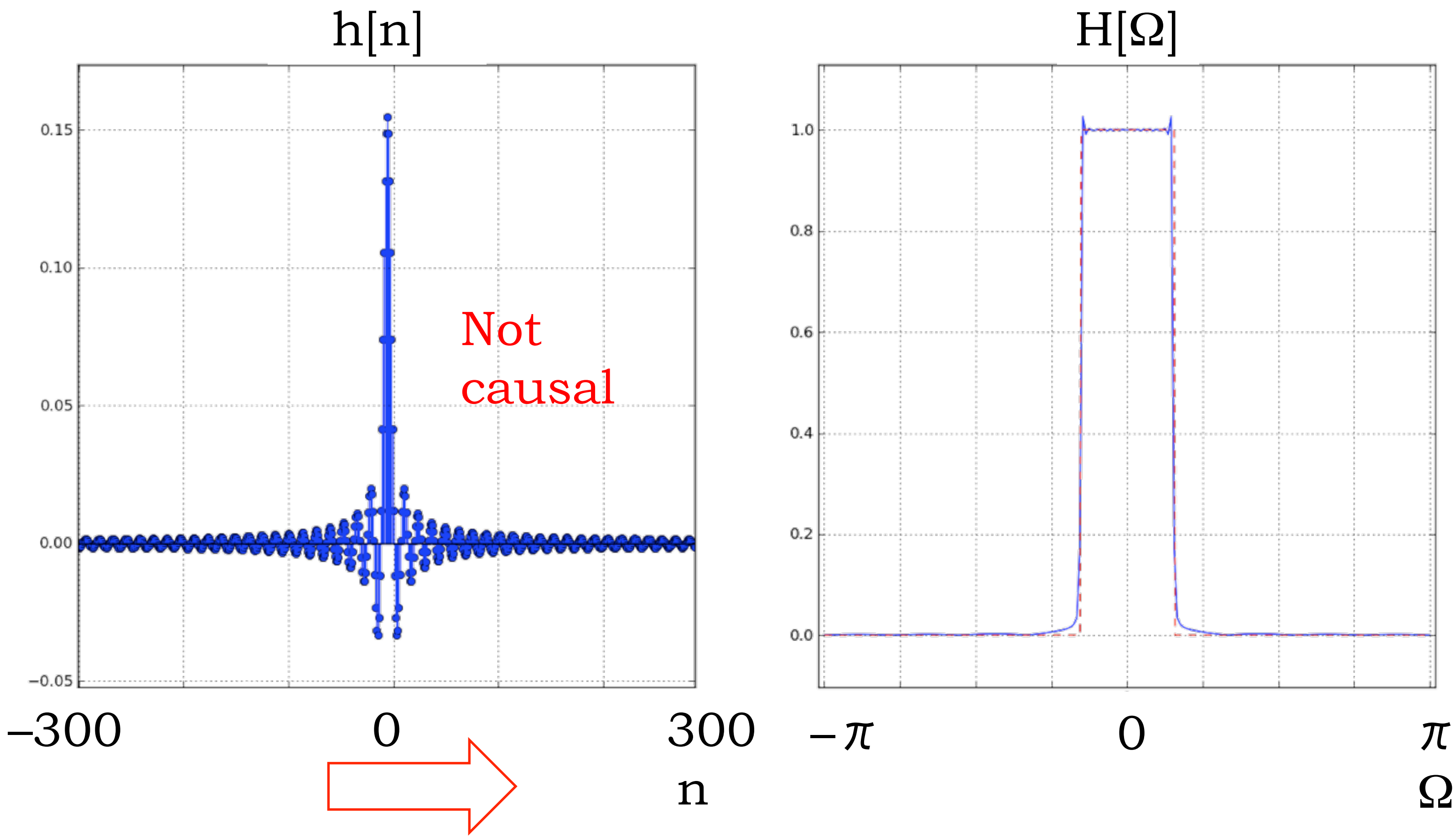
$$= \frac{\sin(\Omega_C n)}{\pi n}, \quad n \neq 0$$

$$= \Omega_C / \pi, \quad n = 0$$



DT "sinc" function
(extends to ±∞ in time, falls off only as 1/n))

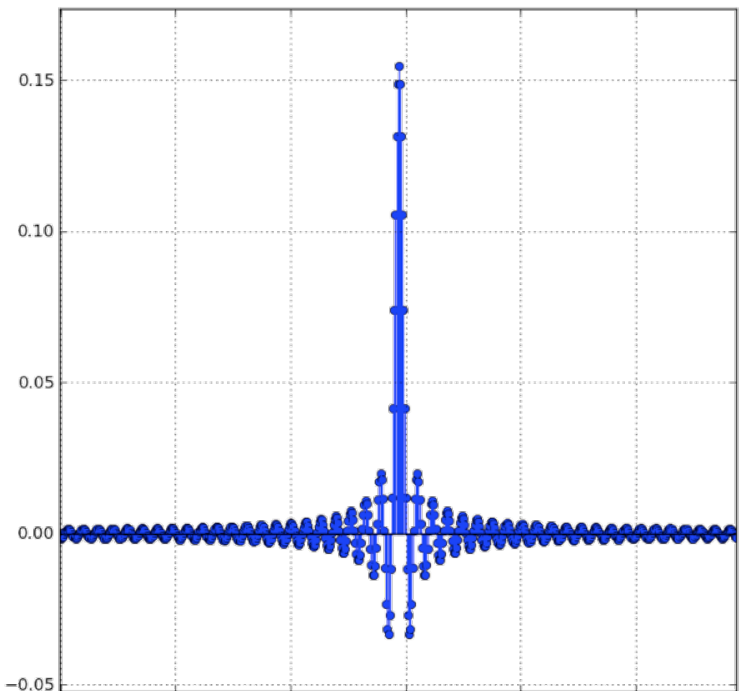
Approximating an ideal lowpass filter



Idea: Delay h[n] to get causal LTI system (after truncation of tails). Will the result still be a lowpass filter?

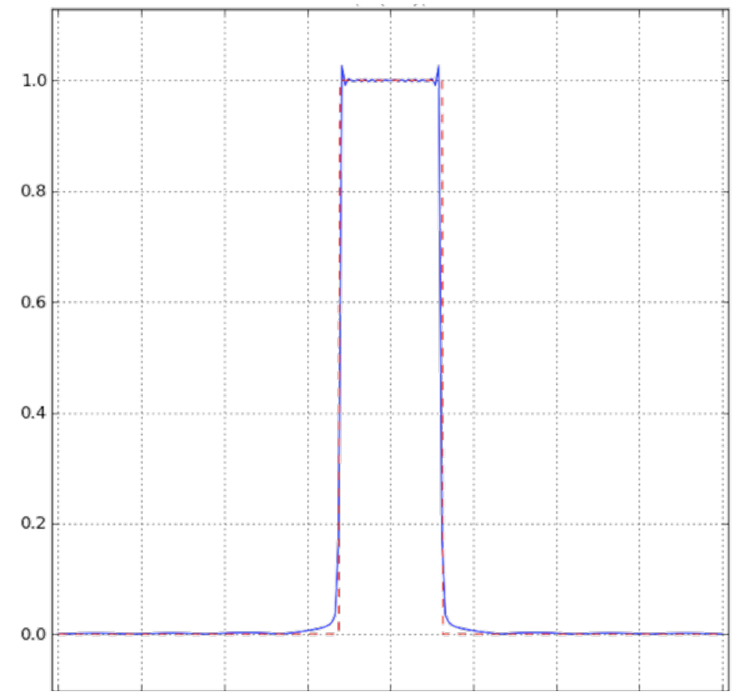
Causal approximation to ideal lowpass filter

$h_C[n] = h[n-300]$



0 300 600
n

$|H_C[\Omega]|$



$-\pi$ 0 π
Ω

Determine $<H_C(\Omega)$

Exercise: Frequency response of h[n-D]

Given an LTI system with unit sample response h[n] and associated frequency response $H(\Omega)$,

determine the frequency response $H_D(\Omega)$ of an LTI system whose unit sample response is

