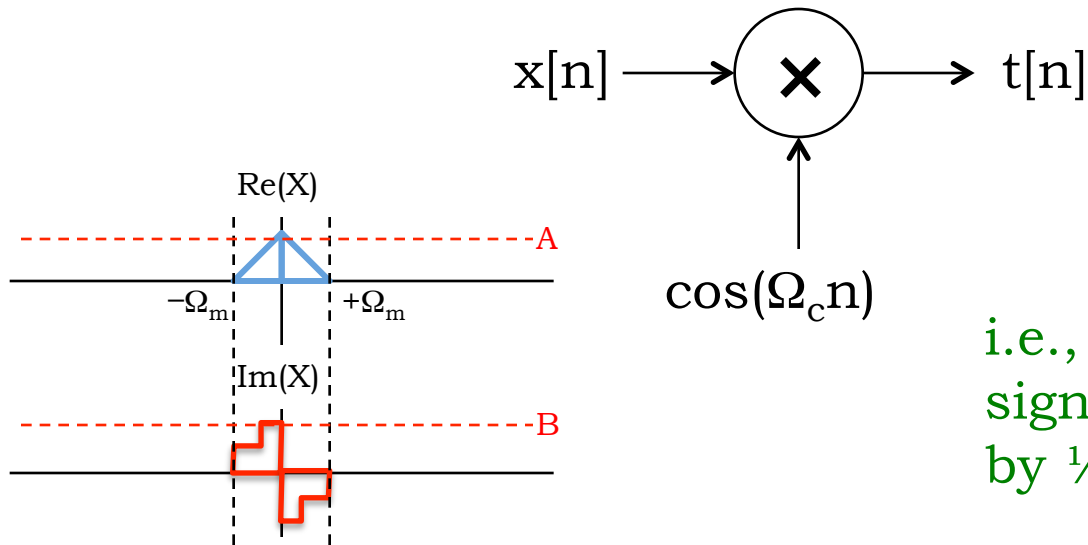


INTRODUCTION TO EECS II  
**DIGITAL  
 COMMUNICATION  
 SYSTEMS**

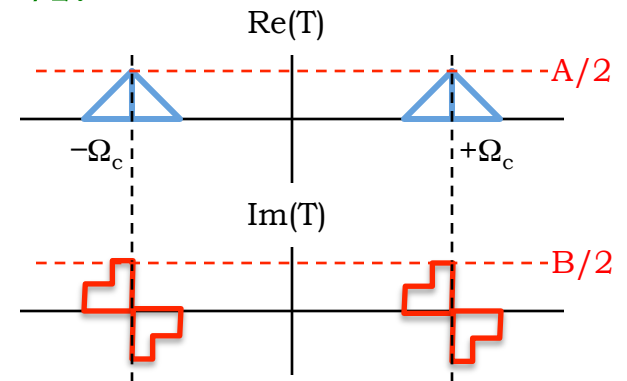
# 6.02 Fall 2013 Lecture #17

- Quadrature Modulation/Demodulation
- Frequency Division Multiplexing (FDM)

# Modulation by Heterodyning\*\* or Amplitude Modulation (AM)



i.e., just replicate baseband signal at  $\pm\Omega_c$ , and scale by  $1/2$ .

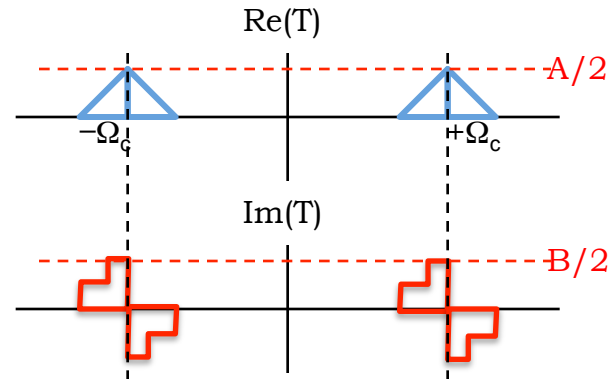


\*\* Reginald Fessenden's invention:

[http://www.ewh.ieee.org/reg/7/millennium/radio/radio\\_unsung.html](http://www.ewh.ieee.org/reg/7/millennium/radio/radio_unsung.html)

