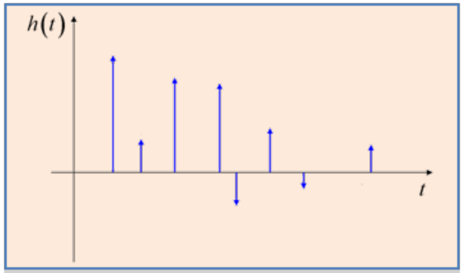
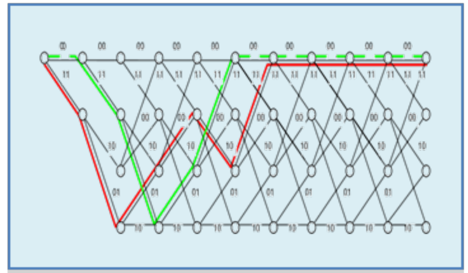
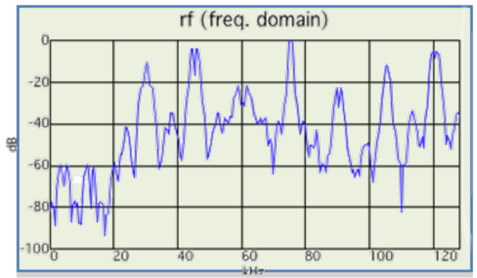
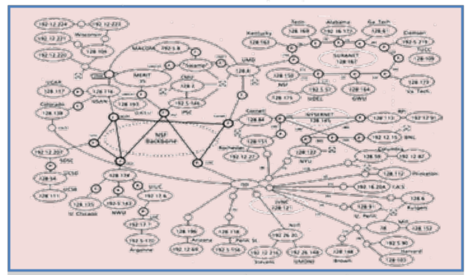








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

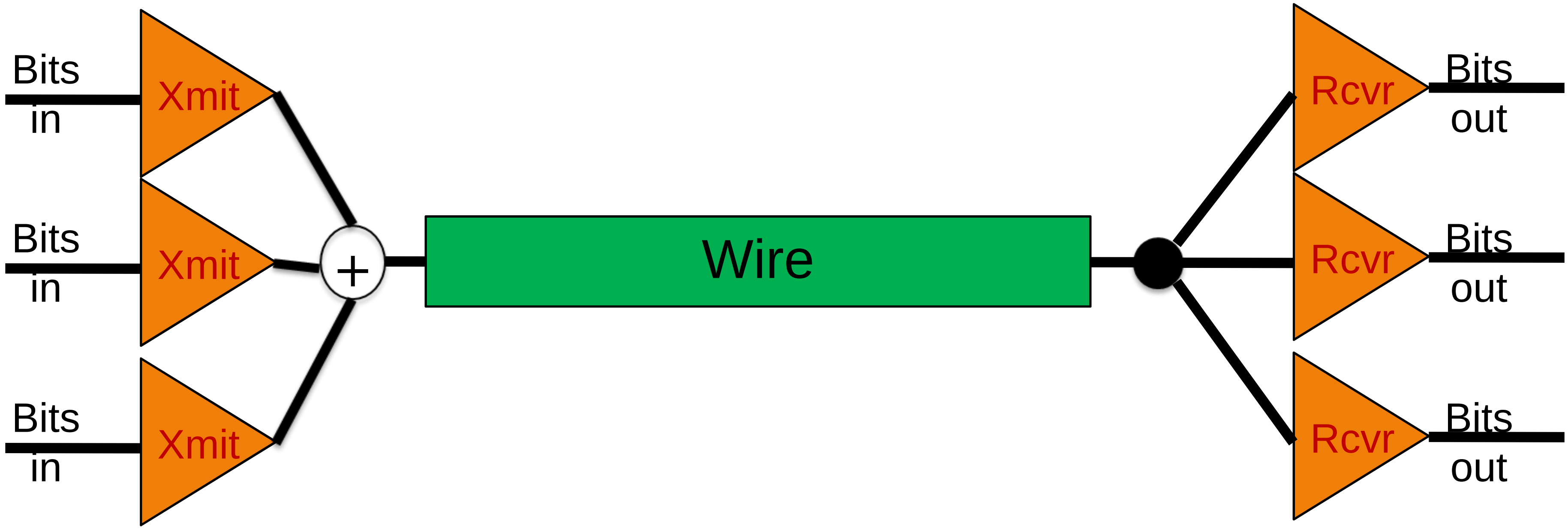
6.02 Spring 2009
Lecture #11

- Summary of progress
- The problem of multiplexing
- Eternal Signals and LTI Systems
- Discrete-Time Sines and Cosines