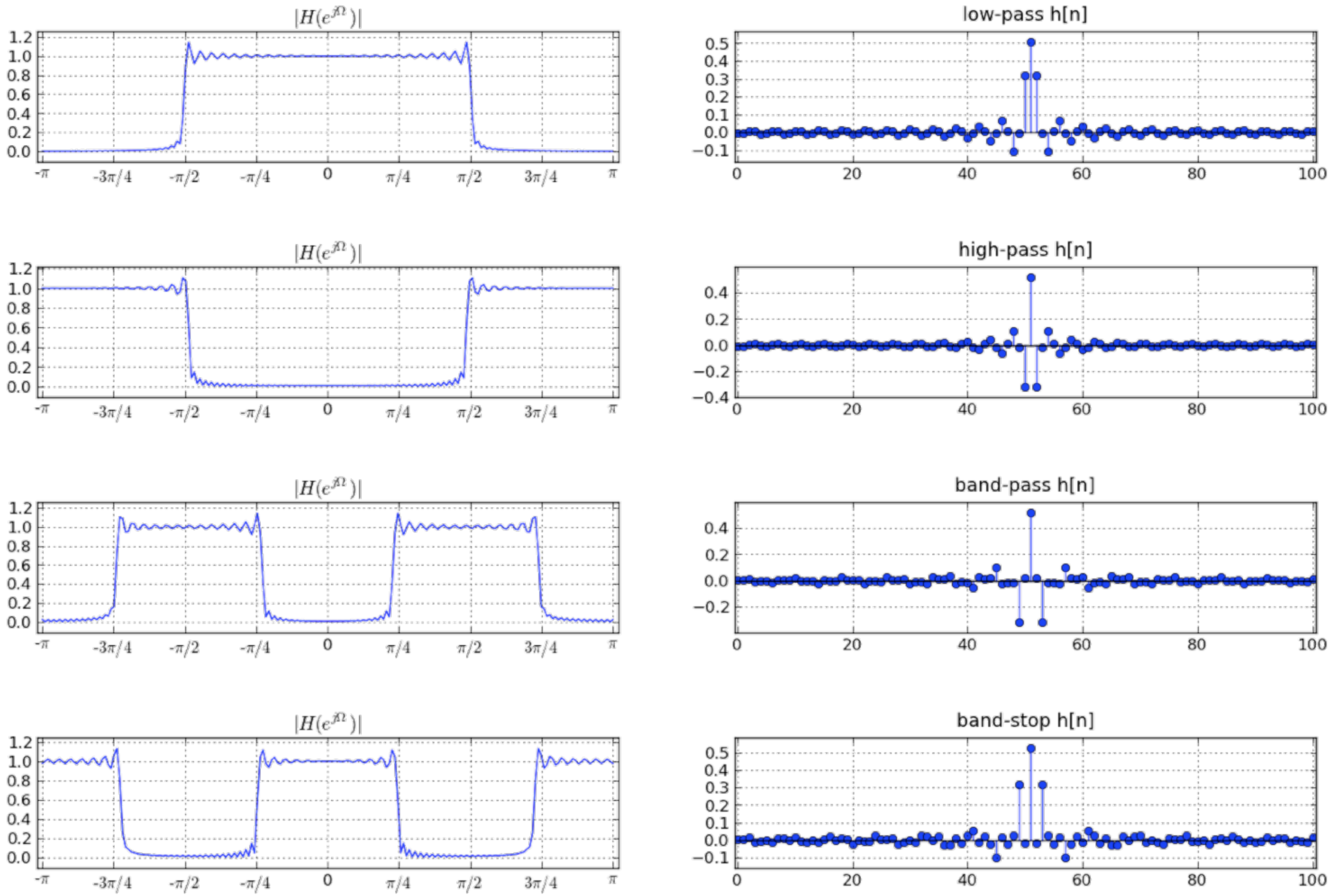
$$h_D[n] = h[n-D].$$

Answer: $H_D(\Omega) = \exp\{-j\Omega D\}.H(\Omega)$

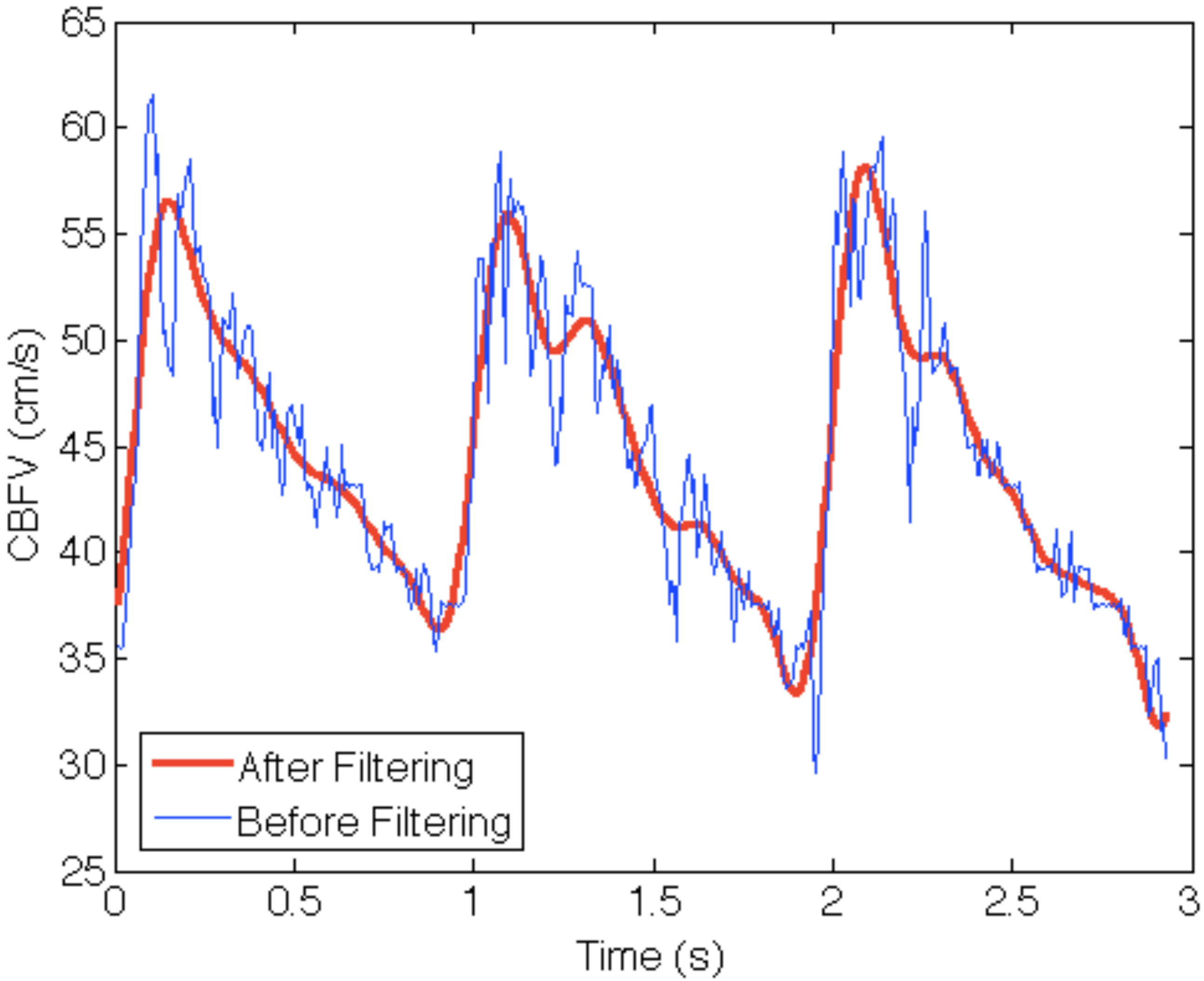
so : $|H_D(\Omega)| = |H(\Omega)|$, i.e., magnitude unchanged

$\angle H_D(\Omega) = -\Omega D + \angle H(\Omega)$, i.e., linear phase term added