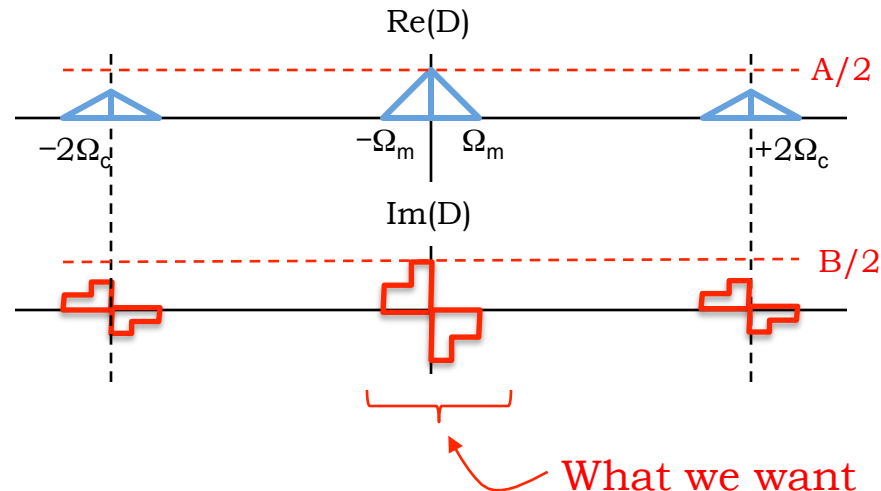
# Demodulation Frequency Diagram

$$R(\Omega) = T(\Omega)$$



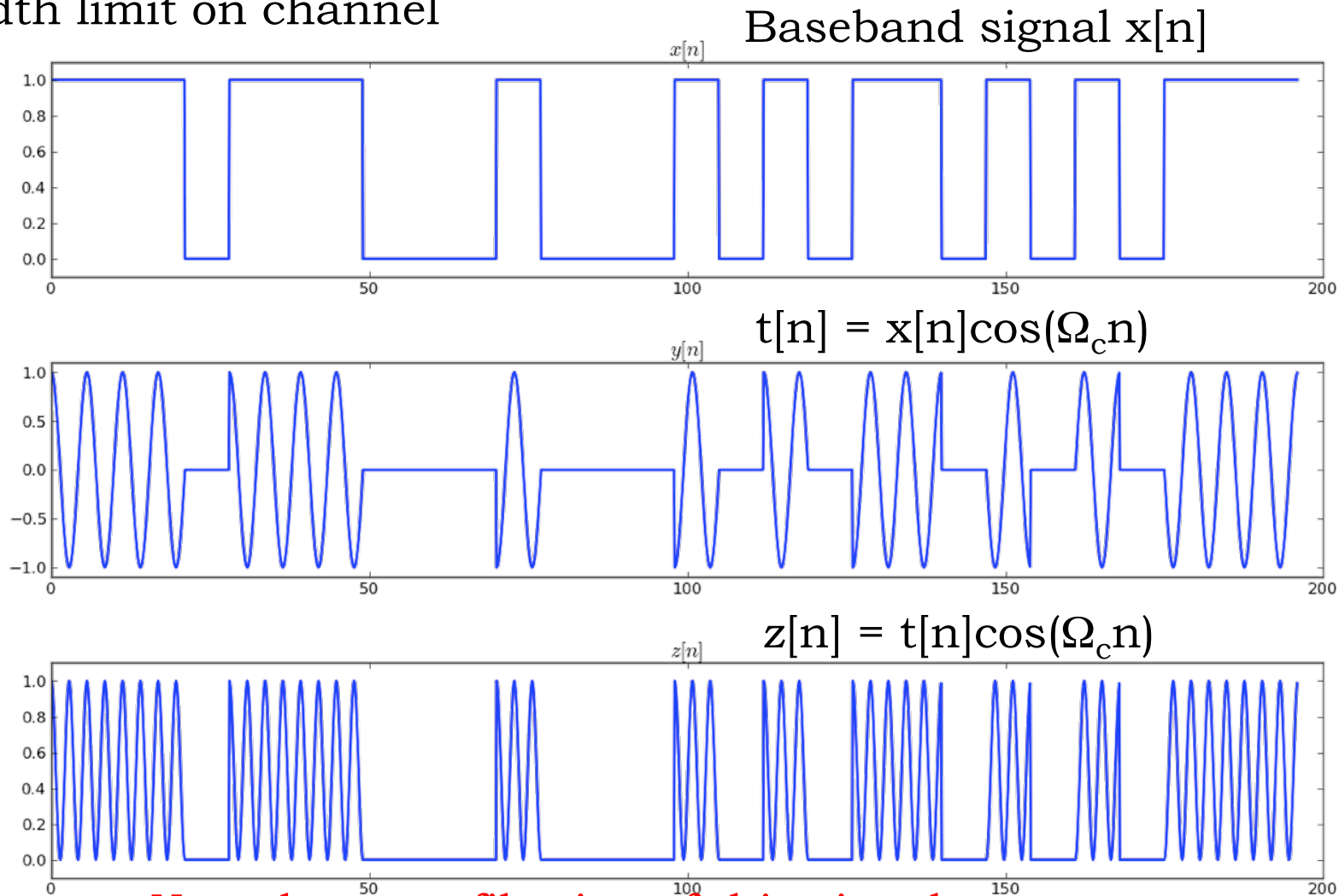
Note combining of signals around 0 results in doubling of amplitude

$$D(\Omega)$$



# Example: Demodulation (time)

Showing idealized signals ---  
no bandwidth limit on channel

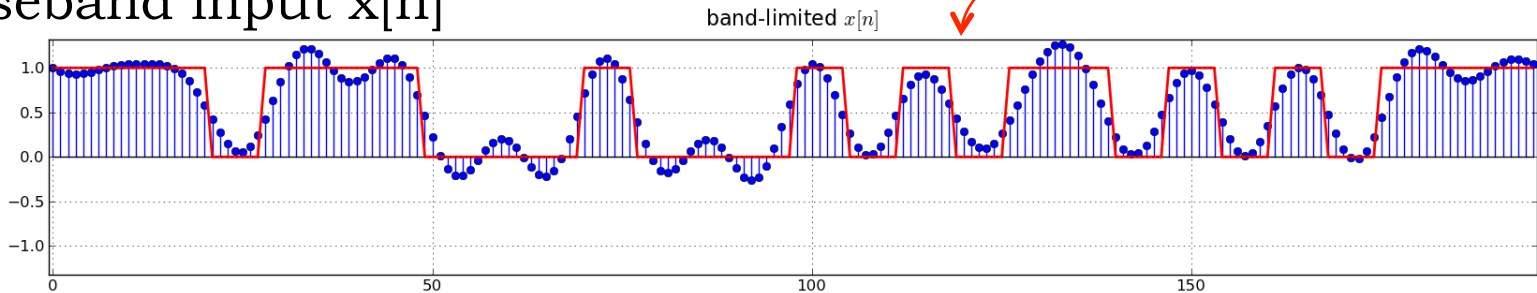


Note: lowpass filtering of this signal  
will yield  $x[n]/2$  !

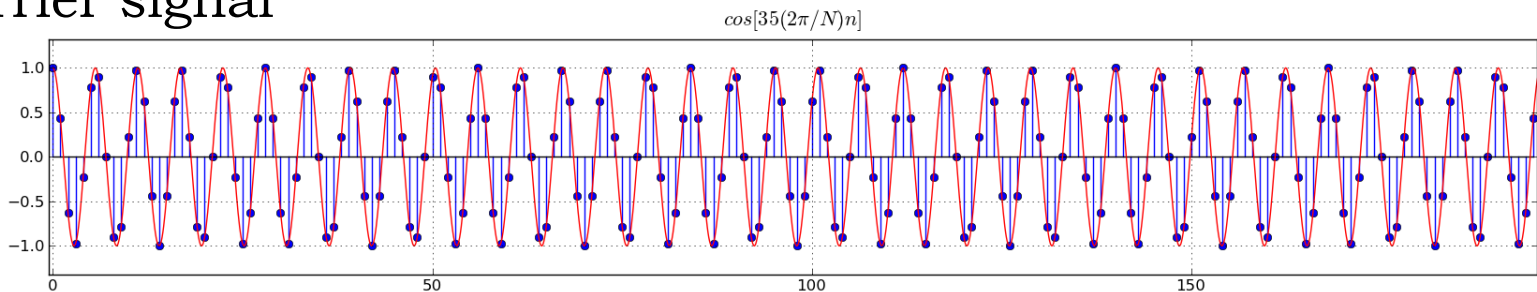
# Example: Modulation (time)

