MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

6.02 Introduction to EECS II
Spring 2011

Quiz 1

Name	*Score*

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☐	12n	Fabian Lim	38-166
☐	1p	John Sun	38-166
☐	2p	John Sun	38-166

Please write your answers legibly in the spaces provided. You can use the backs of the pages if you need extra room for your answer or scratch work. Make sure we can find your answer!

You can use a calculator and one 8.5” x 11” cribsheet.

Partial credit will only be given in cases where you show your work and (very briefly) explain your approach.

Problem	Score
#1 (30 points)	
#2 (20 points)	
#3 (50 points)	

Problem 1. Information, Entropy and Huffman Codes (30 points)

There's a weekly surprise party at a local independent living group with an equal probability that the event will happen on any of the seven days.

(A) (3 points) You learn that party won't be on the weekend, i.e., not Saturday or Sunday. Give an expression for the number of bits of information you have received.

Expression for number of bits of information received: ____________________

(B) (4 points) Give an expression for the expected length in bits of a Huffman encoding of a message that lists the day of the party for each week of the 52-week year, i.e., a message consisting of 52 variable-length symbols, where each day is encoded separately using the Huffman code. The choice for each week is independent of the choices for other weeks.

Expression for expected length of message in bits: ____________________

Examining the historical record, you discover that the probabilities for party days aren't in fact equal – weekends are very popular and the party is never held on Wednesday when 6.02 psets are due. You prepare the following table showing the updated probabilities, which should be used when answering the following questions.

day	*Mon*	*Tue*	*Wed*	*Thu*	*Fri*	*Sat*	*Sun*
p(day)	0.125	0.125	0	0.125	0.125	0.25	0.25
$log_2(1/p)$	3	3	--	3	3	2	2
$p*log_2(1/p)$	0.375	0.375	--	0.375	0.375	0.5	0.5
Encoding from part (C)			--				

(C) (6 points) Using the updated probabilities, create a variable-length Huffman code for sending messages listing party days. Note that no code is required for Wednesday. Please enter the encoding for each day in the last row of the table above.

Fill in last table row

(D) (4 points) Compute the expected length in bits to encode message containing one day using your code from part (C). Please give a numeric answer.

Expected length in bits: __________

(E) (4 points) Using the updated probabilities, compute the entropy of the underlying probability distribution. Please give a numeric answer. Hint: much of the computation has already been performed for you!

Entropy: __________

(F) (4 points) By changing the encoding scheme (say, by encoding pairs of days), would it be possible to improve the expected length of messages? Briefly explain why or why not.

Brief explanation

(G) (5 points) A phone call from a friend causes you to revise the probabilities for the coming week as follows:

day	*Mon*	*Tue*	*Wed*	*Thu*	*Fri*	*Sat*	*Sun*
p(day)	0.1	0.1	0	0.1	0.1	0.6	0
$log_2(1/p)$	3.322	3.322	--	3.322	3.322	0.737	--
$p*log_2(1/p)$	0.332	0.332	--	0.332	0.332	0.442	--

How many bits of information did the phone call deliver? Please give a numeric answer.

Bits of information from phone call: __________

Problem 2. LZW compression (20 points)

An 8-character message was encoded using the LZW encoder whose pseudo-code is shown below:

```
STRING = get input symbol
WHILE there are still input symbols DO
   SYMBOL = get input symbol
   IF STRING + SYMBOL is in the string table THEN
       STRING = STRING + SYMBOL
   ELSE
       output the code for STRING
       add STRING + SYMBOL to the string table
       STRING = SYMBOL
   END
END
output the code for STRING
```

When the encoding process was complete the following additions had been made to the string table:

table[256] = ho
table[257] = oh
table[258] = hoh
table[259] = hoho

(A) (10 points) What was the original 8-character message?

Original message: ____________________

(B) (10 points) Recall that the encoder only sends indices into the string table. What indices did the encoder send? Hint: everything can be figured out from the string entries and their order. The index of 'h' is 104 and of 'o' is 111.

Indices sent by encoder:______________________________

Problem 3. LTI Models for Communication Channels (50 points)

Consider a communications channel *C1* that is accurately modeled as a noise-free linear time invariant system with the following causal unit sample response:

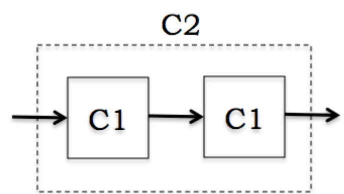
$h_{C1}[0]$	$h_{C1}[1]$	$h_{C1}[2]$	$h_{C1}[3]$	$h_{C1}[4]$	$h_{C1}[\geq 5]$
0.0	0.0	1.8	0.5	0.7	0.0

(A) (4 points) The unit step response for this channel, $s_{C1}[n]$, eventually reaches a steady state value v. What is v and what is the smallest k such that $s_{C1}[k] = v$?