Useful Filters

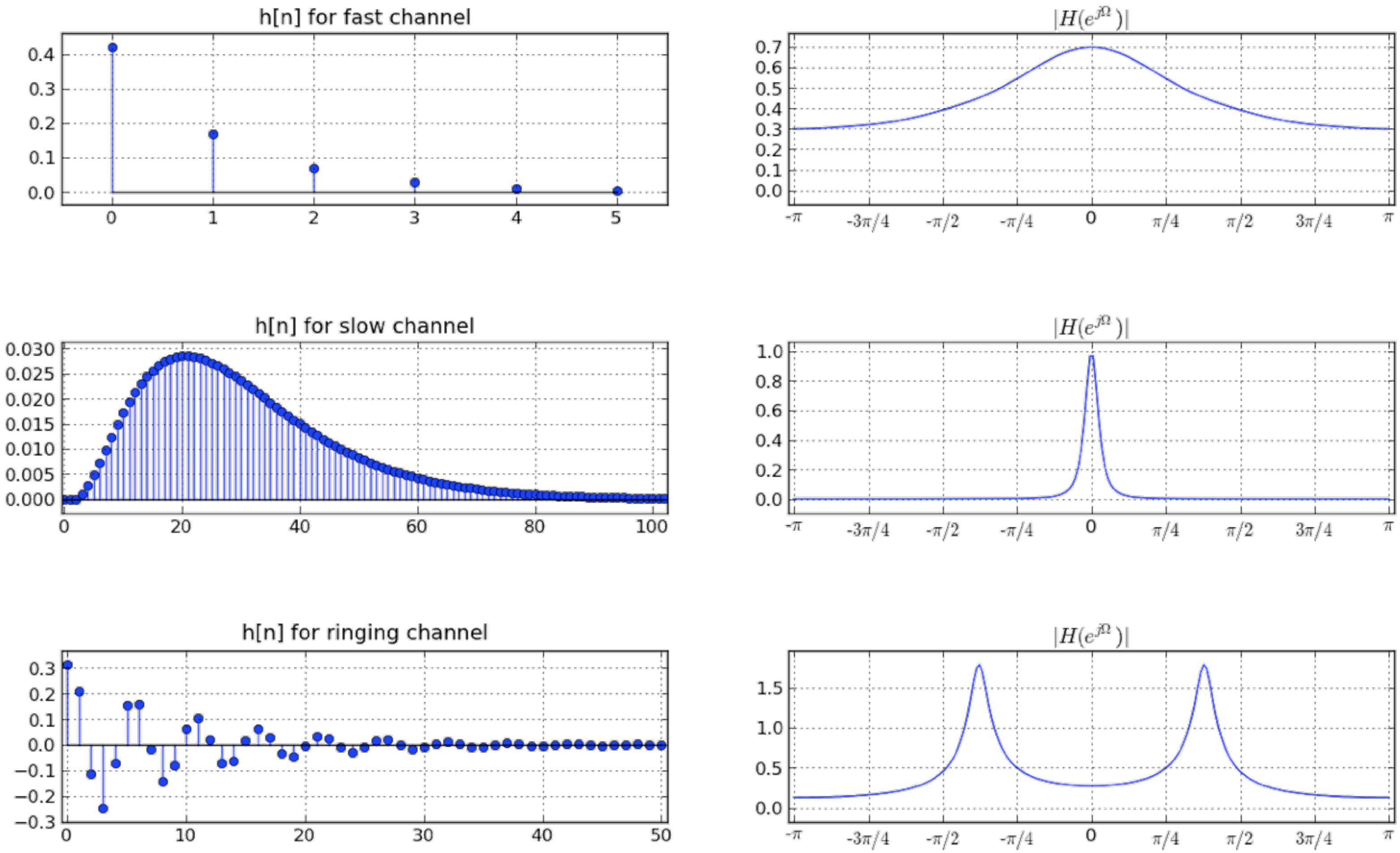


Filtering

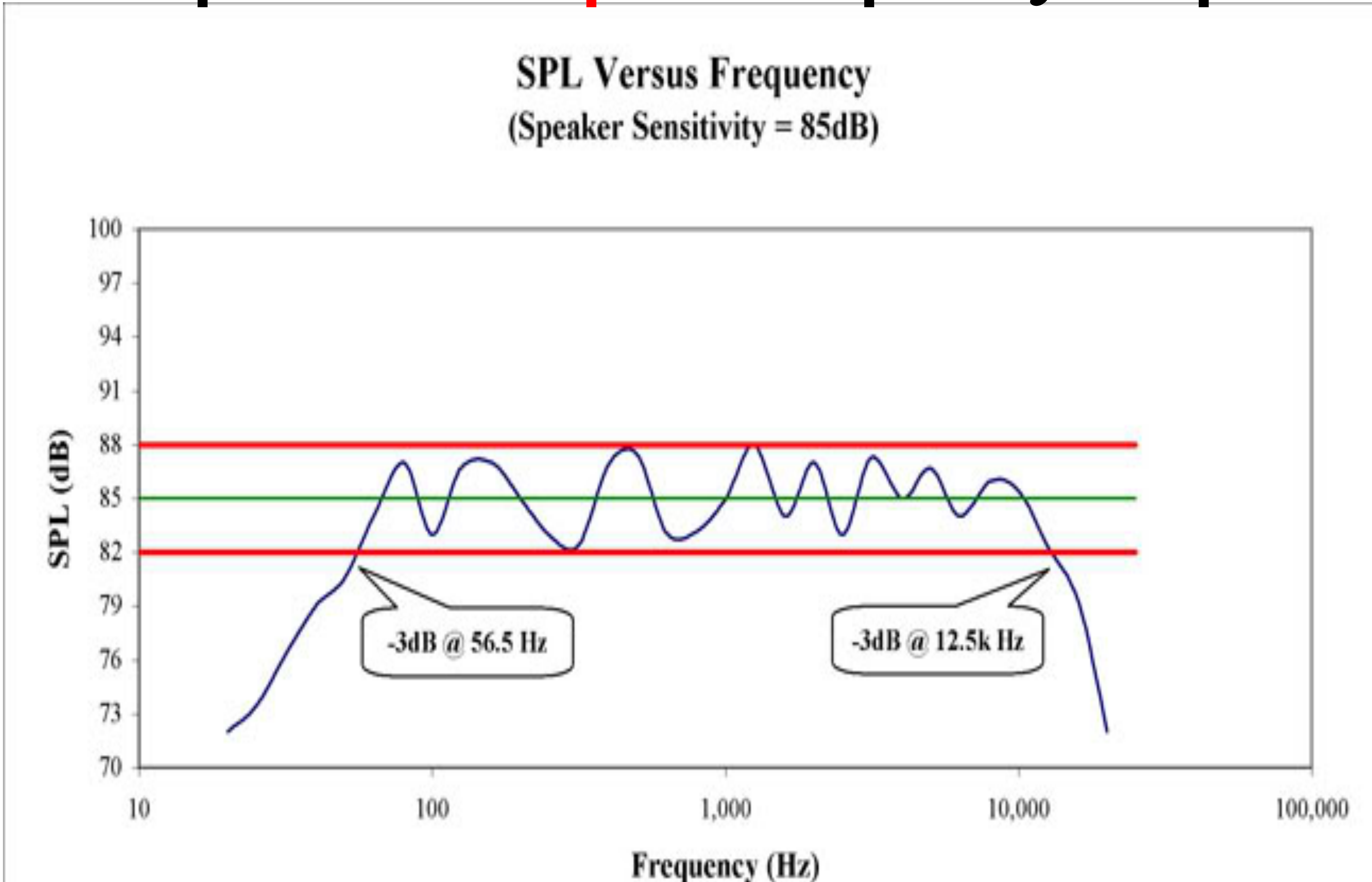


Lowpass filtering (10 Hz cutoff) of blood flow velocity in middle cerebral artery, measured using transcranial Doppler ultrasound