*Shaped pulses! Chosen because we know the channel is bandlimited*

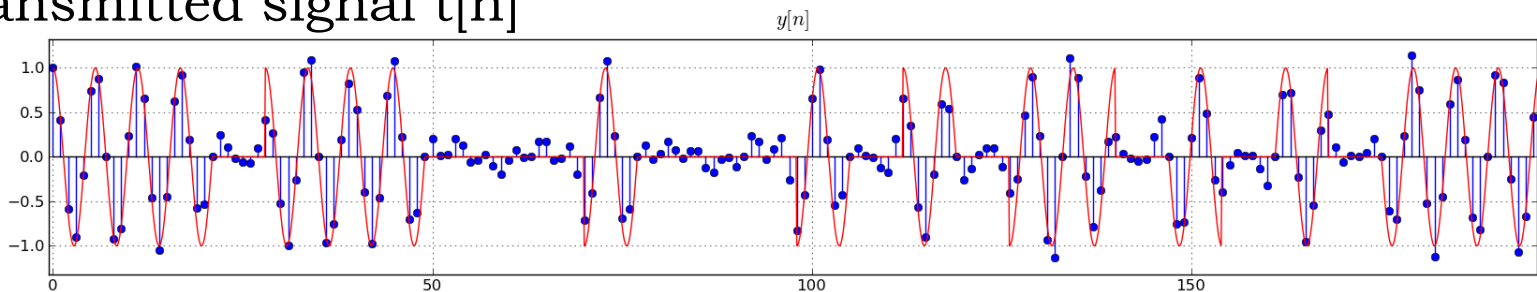
Baseband input  $x[n]$



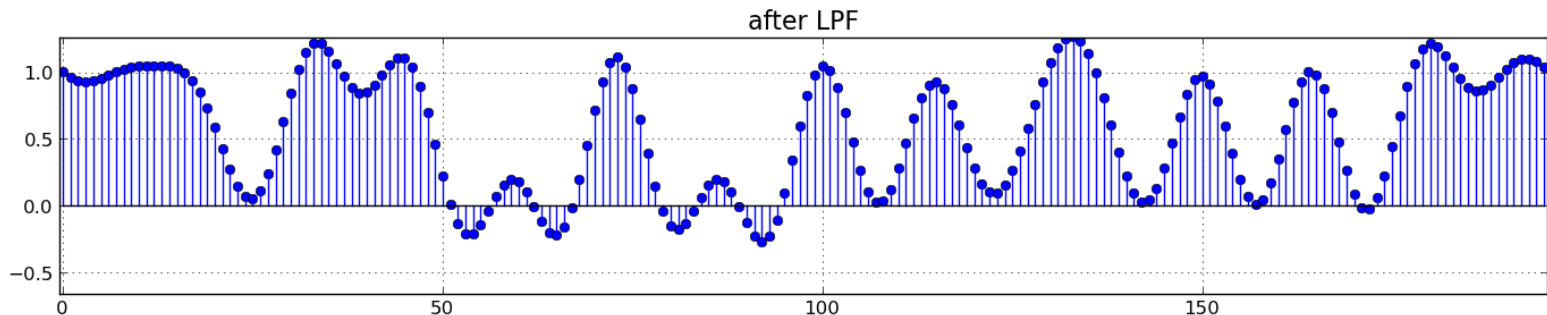
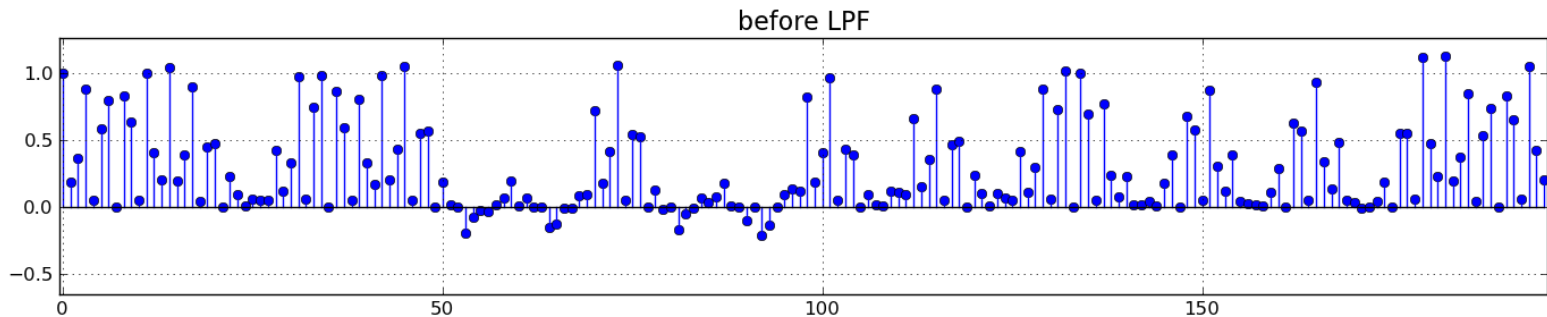
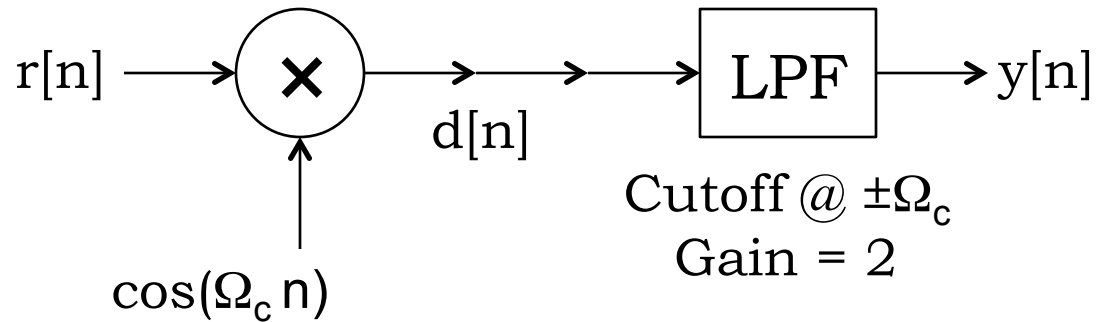
Carrier signal