So Far - Low BER Transmission on Wires

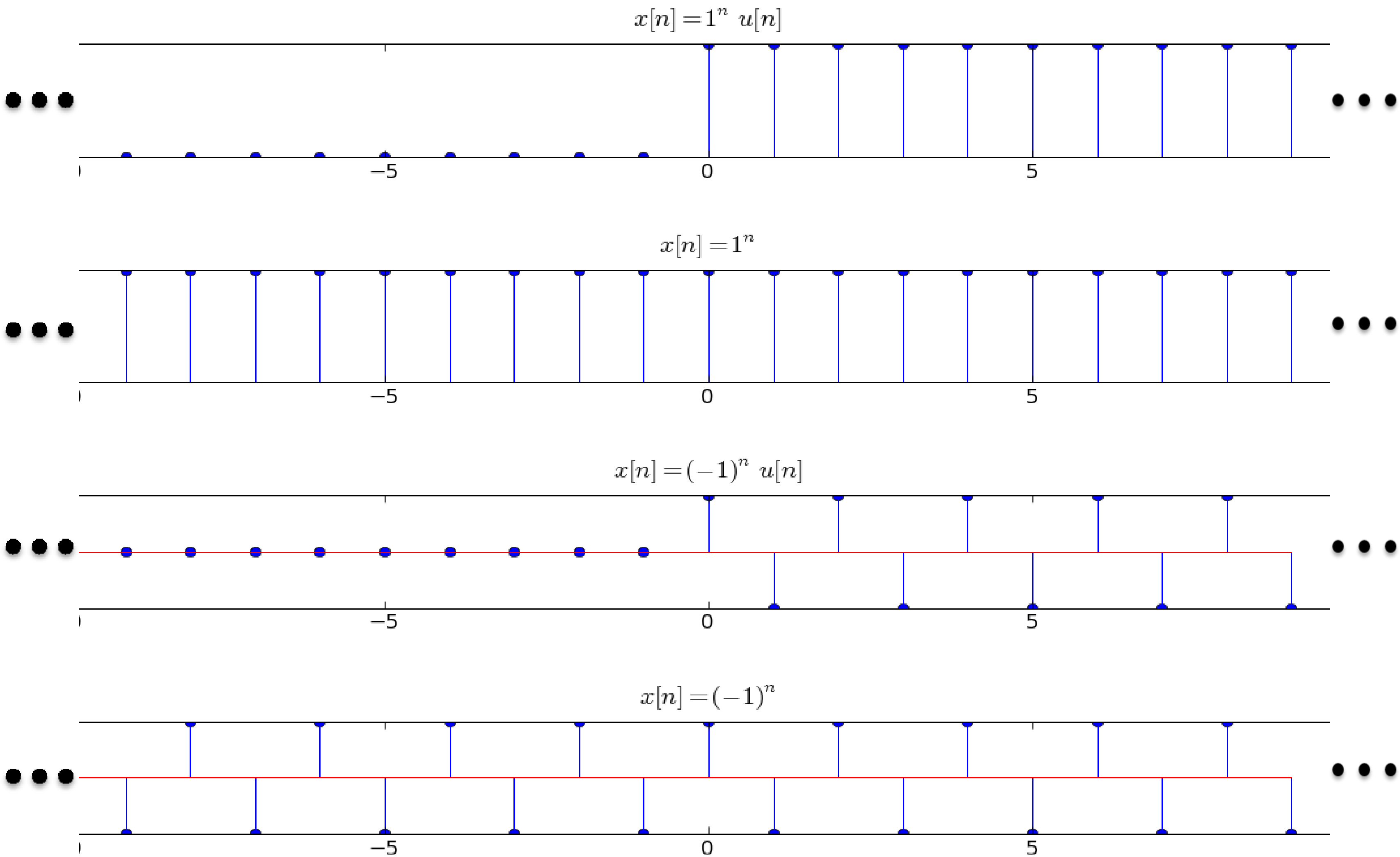
- Problems and Analysis Techniques:
  - Intersymbol Interference (ISI)
    - LTI Systems and Unit Sample Responses
    - Eye Diagrams
  - Noise
    - Probability Density and Cumulative Distribution Functions
    - Normal (Gaussian) random variables
- Solution Approaches
  - Techniques based on LTI models
    - Deconvolution and Decision Feedback Equalization
  - Error Detection and Correction Codes
    - Parity bits, Reed-Solomon Codes, Viterbi Algorithm