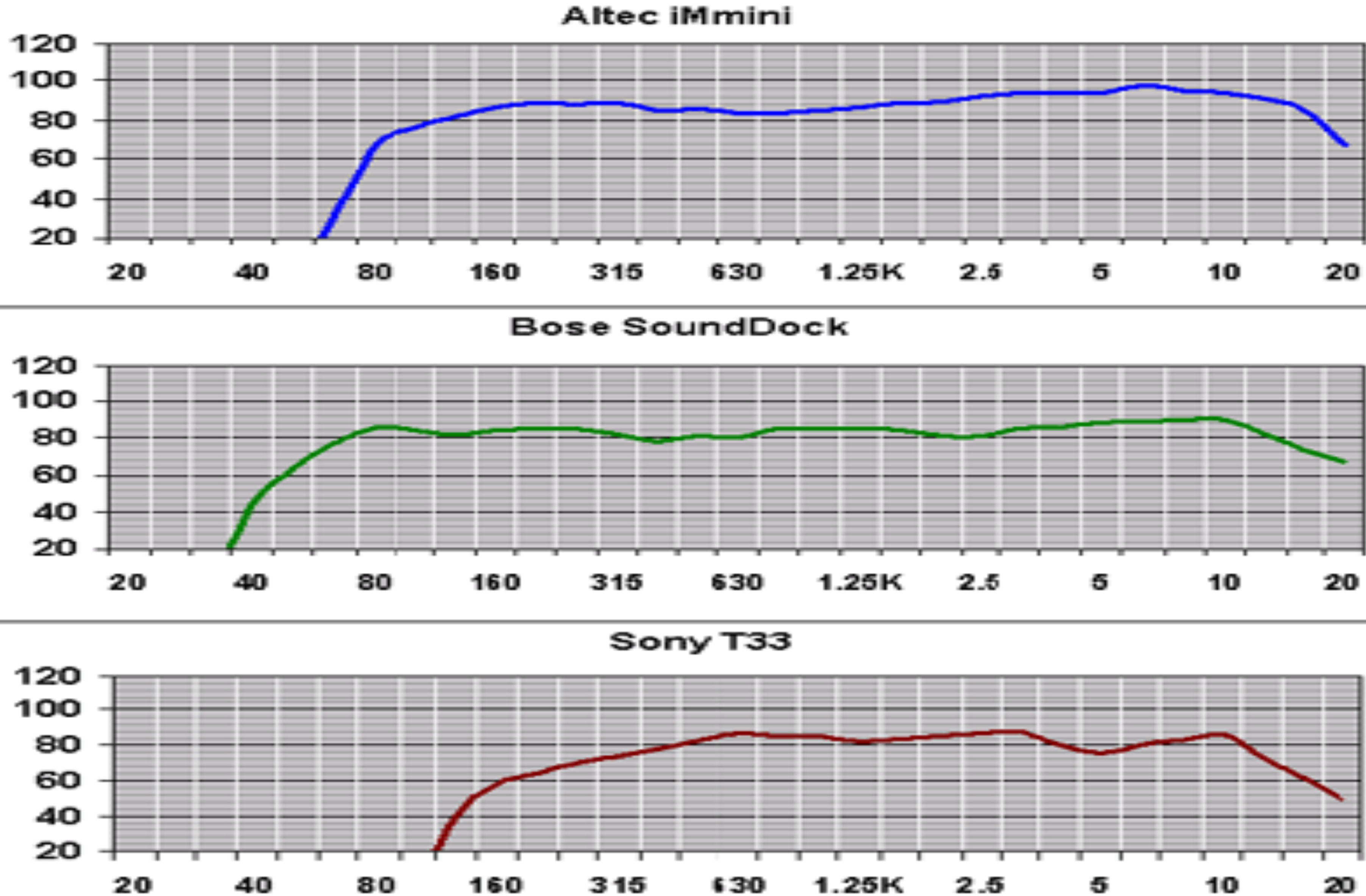
Frequency Response of Channels



Loudspeaker Bandpass Frequency Response



http://forum.blu-ray.com/showthread.php?t=150915



http://www.pcmag.com/article2/0,2817,1769243,00.asp

Connection between CT and DT

The continuous-time (CT) signal

$$x(t) = \cos(\omega t) = \cos(2\pi f t)$$

sampled every T seconds, i.e., at a sampling frequency of $f_s = 1/T$, gives rise to the discrete-time (DT) signal

$$x[n] = x(nT) = \cos(\omega nT) = \cos(\Omega n)$$

So $\Omega = \omega T$

and $\Omega = \pi$ corresponds to $\omega = \pi/T$ or $f = 1/(2T) = f_s/2$

A Deeper Reason for Interest in Sinusoidal Inputs

- General inputs x[.] can we written as “sums” of sinusoids
- Each input sinusoidal component is mapped via the frequency response $H(\Omega)$ to its corresponding sinusoidal output component
- Superposition of these output components yields the general response y[.]

DT Fourier Transform (DTFT) for Spectral Representation of General x[n]

If we can write

$$h[n] = \frac{1}{2\pi}\int_{<2\pi>} H(\Omega)e^{j\Omega n}\,d\Omega \qquad \text{where} \qquad H(\Omega) = \sum_m h[m]e^{-j\Omega m}$$

Any contiguous interval of length 2π

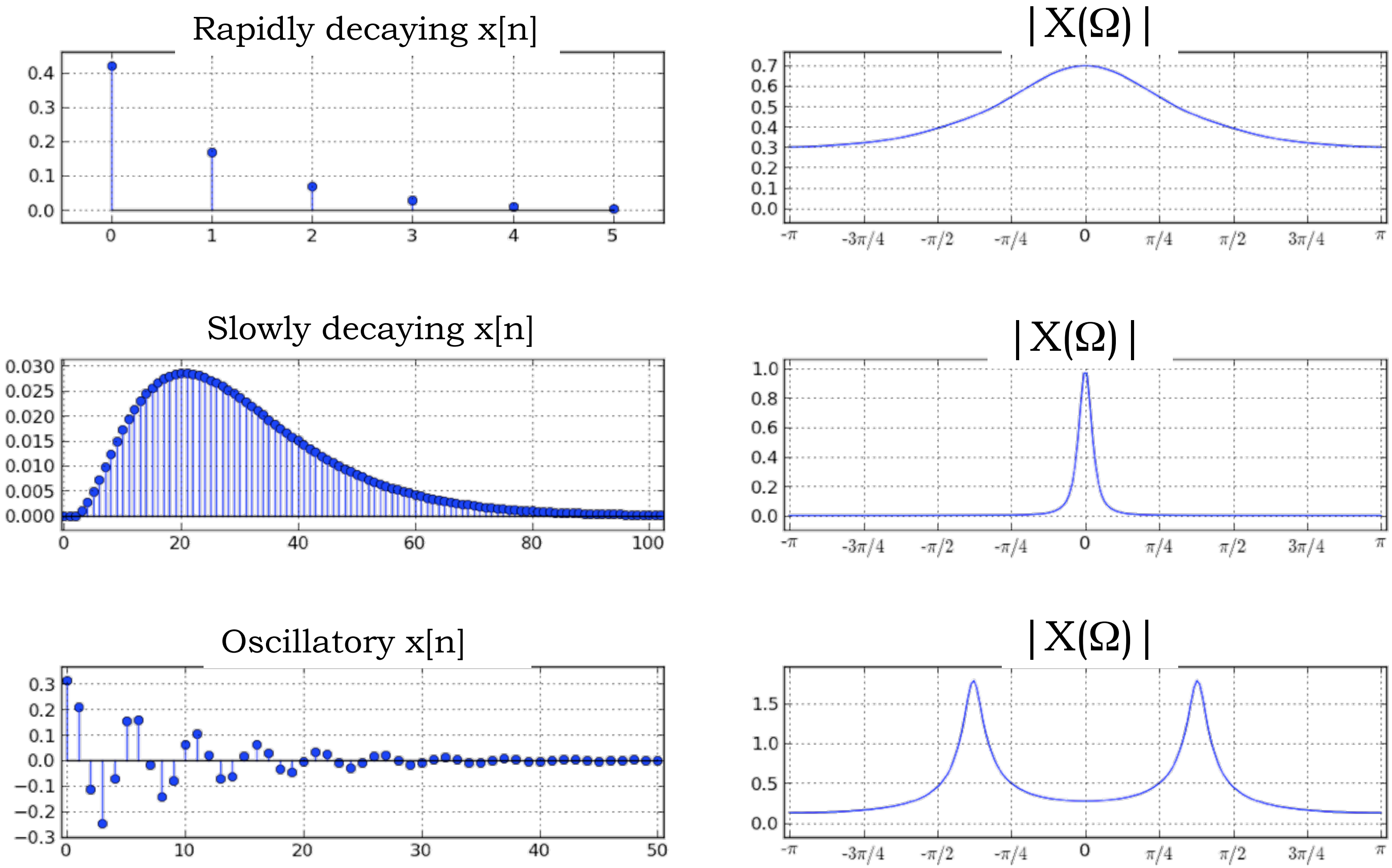
then we can write

$$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(\Omega)e^{j\Omega n}\,d\Omega \qquad \text{where} \qquad X(\Omega) = \sum_m x[m]e^{-j\Omega m}$$

This Fourier representation expresses x[n] as a weighted combination of $e^{j\Omega n}$ for all Ω in $[-\pi, \pi]$.

$X(\Omega_o)d\Omega$ indicates the **spectral content** of x[n] in the frequency interval $[\Omega_o, \Omega_o + d\Omega]$

x[n] and X(Ω)


Rapidly decaying x[n]
|X(Ω)|
Slowly decaying x[n]
|X(Ω)|
Oscillatory x[n]
|X(Ω)|

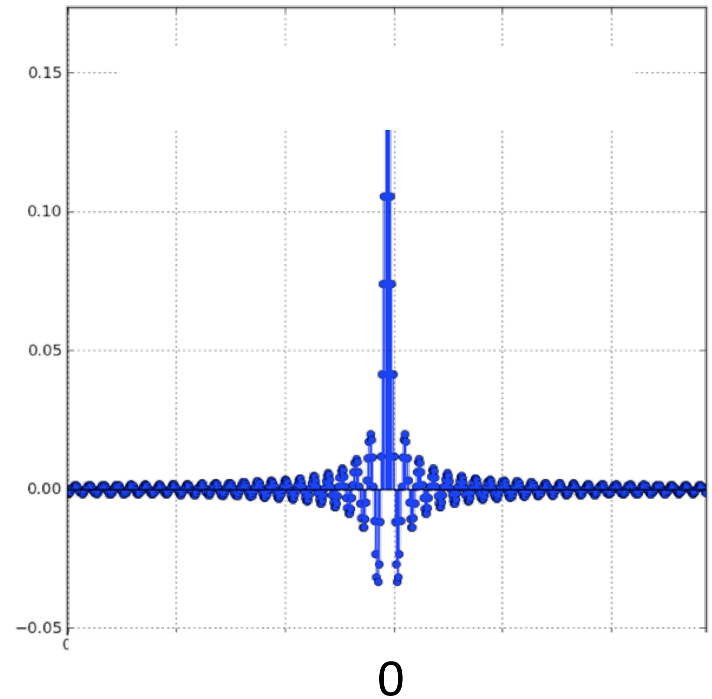
Signal x[n] that has its frequency content uniformly distributed in [-Ω_c , Ω_c]