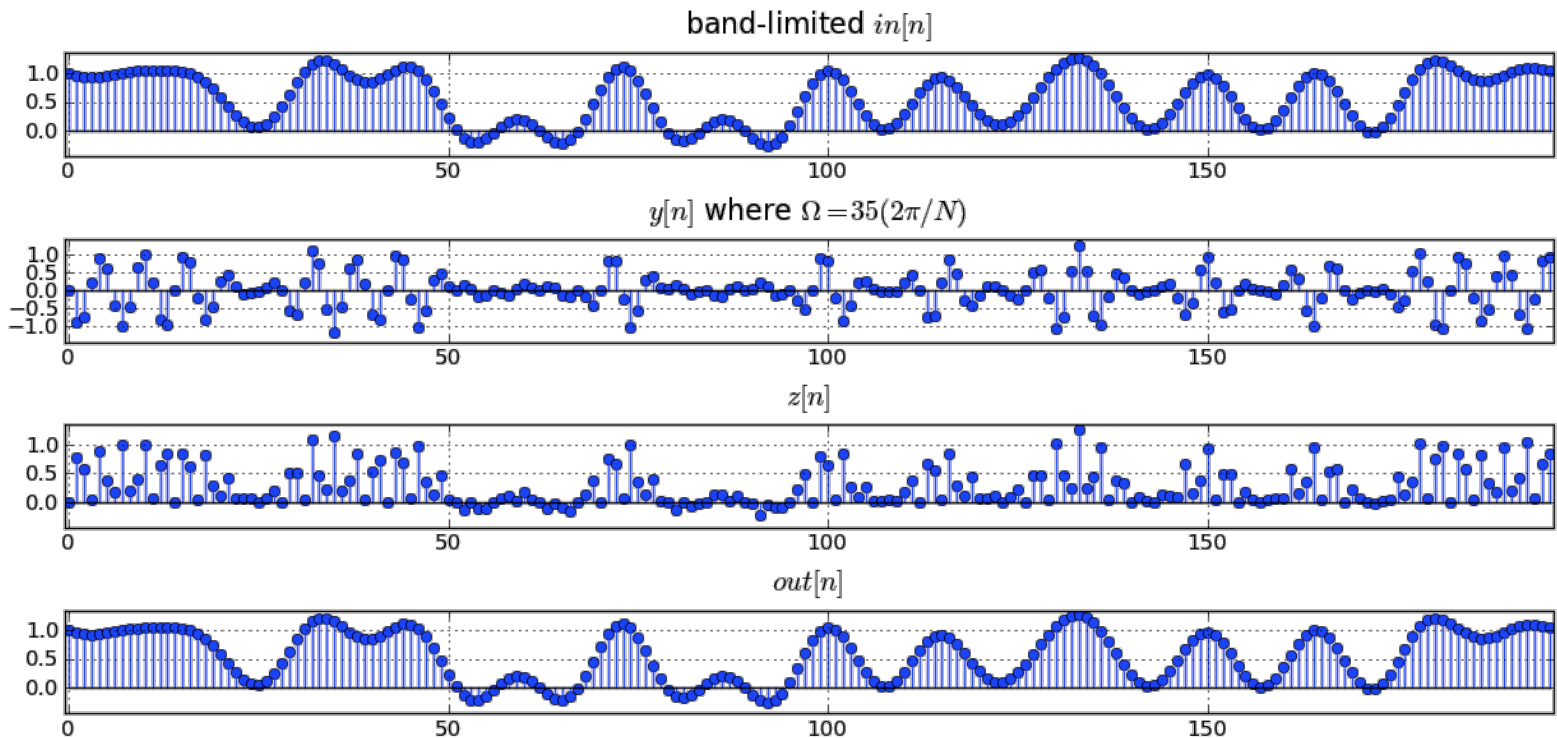
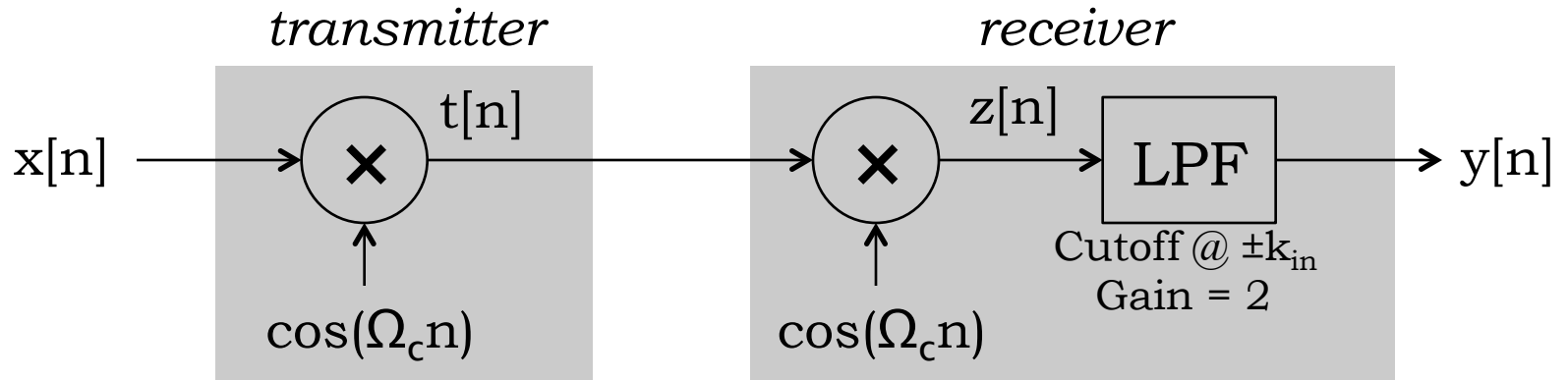
Transmitted signal  $t[n]$



# Demodulation



# Ideal Modulation/Demodulation



# Phase Error In Demodulation

When the receiver oscillator is out of phase with the transmitter:

$$d[n] = r[n] \cdot \cos(\Omega_c n - \varphi) = x[n] \cdot \cos(\Omega_c n) \cdot \cos(\Omega_c n - \varphi)$$

But

$$\cos(\Omega_c n) \cdot \cos(\Omega_c n - \varphi) = 0.5 \{ \cos(\varphi) + \cos(2\Omega_c n - \varphi) \}$$