New Problem – Resource Sharing



- Two Approaches
  - Time Division Multiplexing (TDM)
    - Each Xmit-Rcvr pair gets a time slot (how to decide?)
    - Used by wired internet
  - Frequency Division Multiplexing (FDM)
    - Each Xmit-Rcvr gets part of the spectrum (explained later)
    - Like Broadcast TV and Radio, many wireless devices
- Next Two Weeks on FDM

Consider Samples for $-\infty < n < \infty$



Eternal Signals

LTI Systems for Eternal Signals

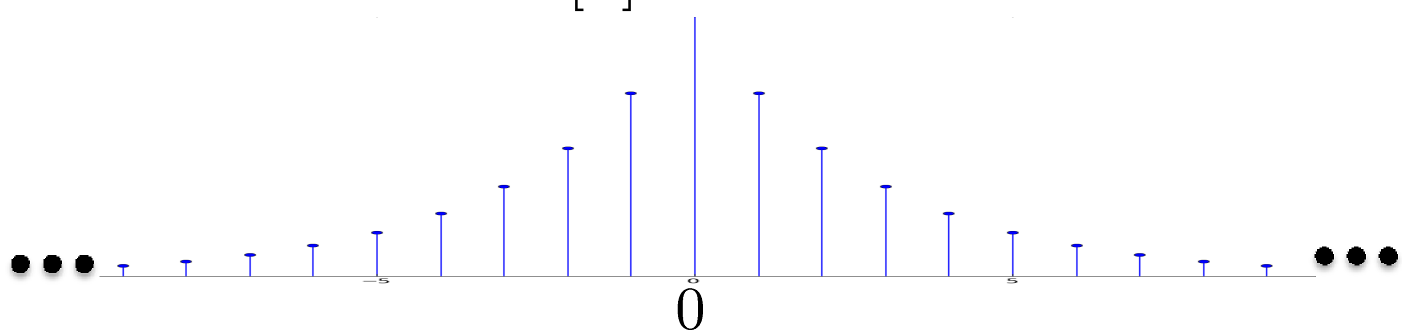
- Convolution Changes a little

x[n] may be eternal

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

h[m] may not be causal

$$h[n] = 0.7^{|n|}$$



Suppose $x[n] = z^n, \quad -\infty < n < \infty$

- Find y using convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m] z^{n-m}$$

Reorganizing

$$y[n] = \left(\sum_{m=-\infty}^{\infty} h[m] z^{-m}\right) z^n$$

Just a number if sum converges = *H(z)*

$$y[n] = H(z) z^n$$

y[n] has the same form as x[n]!

So What

- Suppose $x[n] = A_1 z_1^n + A_2 z_2^n$

$$y[n] = \left(\sum_{m=-\infty}^{\infty} h[m] z_1^{-m}\right) A_1 z_1^n + \left(\sum_{m=-\infty}^{\infty} h[m] z_2^{-m}\right) A_2 z_2^n$$