$$x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(\Omega) e^{j\Omega n} d\Omega$$

$$= \frac{1}{2\pi} \int_{-\Omega_C}^{\Omega_C} 1 \cdot e^{j\Omega n} d\Omega$$

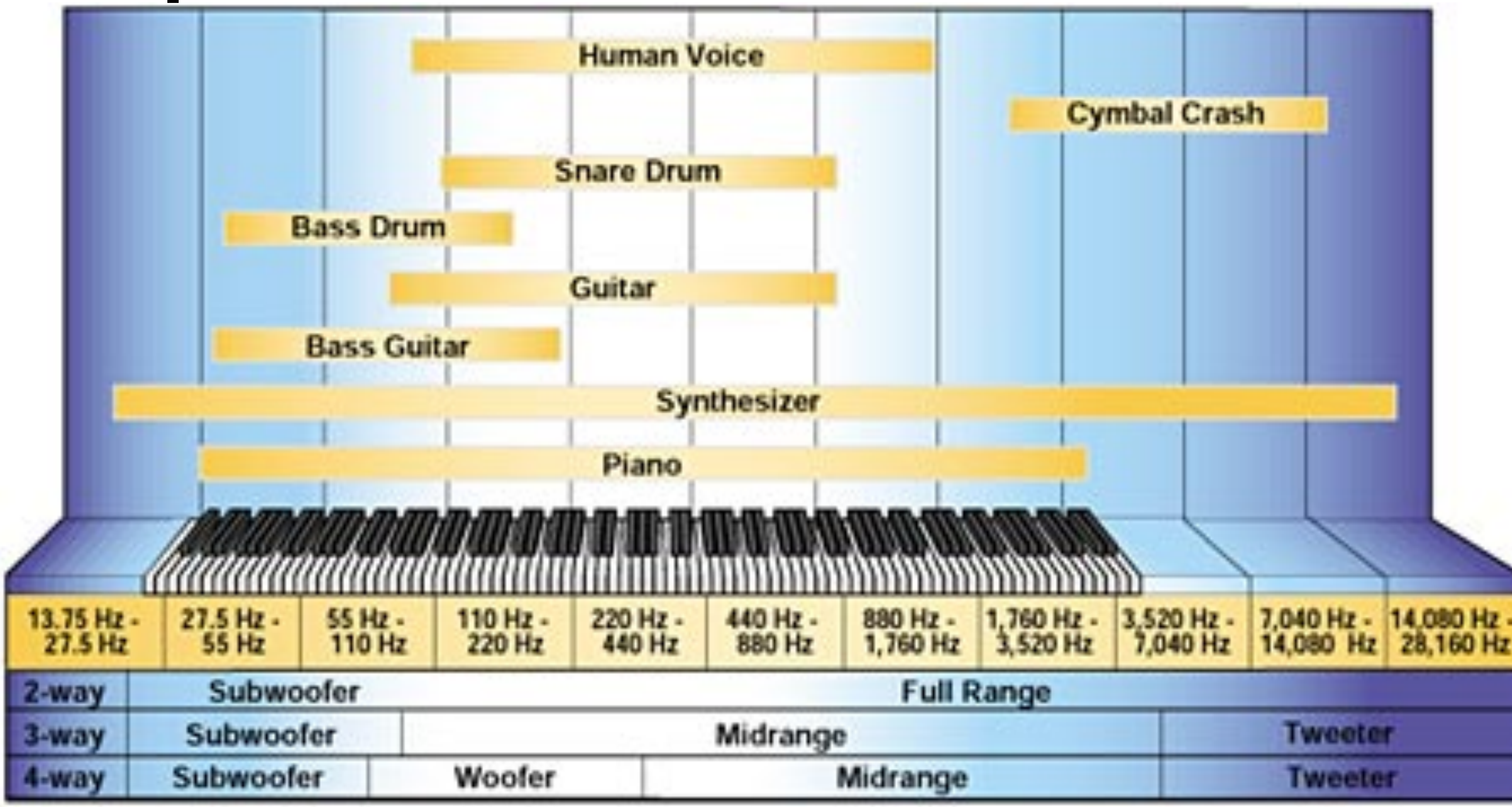
$$= \frac{\sin(\Omega_C n)}{\pi n}, \quad n \neq 0$$

$$= \Omega_C / \pi \quad , \quad n = 0$$



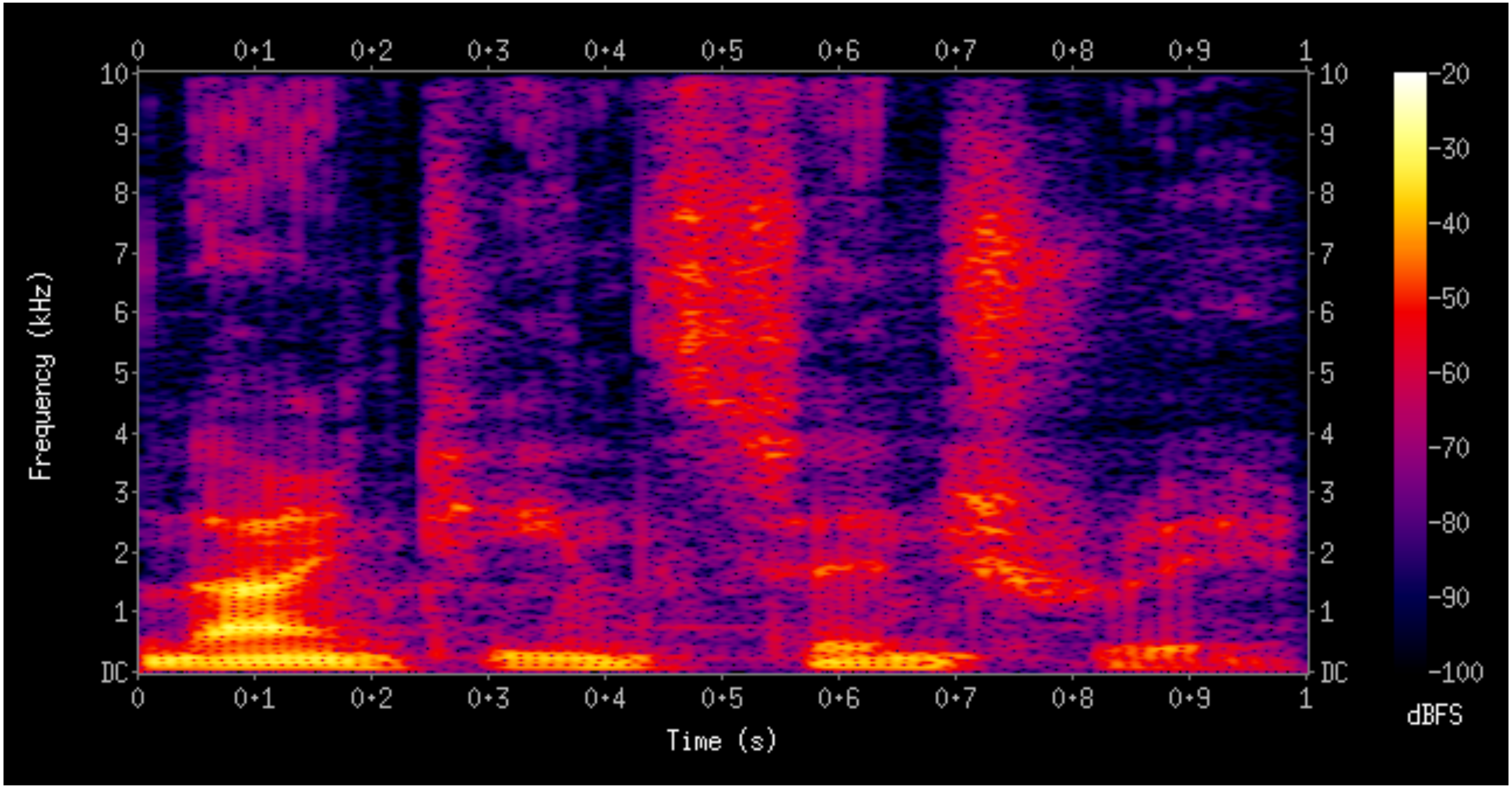
DT "sinc" function
(extends to ±∞ in time, falls off only as 1/n)

