• Quadrature Modulation/Demodulation
• Frequency Division Multiplexing (FDM)
Modulation by Heterodyning** or Amplitude Modulation (AM)

\[ x[n] \times \cos(\Omega_c n) \rightarrow t[n] \]

i.e., just replicate baseband signal at \( \pm \Omega_c \), and scale by \( \frac{1}{2} \).

** Reginald Fessenden’s invention:
http://www.ewh.ieee.org/reg/7/millennium/radio/radio_unsung.html
Demodulation Frequency Diagram

\[ R(\Omega) = T(\Omega) \]

What we want

Note combining of signals around 0 results in doubling of amplitude

What we want
Example: Demodulation (time)

Showing idealized signals ---
no bandwidth limit on channel

Baseband signal $x[n]$

t[n] = $x[n]\cos(\Omega_c n)$

$y[n] = x[n]\cos(\Omega_c n)$

$z[n] = t[n]\cos(\Omega_c n)$

Note: lowpass filtering of this signal will yield $x[n]/2$ !
Example: Modulation (time)

Shaped pulses! Chosen because we know the channel is bandlimited.

Baseband input $x[n]$

Carrier signal

Transmitted signal $t[n]$
Demodulation

\[ r[n] \times \cos(\Omega_c n) \rightarrow d[n] \rightarrow \text{LPF} \rightarrow y[n] \]

Cutoff @ $\pm \Omega_c$
Gain = 2

*Diagram showing signal processing flow with demodulation.*
Ideal Modulation/Demodulation

transmitter

\[ x[n] \rightarrow t[n] \rightarrow \cos(\Omega_c n) \]

receiver

\[ z[n] \rightarrow \cos(\Omega_c n) \rightarrow y[n] \]

LPF

Cutoff @ ±k_{in}

Gain = 2

\[ x[n] \rightarrow t[n] \rightarrow y[n] \]

\[ \text{band-limited } \text{in}[n] \]

\[ y[n] \text{ where } \Omega = 35(2\pi/N) \]

\[ z[n] \]

\[ \text{out}[n] \]
Phase Error In Demodulation

When the receiver oscillator is out of phase with the transmitter:

\[ d[n] = r[n] \cdot \cos(\Omega_c n - \varphi) = x[n] \cdot \cos(\Omega_c n) \cdot \cos(\Omega_c n - \varphi) \]

But

\[
\cos(\Omega_c n) \cdot \cos(\Omega_c n - \varphi) = 0.5\{\cos(\varphi) + \cos(2\Omega_c n - \varphi)\}
\]

It follows that the demodulated output, after the LPF of gain 2, is

\[ y[n] = x[n] \cdot \cos(\varphi) \]

So a phase error of \( \varphi \) results in amplitude scaling by \( \cos(\varphi) \).

Note: in the extreme case where \( \varphi = \pi/2 \), we are demodulating by a sine rather than a cosine, and we get \( y[n] = 0 \).
**Phase Error in Demodulator**

**transmitter**

\[ x[n] \rightarrow t[n] \rightarrow \cos(\Omega c n) \]

**receiver**

\[ z[n] \rightarrow \cos(\Omega c n - \varphi) \rightarrow \text{LPF} \]

- Cutoff @ \( \pm k_in \)
- Gain = 2

Graphs showing:
- Band-limited \( in[n] \)
- \( out[n] \) with \( \phi = \pi/4 \)
- \( out[n] \) with \( \phi = \pi/2 \)
- \( out[n] \) with \( \phi = 3\pi/4 \)
Demodulation with $\sin(\Omega_c n)$

\[
\begin{align*}
R(\Omega) & \quad \text{Re}(T) \\
\quad -\Omega_c & \quad +\Omega_c \\
\quad \text{Im}(T) & \quad A/2 \\
\quad B/2 & \quad -2\Omega_c \\
\quad +2\Omega_c & \\
D(\Omega) & \\
\end{align*}
\]
… produces

Note combining of signals around 0 results in cancellation!
Channel Delay

Very similar math to the previous “phase error” case:

\[ d[n] = t[n - D] \cdot \cos(\Omega_c n) = x[n - D] \cdot \cos[\Omega_c (n - D)] \cdot \cos(\Omega_c n) \]

Passing this through the LPF:

\[ y[n] = x[n - D] \cdot \cos(\Omega_c D) \]

If \( \Omega_c D \) is an odd multiple of \( \pi / 2 \), then \( y[n] = 0 \) !!
Fixing Phase Problems in the Receiver

So phase errors and channel delay both result in a scaling of the output amplitude, where the magnitude of the scaling can’t necessarily be determined at system design time:
- channel delay varies on mobile devices
- phase difference between transmitter and receiver is arbitrary