Steady state value v: __________

Smallest k: __________

(B) (10 points) Suppose we built a communications channel *C2* composed of two C1 channels connected in series:

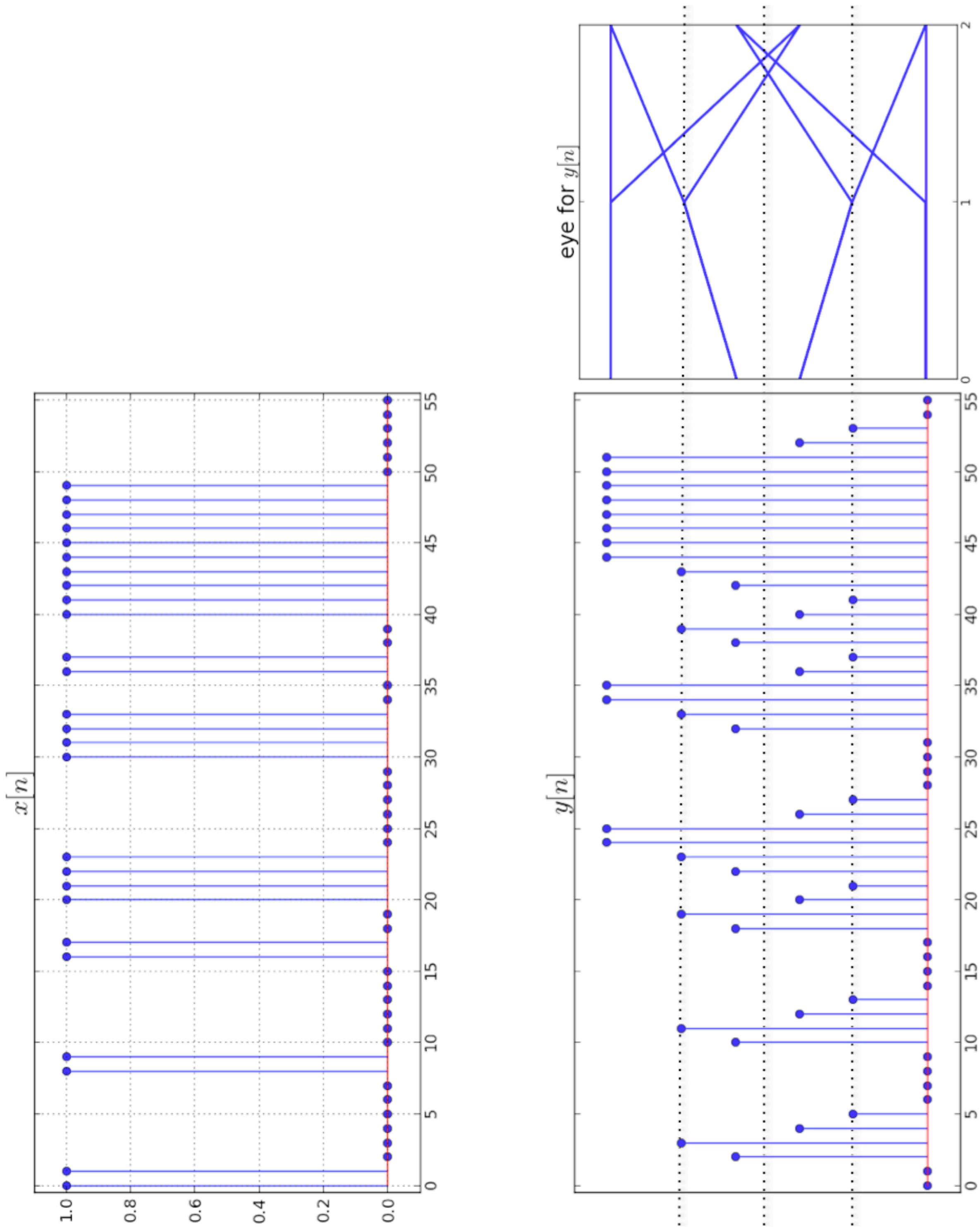


Please fill in the following table, giving the first 10 values of the unit sample response for the C2 channel.

Fill in table

$h_{C2}[0]$	$h_{C2}[1]$	$h_{C2}[2]$	$h_{C2}[3]$	$h_{C2}[4]$	$h_{C2}[5]$	$h_{C2}[6]$	$h_{C2}[7]$	$h_{C2}[8]$	$h_{C2}[9]$

Consider digital transmissions over the original channel C1 where we use *2 samples/bit*. The following figure shows a test sequence x[n], the channel's response y[n] and an eye diagram constructed from y[n]. Assume x[i] = 0 for i < 0. Note that there are no vertical scales on the plots for y[n] and the eye diagram, but both plots use the same vertical scale (which is *not* the same vertical scale used to plot x[n] – you can't get the answers by measuring!). The receiver will periodically sample y[n] at the widest part of the eye and compare those voltages against a digitization threshold V_{th} to determine the message bits.



(C) (10 points) What are the possible voltages the receiver will see when it periodically samples y[n] at the widest part of the eye? Since the diagrams have no scale, you will need to compute the voltage values. To receive credit for this part you must *show your work*.

Possible voltage values at sample point: ______________________________

(D) (6 points) Referring to the figure for y[n], give the first three indices for y[n] where the receiver will sample to determine the first 3 bits of the message.

First index: __________ Second index: __________ Third index: __________

(E) (3 points) Assuming there is an equal probability of sending 0's and 1's, what value of V_{th} will maximize the noise margins at the receiver?

Value of V_{th}: __________

(F) (3 points) What is the noise margin in volts using your threshold of part (E)?

Noise margin: __________

(G) (9 points) Since the C1 channel is noise-free (obviously this a work of fiction), it is possible to reliably use deconvolution to construct a perfect estimate, w[n], of the input waveform given y[n] and $h_{C1}[n]$. Give an equation for w[n] where the only variables are from the response (y[n], y[n-1], y[n+1], …) and earlier values of w (w[n-1], w[n-2], …), everything else must be numeric. In other words, use numeric values for any h_{C1} elements appearing in the equation.

Give equation for w[n]

(H) (5 points) The lecture slides and notes discuss some criteria under which the deconvolution equation will be stable in the presence of noise, i.e., where the estimate w[n] will not grow without bound if some of the y[n] have been affected by noise. Does $h_{C1}[n]$ meet this criteria? Briefly explain.

Brief explanation

END OF QUIZ 1!