Spectral Content of Various Sounds



http://forum.blu-ray.com/showthread.php?t=150915

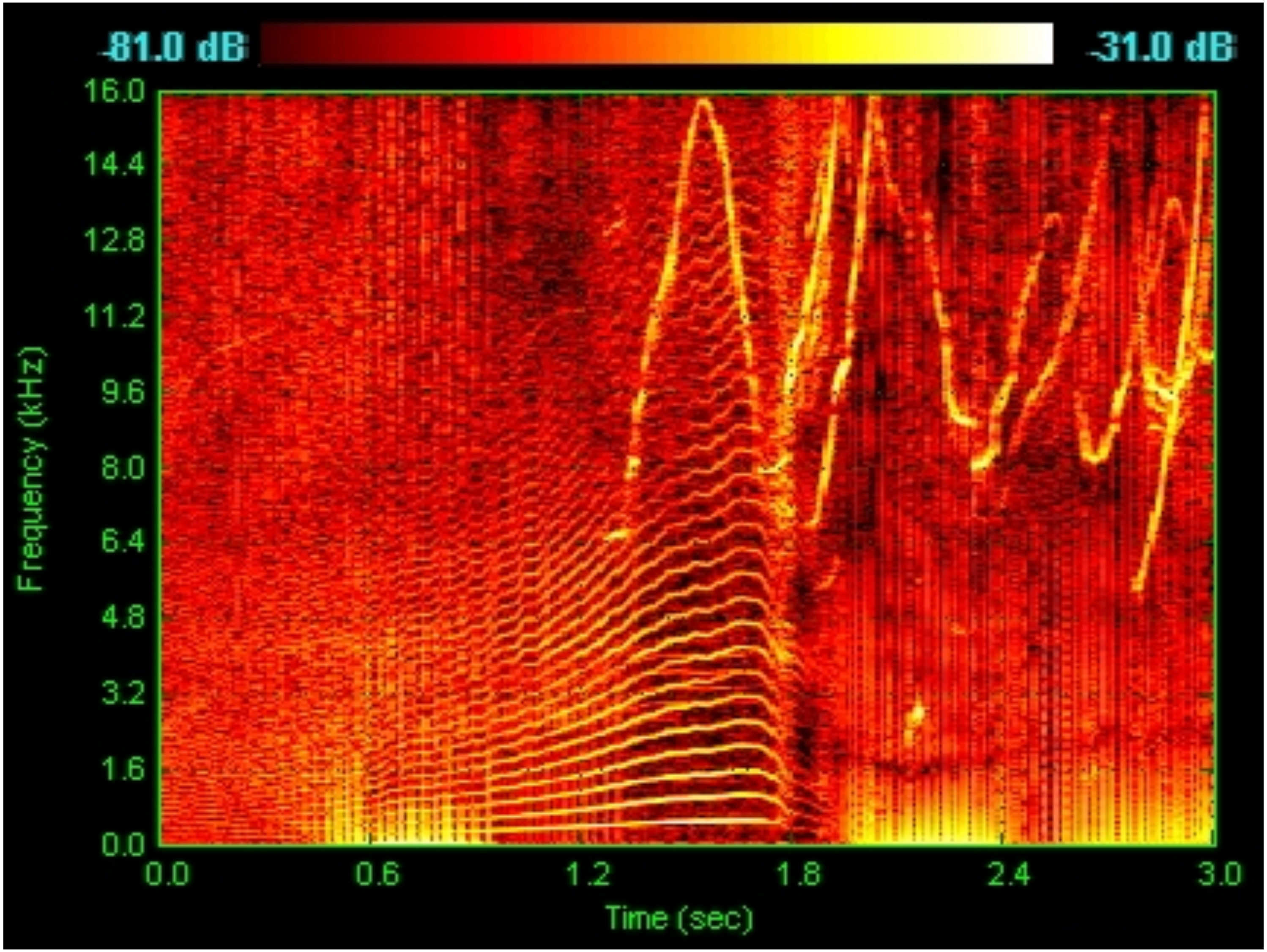
Ask Prof. Zue** to read this speech spectrogram



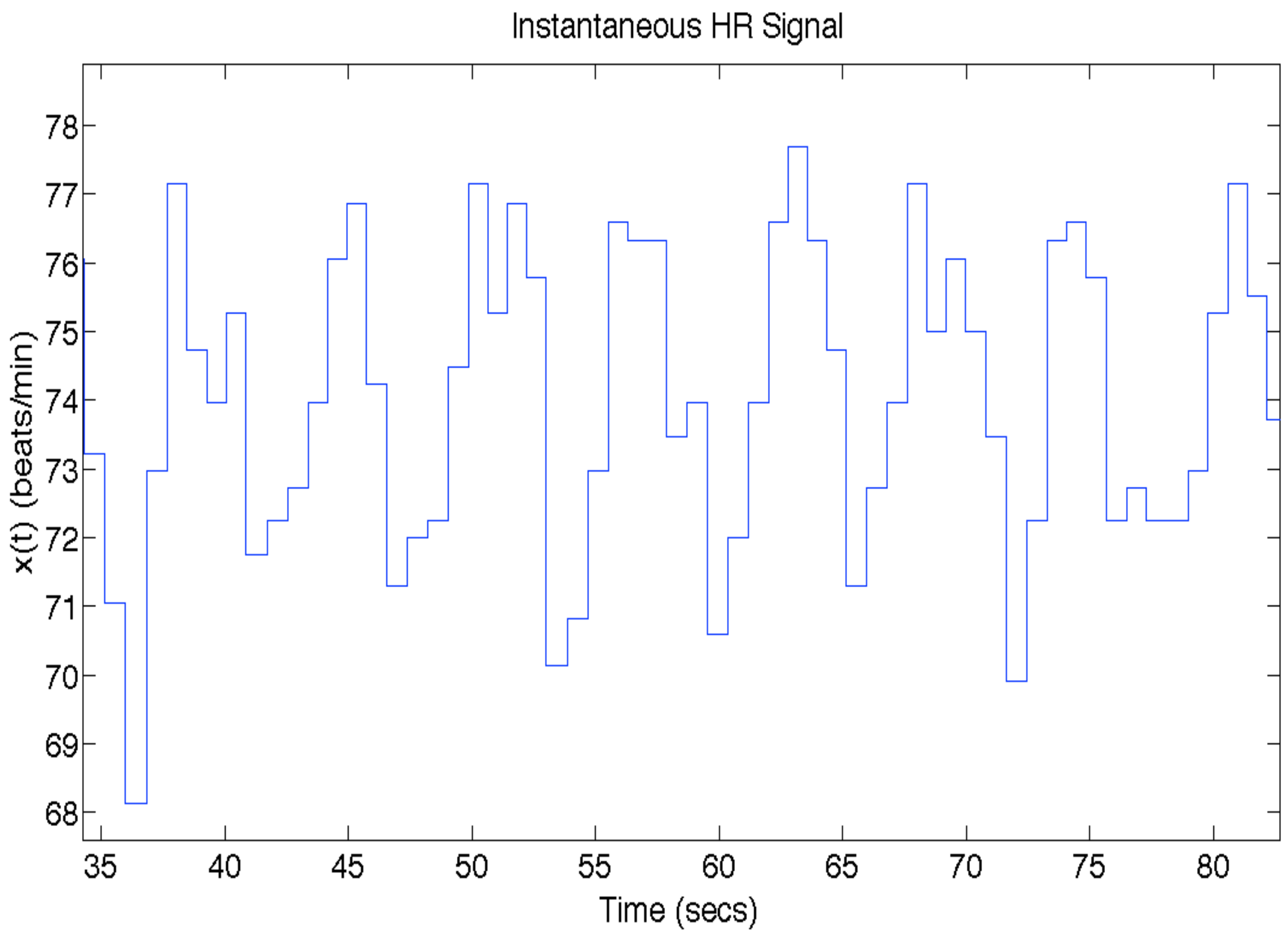
http://en.wikipedia.org/wiki/Spectrogram

**Victor was the first person to be able to read these! Get a sense of his achievements at http://www.okawa-foundation.or.jp/en/activities/prize/data/2012_evi.pdf