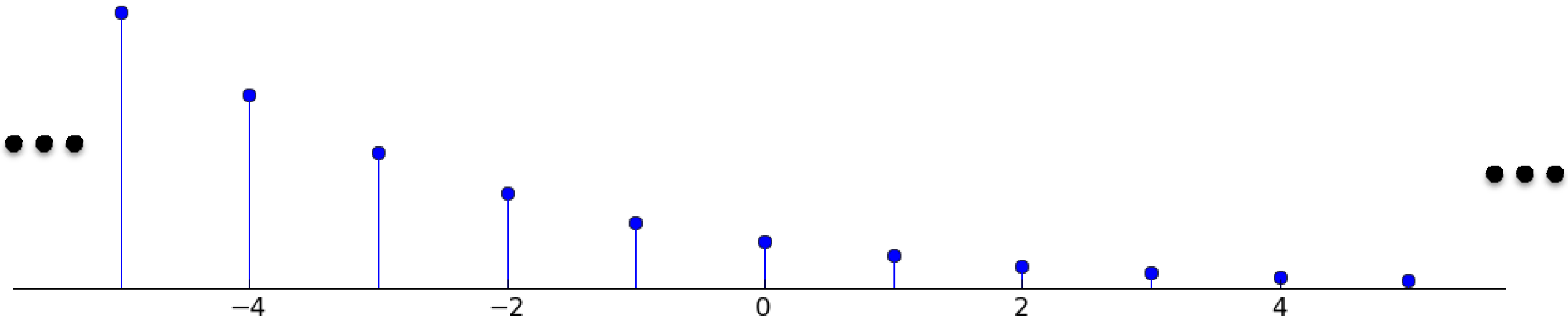
$$\Downarrow$$

$$y[n] = H(z_1) A_1 z_1^n + H(z_2) A_2 z_2^n$$

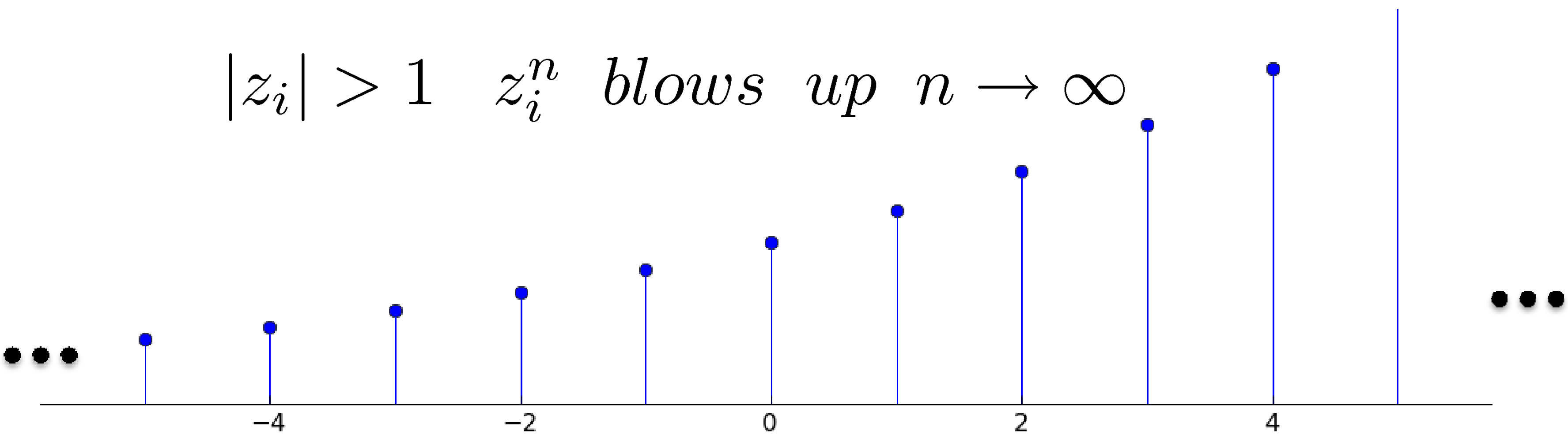
- If we can separate z_1^n part from z_2^n part in y[n]
  - Can recover two messages
- Could then use lots of z_i's
  - Multiple channels!

A problem with eternal signals

$$|z_i| < 1 \quad z_i^n \;\; blows \;\; up \;\; n \rightarrow -\infty$$



$$|z_i| > 1 \quad z_i^n \;\; blows \;\; up \;\; n \rightarrow \infty$$



Use $|z_i| = 1$

- How many z's are there with unit magnitude?
  - z=1, z=-1, any others?