It follows that the demodulated output, after the LPF of gain 2, is

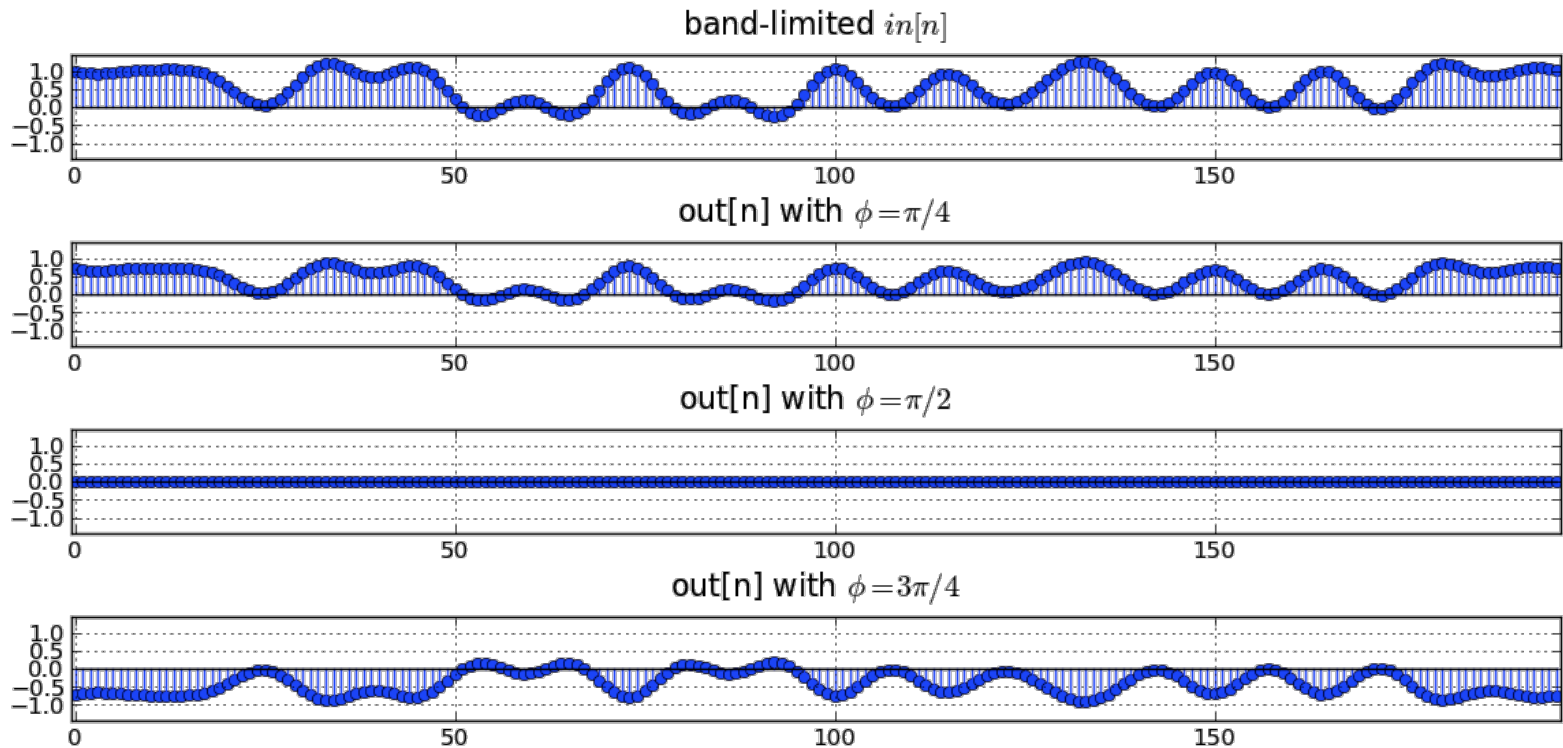
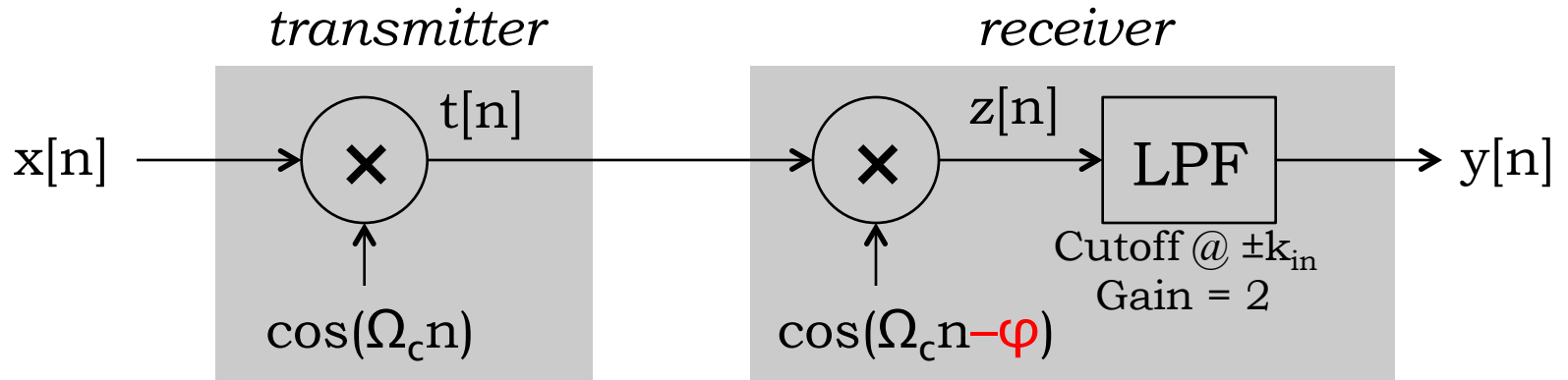
$$y[n] = x[n] \cdot \cos(\varphi)$$

So a phase error of  $\varphi$  results in amplitude scaling by  $\cos(\varphi)$ .

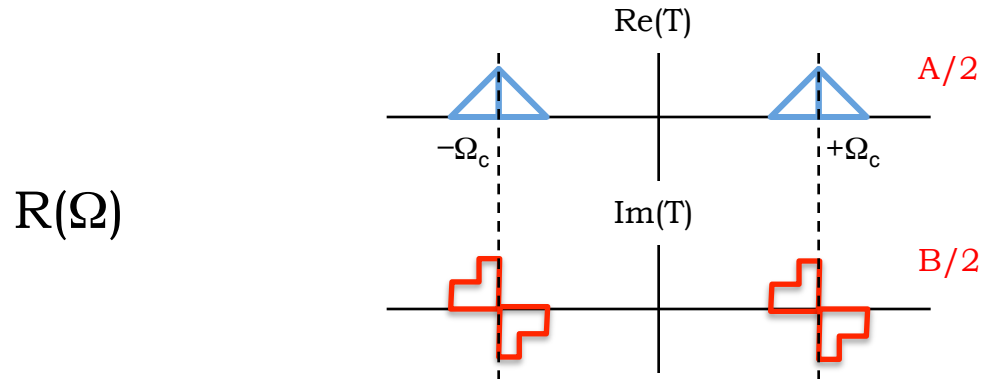
**Note:** in the extreme case where  $\varphi = \pi/2$ , we are demodulating by a sine rather than a cosine, and we get  $y[n] = 0$ .