Dolphin sounds

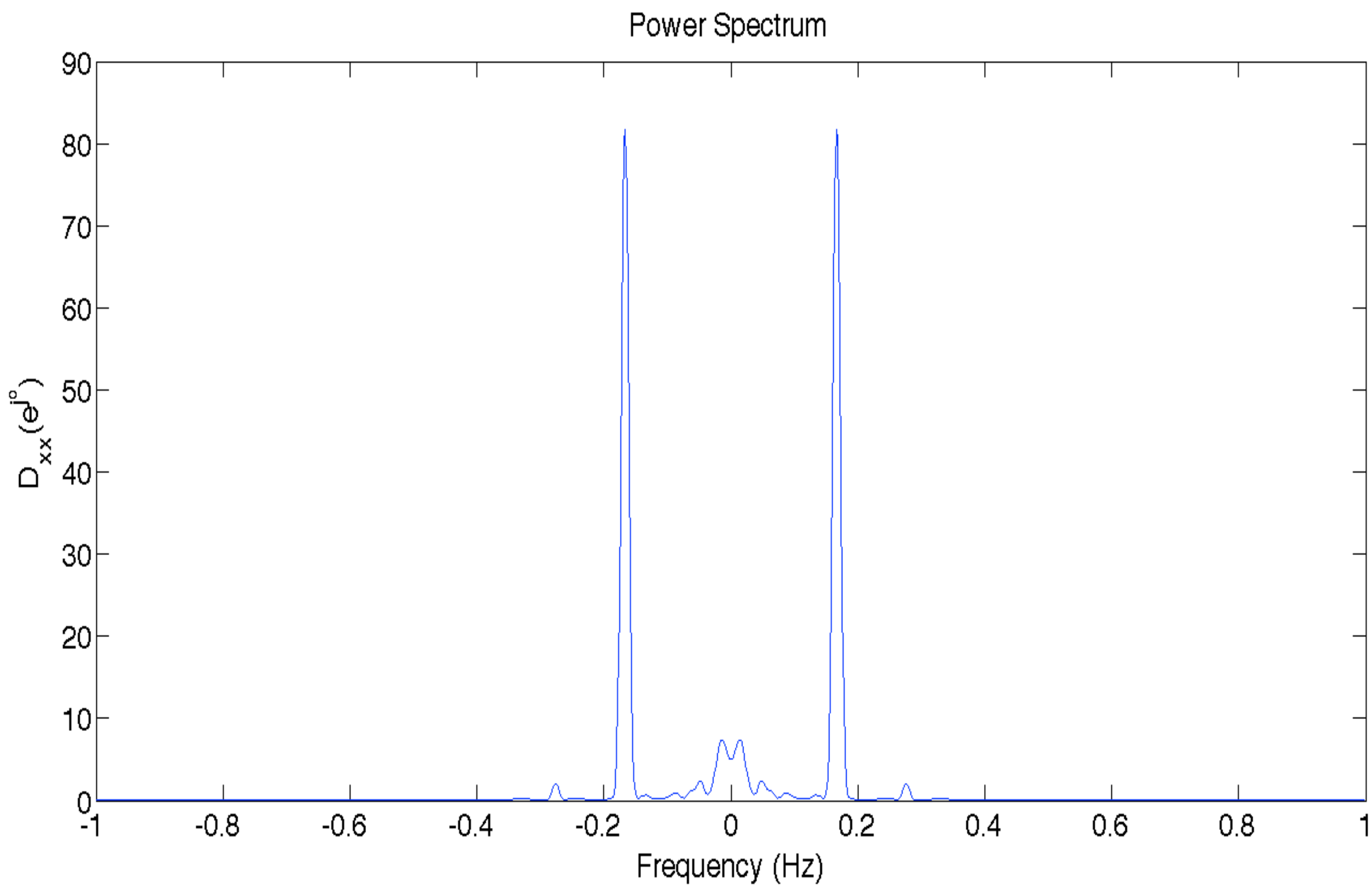


Instantaneous Heart Rate



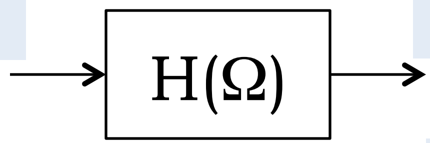
Heart-Rate Power Spectral Density $|X(\Omega)|^2$ averaged over many time-windows

- Breathing frequency: 0.18Hz



Relating Output Spectral Content to Input Spectral Content for an LTI System

$$x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(\Omega) e^{j\Omega n} d\Omega$$



$$y[n] = \frac{1}{2\pi} \int_{<2\pi>} H(\Omega) X(\Omega) e^{j\Omega n} d\Omega$$

$$y[n] = \frac{1}{2\pi} \int_{<2\pi>} Y(\Omega) e^{j\Omega n} d\Omega$$

$$Y(\Omega) = H(\Omega) X(\Omega)$$

Compare with y[n]=(h*x)[n]

Again, convolution in time
has mapped to
multiplication in frequency

Magnitude and Angle

$$Y(\Omega) = H(\Omega)X(\Omega)$$

$\Updownarrow$

$$|Y(\Omega)| = |H(\Omega)| . |X(\Omega)|$$

and

$$< Y(\Omega) = < H(\Omega) + < X(\Omega)$$

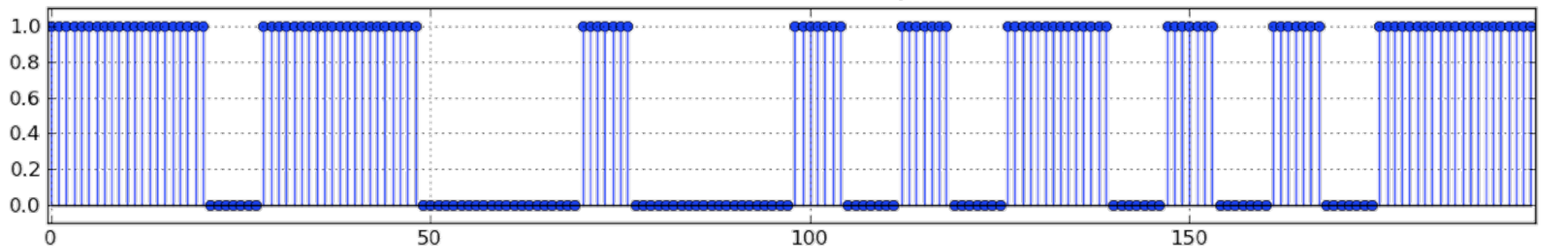
Core of the Story

1. A huge class of DT and CT signals
can be written --- using **Fourier** transforms --- as a **weighted sums of sinusoids** (ranging from very slow to very fast) or (equivalently, but more compactly) complex exponentials. The sums can be discrete ∑ or continuous ∫ (or both).

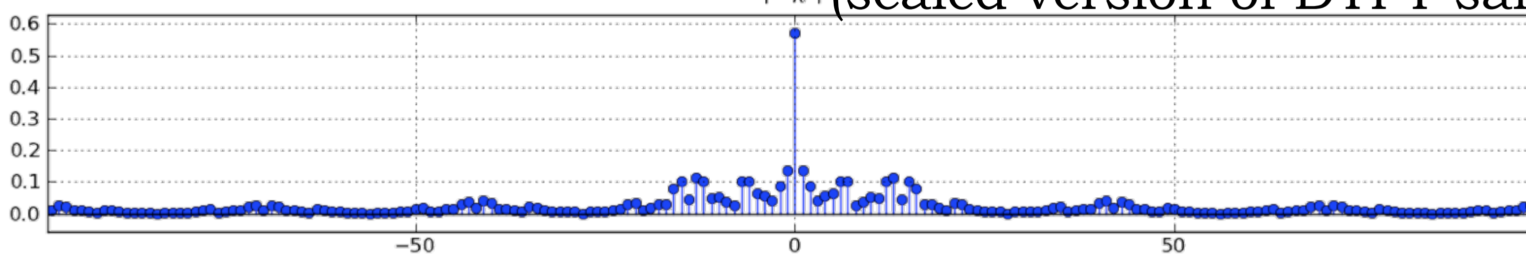
2. **LTI** systems act very simply on sums of sinusoids: **superposition** of responses to each sinusoid, with the **frequency response** determining the frequency-dependent scaling of magnitude, shifting in phase.

Spectrum of Digital Transmissions

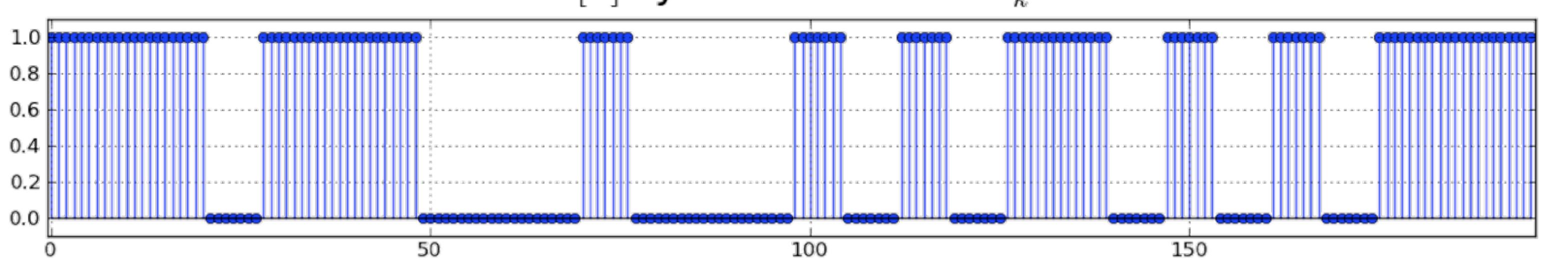
transmit @ 7 samples/bit



$|a_k|$ (scaled version of DTFT samples)