One solution: *quadrature demodulation*

\[
\begin{align*}
I[n] &= x[n-D] \cdot \cos(\theta) \\
Q[n] &= x[n-D] \cdot \sin(\theta)
\end{align*}
\]

\[\theta = \phi - \Omega_c D\]
Quadrature Demodulation

If we let

\[ w[n] = I[n] + jQ[n] \]

then

\[ |w[n]| = \sqrt{I[n]^2 + Q[n]^2} \]

\[ = |x[n-D]| \sqrt{\cos^2 \theta + \sin^2 \theta} \]

\[ = |x[n-D]| \]

OK for recovering \( x[n] \) if it never goes negative, as in on-off keying
Dealing With Phase Ambiguity in Bipolar Modulation

In bipolar modulation ($x[n]=\pm 1$), also called Binary Phase Shift Keying (BPSK) since the modulated carrier changes phase by $\pi/2$ when $x[n]$ switches levels, the received constellation will be rotated with respect to the transmitter’s constellation. Which phase corresponds to which bit?

Different fixes:

1. Send an agreed-on sign-definite preamble
2. Transmit differentially encoded bits, e.g., transmit a “1” by stepping the phase by $\pi$, transmit a “0” by not changing the phase. (Agreed starting symbol, say “0”.)
QPSK Modulation

We can use the quadrature scheme at the transmitter too:

\[ I[n] = \cos(\Omega_c n) \times \text{Samples from first bit stream} \]

\[ Q[n] = \sin(\Omega_c n) \times \text{Samples from second bit stream} \]

\[ t[n] = I[n] + Q[n] \]

Diagram:
Phase Shift Keying underlies many familiar modulation schemes

The wireless LAN standard, IEEE 802.11b-1999,[1][2] uses a variety of different PSKs depending on the data-rate required. At the basic-rate of 1 Mbit/s, it uses DBPSK (differential BPSK). To provide the extended-rate of 2 Mbit/s, DQPSK is used. In reaching 5.5 Mbit/s and the full-rate of 11 Mbit/s, QPSK is employed, but has to be coupled with complementary code keying. The higher-speed wireless LAN standard, IEEE 802.11g-2003[1][3] has eight data rates: 6, 9, 12, 18, 24, 36, 48 and 54 Mbit/s. The 6 and 9 Mbit/s modes use OFDM modulation where each sub-carrier is BPSK modulated. The 12 and 18 Mbit/s modes use OFDM with QPSK. The fastest four modes use OFDM with quadrature amplitude modulation.

Because of its simplicity BPSK is appropriate for low-cost passive transmitters, and is used in RFID standards such as ISO/IEC 14443 which has been adopted for biometric passports, credit cards such as American Express's ExpressPay, and many other applications.[4] Bluetooth 2 will use (p/4)-DQPSK at its lower rate (2 Mbit/s) and 8-DPSK at its higher rate (3 Mbit/s) when the link between the two devices is sufficiently robust. Bluetooth 1 modulates with Gaussian minimum-shift keying, a binary scheme, so either modulation choice in version 2 will yield a higher data-rate. A similar technology, IEEE 802.15.4 (the wireless standard used by ZigBee) also relies on PSK. IEEE 802.15.4 allows the use of two frequency bands: 868–915 MHz using BPSK and at 2.4 GHz using OQPSK.

http://en.wikipedia.org/wiki/Phase-shift_keying
Multiple Transmitters: 
*Frequency Division Multiplexing (FDM)*

Choose bandwidths and $\Omega_c$’s so as to avoid overlap! Once signals combine at a given frequency, can’t be undone... LPF cutoff needs to be half the minimum separation between carriers.

Channel “performs addition” by superposing signals (“voltages”) from different frequency bands.
Edwin Howard Armstrong**
(1890-1954)

Invented and patented first positive feedback (“regenerative”) electronic oscillator and tuned amplifier while undergrad at Columbia (1914); superheterodyne reception for FDM (1918, see block diagram below); frequency modulation (FM, 1930’s); ... (42 patents)

**http://en.wikipedia.org/wiki/Edwin_Armstrong
http://en.wikipedia.org/wiki/Superheterodyne_receiver
Superhet Reception: AM and FM Radio

AM radio receivers span 510 – 1655 kHz in the US, with carrier frequencies of different stations spaced 10 kHz apart. The usual IF frequency is 455 kHz.

FM radio receivers span 88 – 108 MHz, with carrier frequencies spaced 200 kHz apart. The usual IF frequency is 10.7 MHz.