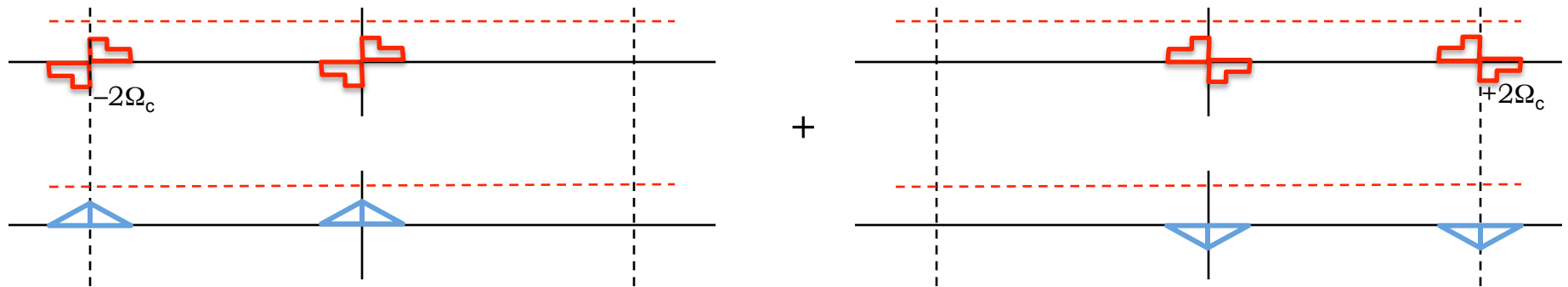
# Phase Error in Demodulator



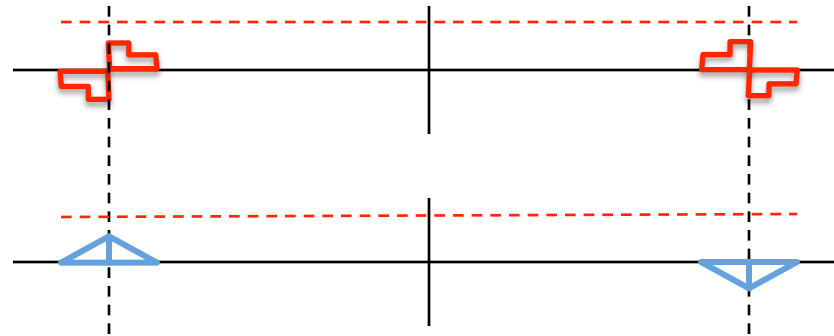
# Demodulation with $\sin(\Omega_c n)$



$D(\Omega)$

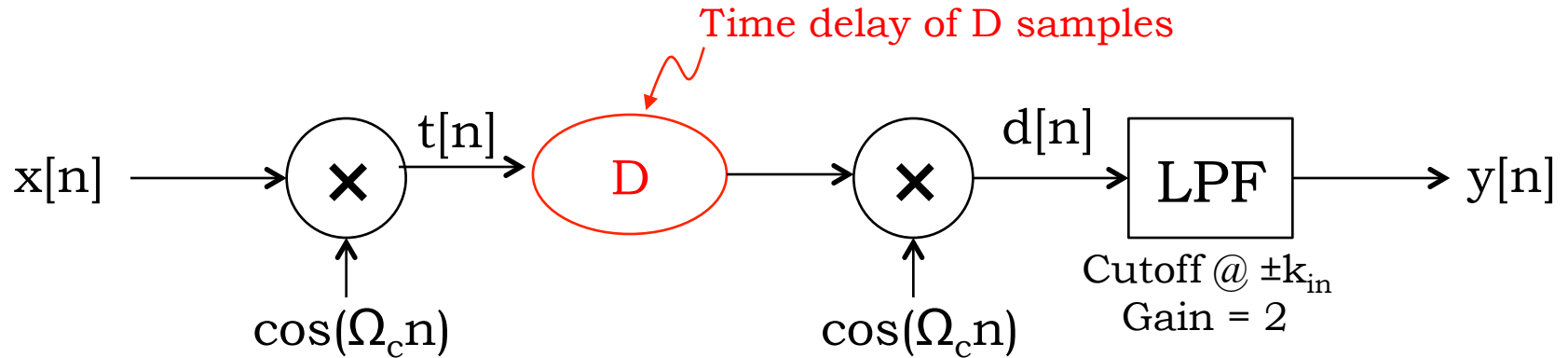


... produces



Note combining of signals around 0 results in cancellation!

# Channel Delay



Very similar math to the previous “phase error” case:

$$\begin{aligned}d[n] &= t[n - D] \cdot \cos(\Omega_c n) \\ &= x[n - D] \cdot \cos[\Omega_c (n - D)] \cdot \cos(\Omega_c n)\end{aligned}$$

Passing this through the LPF:

$$y[n] = x[n - D] \cdot \cos(\Omega_c D)$$

Looks like a phase error  
of  $\Omega_c D$

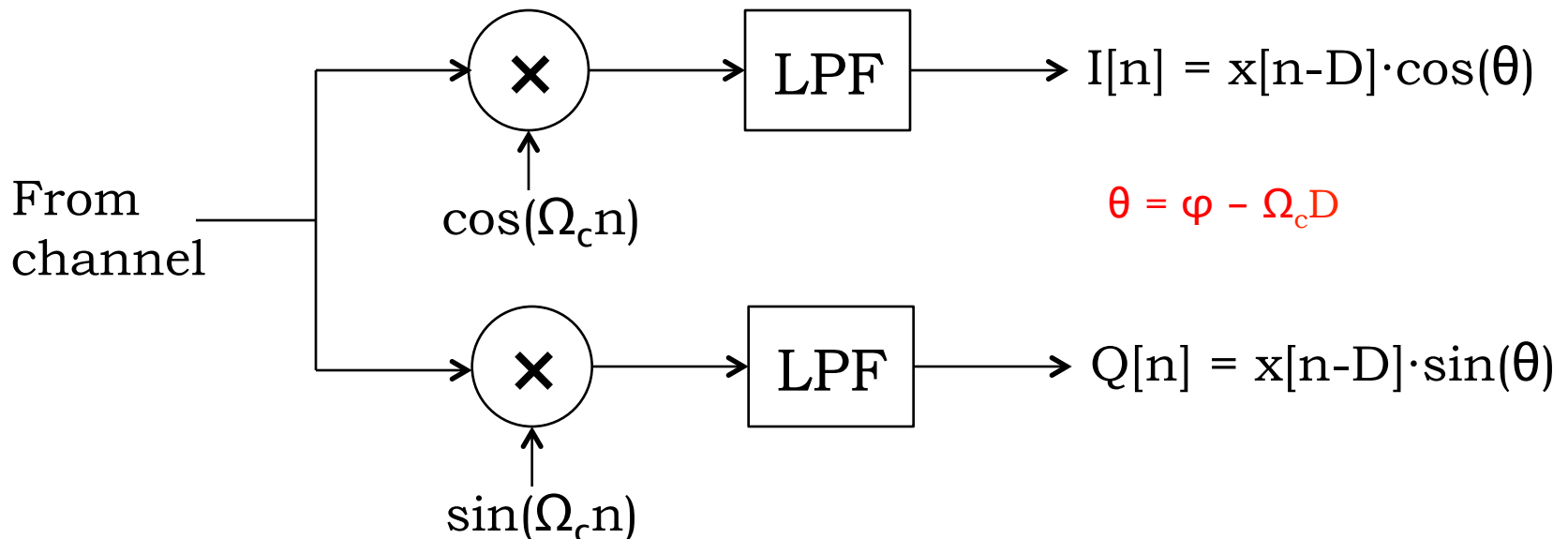
If  $\Omega_c D$  is an odd multiple of  $\pi / 2$ , then  $y[n]=0$  !!