$x[n]$ synthesized from a_k



Observations on previous figure

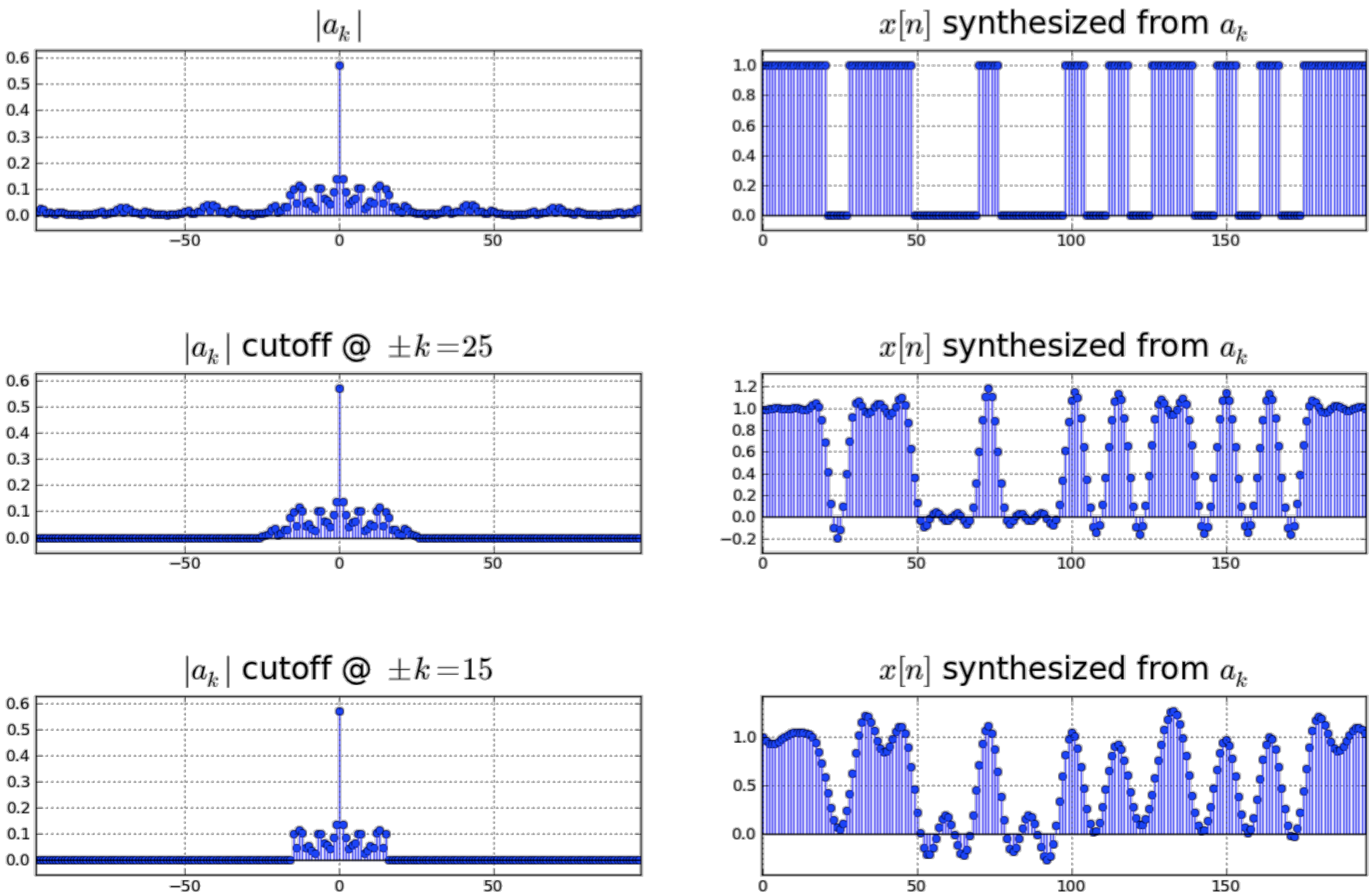
- The waveform x[n] cannot vary faster than the step change every 7 samples, so we expect the highest frequency components in the waveform to have a period around 14 samples. (The is rough and qualitative, as x[n] is not sinusoidal.)

- A period of 14 corresponds to a frequency of $2\pi/14 = \pi/7$, which is 1/7 of the way from 0 to the positive end of the frequency axis at π (so k approximately 100/7 or 14 in the figure). And that indeed is the neighborhood of where the Fourier coefficients drop off significantly in magnitude.

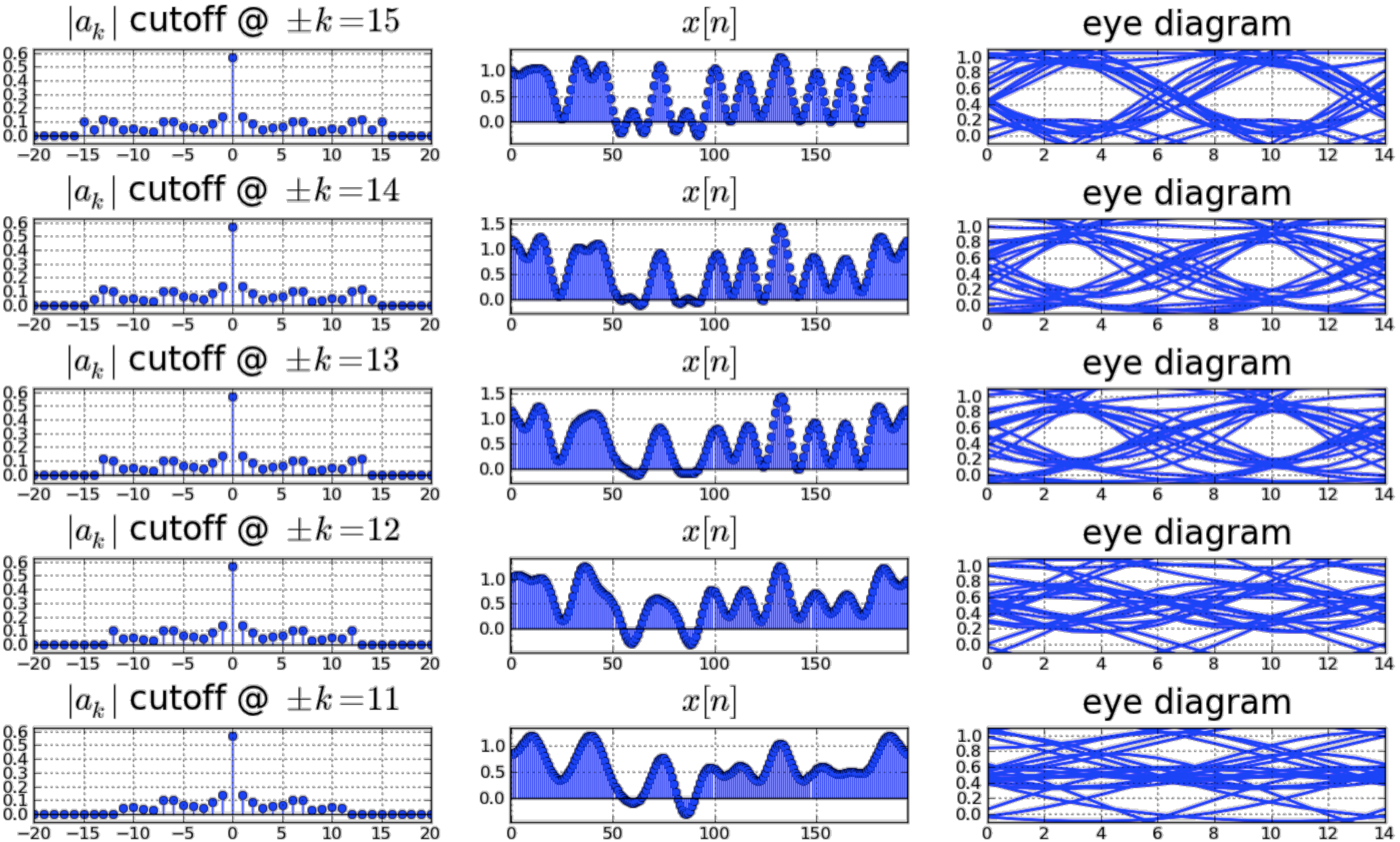
- There are also lower-frequency components corresponding to the fact that the 1 or 0 level may be held for several bit slots.

- And there are higher-frequency components that result from the transitions between voltage levels being sudden, not gradual.

Effect of Low-Pass Channel



How Low Can We Go?



7 samples/bit → 14 samples/period → k=(N/14)=(196/14)=14