- Can generate more z's using complex numbers

$$z = e^{j\Omega} = cos\Omega + jsin\Omega$$

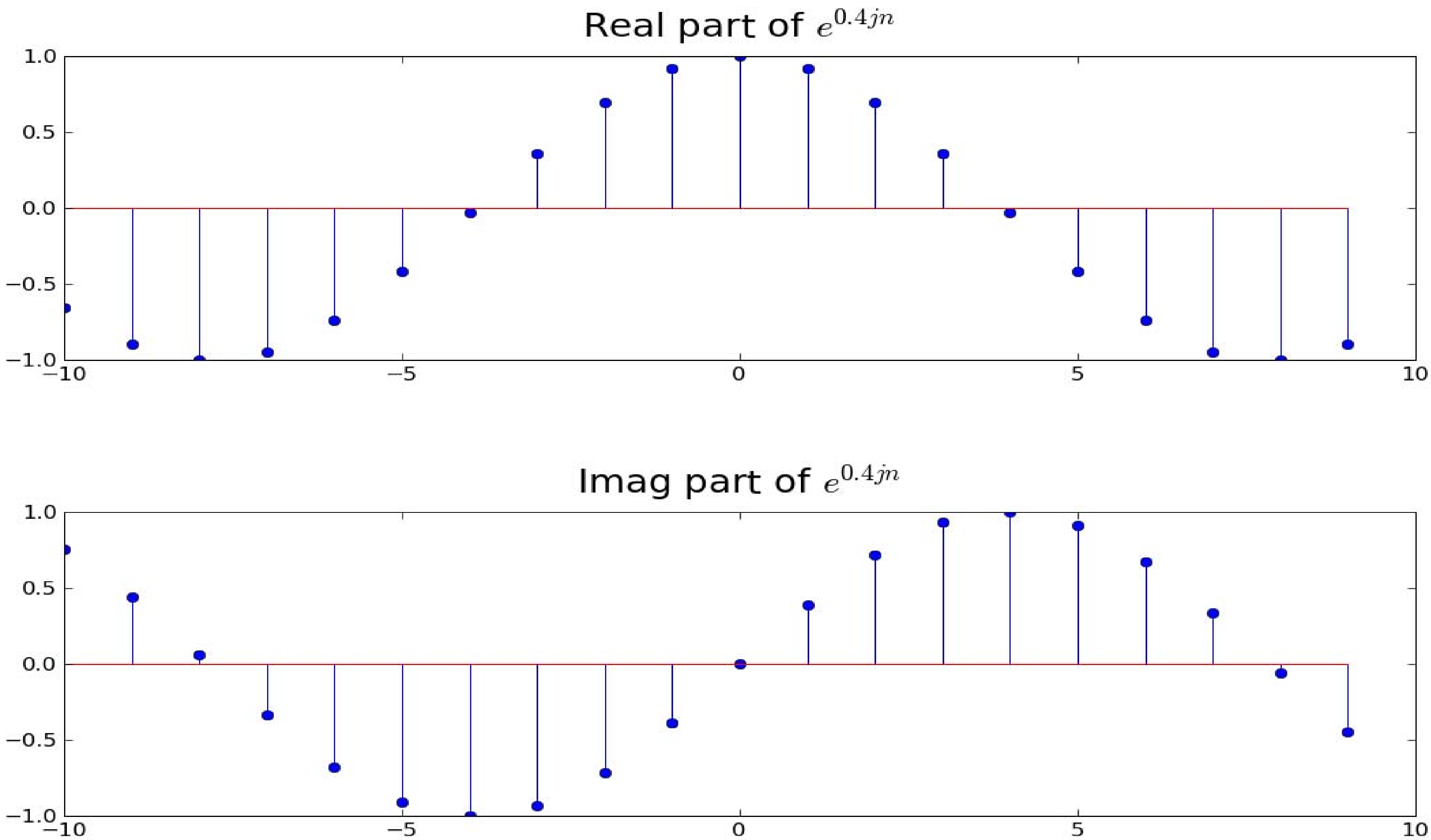
- Magnitude is one for any frequency

$$|z| = |e^{j\Omega}| = |\cos\Omega + j\sin\Omega| = \sqrt{\cos^2\Omega + \sin^2\Omega} = 1$$

- Know how to evaluate z^n

$$z^n = e^{j\Omega n} = \cos\Omega n + j\sin\Omega n$$

Example: $z^n = e^{0.4jn} = \cos 0.4n + j \sin 0.4n$



Frequency Response

- From convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m] e^{j\Omega(n-m)}$$

Reorganizing

$$y[n] = \left(\sum_{m=-\infty}^{\infty} h[m] e^{-j\Omega m}\right) e^{j\Omega n}$$

A *complex* number if the sum converges

$$y[n] = H\left(e^{j\Omega}\right) e^{j\Omega n}$$

$H(e^{j\Omega})$, $-\pi < \Omega < \pi$, is the frequency response

Summary and Next Time

- Frequency Division Multiplexing
  - Eternal z^n 's do not mix when input to LTI systems
  - Need many eternal z^n 's that do not blow up, $|z| = 1$
  - Use $z_i = e^{j\Omega_i}, \quad -\pi < \Omega_i \leq \pi$

- Next Week
  - How do we separate the different frequencies in y[n]
    - Filters and Fourier Analysis
- After Spring Break
  - Encoding Information using different frequencies
    - What happens when we use

$$x[n] = A_1[n]e^{j\Omega_1 n} + A_2[n]e^{j\Omega_2 n}$$

    - Do the modulated complex exponentials still stay separated?