# Fixing Phase Problems in the Receiver

So phase errors and channel delay both result in a scaling of the output amplitude, where the magnitude of the scaling can't necessarily be determined at system design time:

- channel delay varies on mobile devices
- phase difference between transmitter and receiver is arbitrary

One solution: *quadrature demodulation*



# Quadrature Demodulation

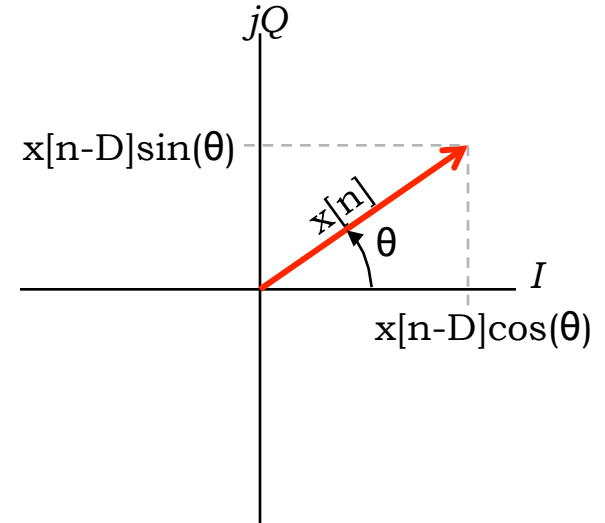
If we let

$$w[n] = I[n] + jQ[n]$$

then

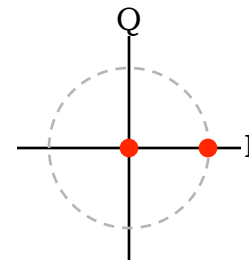
$$\begin{aligned} |w[n]| &= \sqrt{I[n]^2 + Q[n]^2} \\ &= |x[n - D]| \sqrt{\cos^2 \theta + \sin^2 \theta} \\ &= |x[n - D]| \end{aligned}$$

OK for recovering  $x[n]$  if it never goes negative, as in on-off keying

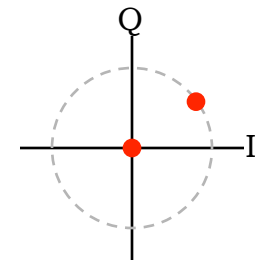


Constellation diagrams:

$$x[n] = \{0, 1\}$$

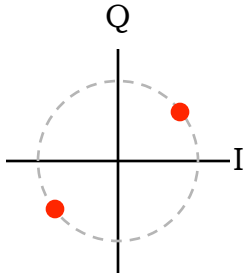


transmitter



receiver

# Dealing With Phase Ambiguity in Bipolar Modulation



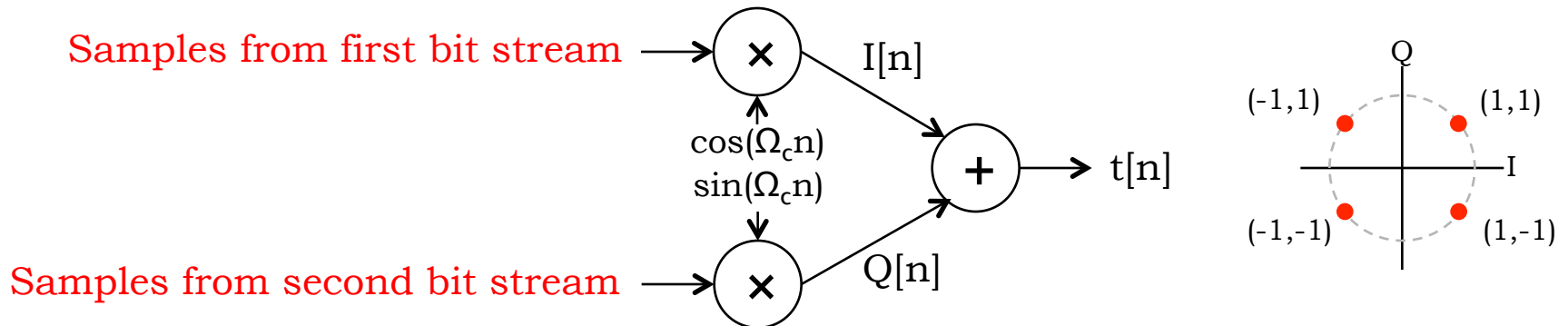
In bipolar modulation ( $x[n]=\pm 1$ ), also called Binary Phase Shift Keying (**BPSK**) since the modulated carrier changes phase by  $\pi/2$  when  $x[n]$  switches levels, the received constellation will be rotated with respect to the transmitter's constellation. Which phase corresponds to which bit?

Different fixes:

1. Send an agreed-on sign-definite preamble
2. Transmit differentially encoded bits, e.g., transmit a “1” by stepping the phase by  $\pi$ , transmit a “0” by not changing the phase. (Agreed starting symbol, say “0”.)

# QPSK Modulation

We can use the quadrature scheme at the transmitter too:



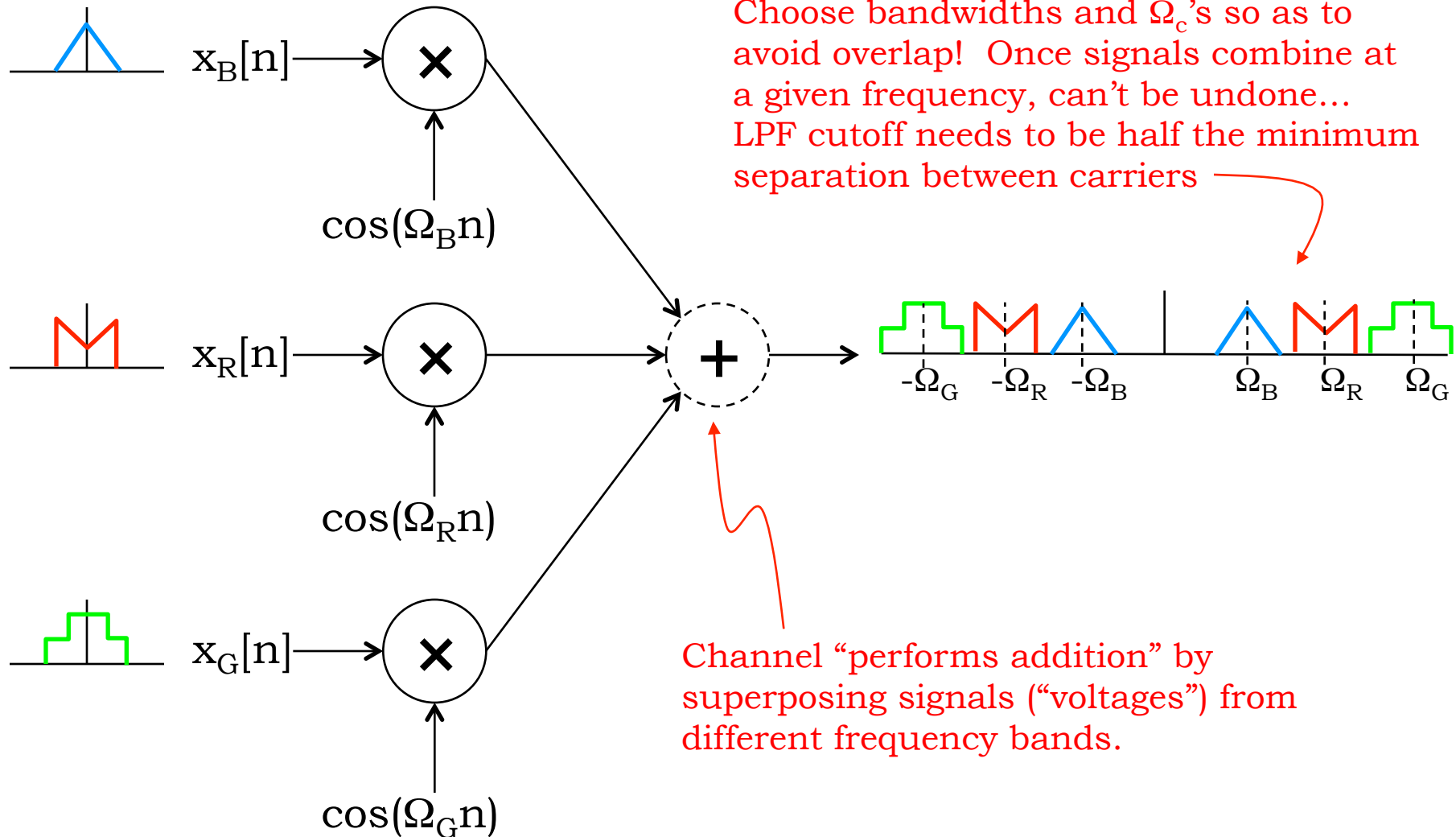


# Phase Shift Keying underlies many familiar modulation schemes

The [wireless LAN](#) standard, [IEEE 802.11b-1999](#),<sup>[1][2]</sup> uses a variety of different PSKs depending on the data-rate required. At the basic-rate of 1 [Mbit/s](#), it uses DBPSK (differential BPSK). To provide the extended-rate of 2 Mbit/s, DQPSK is used. In reaching 5.5 Mbit/s and the full-rate of 11 Mbit/s, QPSK is employed, but has to be coupled with [complementary code keying](#). The higher-speed wireless LAN standard, [IEEE 802.11g-2003](#)<sup>[1][3]</sup> has eight data rates: 6, 9, 12, 18, 24, 36, 48 and 54 Mbit/s. The 6 and 9 Mbit/s modes use [OFDM](#) modulation where each sub-carrier is BPSK modulated. The 12 and 18 Mbit/s modes use OFDM with QPSK. The fastest four modes use OFDM with forms of [quadrature amplitude modulation](#). Because of its simplicity BPSK is appropriate for low-cost passive transmitters, and is used in [RFID](#) standards such as [ISO/IEC 14443](#) which has been adopted for [biometric passports](#), credit cards such as [American Express's ExpressPay](#), and many other applications.<sup>[4]</sup> [Bluetooth](#) 2 will use (p/4)-DQPSK at its lower rate (2 Mbit/s) and 8-DPSK at its higher rate (3 Mbit/s) when the link between the two devices is sufficiently robust. Bluetooth 1 modulates with [Gaussian minimum-shift keying](#), a binary scheme, so either modulation choice in version 2 will yield a higher data-rate. A similar technology, [IEEE 802.15.4](#) (the wireless standard used by [ZigBee](#)) also relies on PSK. IEEE 802.15.4 allows the use of two frequency bands: 868–915 [MHz](#) using BPSK and at 2.4 [GHz](#) using OQPSK.

[http://en.wikipedia.org/wiki/Phase-shift\\_keying](http://en.wikipedia.org/wiki/Phase-shift_keying)

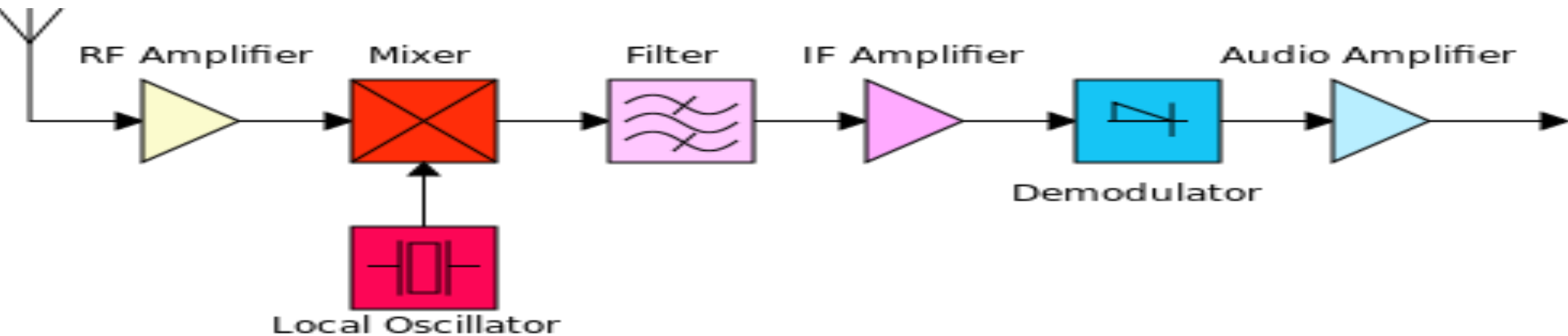
# Multiple Transmitters: *Frequency Division Multiplexing (FDM)*



# Edwin Howard Armstrong\*\* (1890-1954)



Invented and patented first positive feedback (“regenerative”) electronic oscillator and tuned amplifier while undergrad at Columbia (1914); superheterodyne reception for FDM (1918, see block diagram below); frequency modulation (FM, 1930’s); ... (42 patents)



\*\*[http://en.wikipedia.org/wiki/Edwin\\_Armstrong](http://en.wikipedia.org/wiki/Edwin_Armstrong)

[http://en.wikipedia.org/wiki/Superheterodyne\\_receiver](http://en.wikipedia.org/wiki/Superheterodyne_receiver)

# Superhhet Reception: AM and FM Radio

AM radio receivers span 510 – 1655 kHz in the US, with carrier frequencies of different stations spaced 10 kHz apart.

The usual IF frequency is 455 kHz.

FM radio receivers span 88 – 108 MHz, with carrier frequencies spaced 200 kHz apart.

The usual IF frequency is 10.7 MHz.