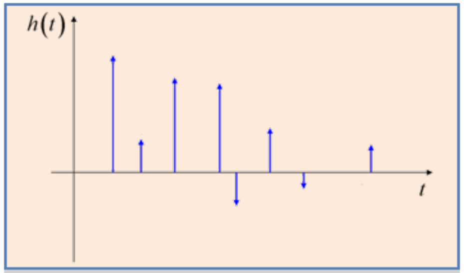
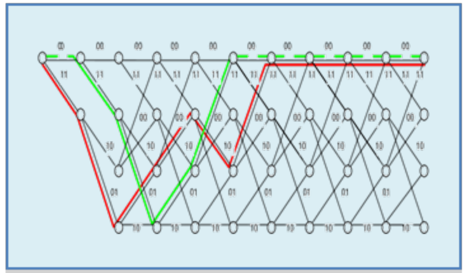
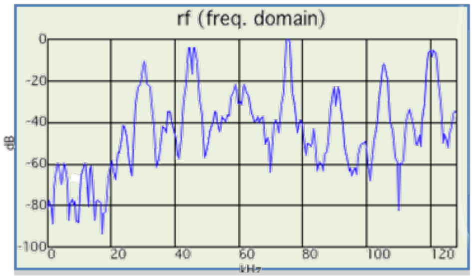
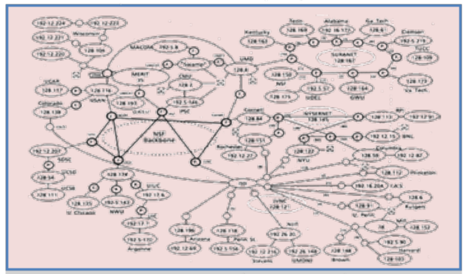








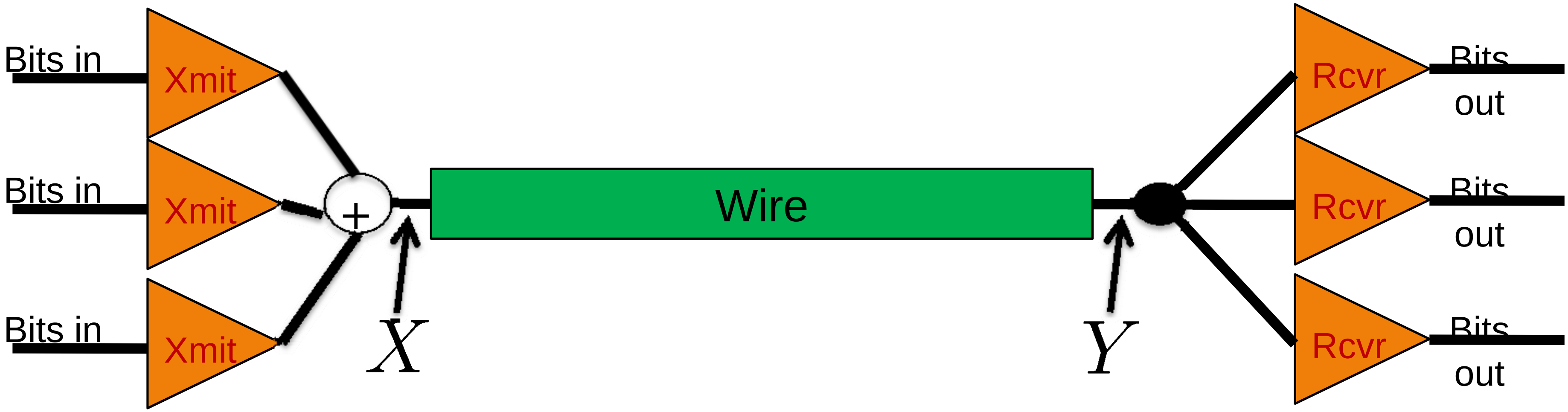
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Spring 2009
Lecture #12

- Frequency Division Multiplexing
- Why Complex Exponentials
- Frequency Response and Filters
- Zeros and Poles

New Problem – Resource Sharing



- Frequency Division Multiplexing Strategy
  - Represent each channel with a different frequency
    - For LTI systems, frequencies do not mix

$$x[n] = A_1 e^{j\Omega_1 n} + \ldots + A_K e^{j\Omega_K n}$$

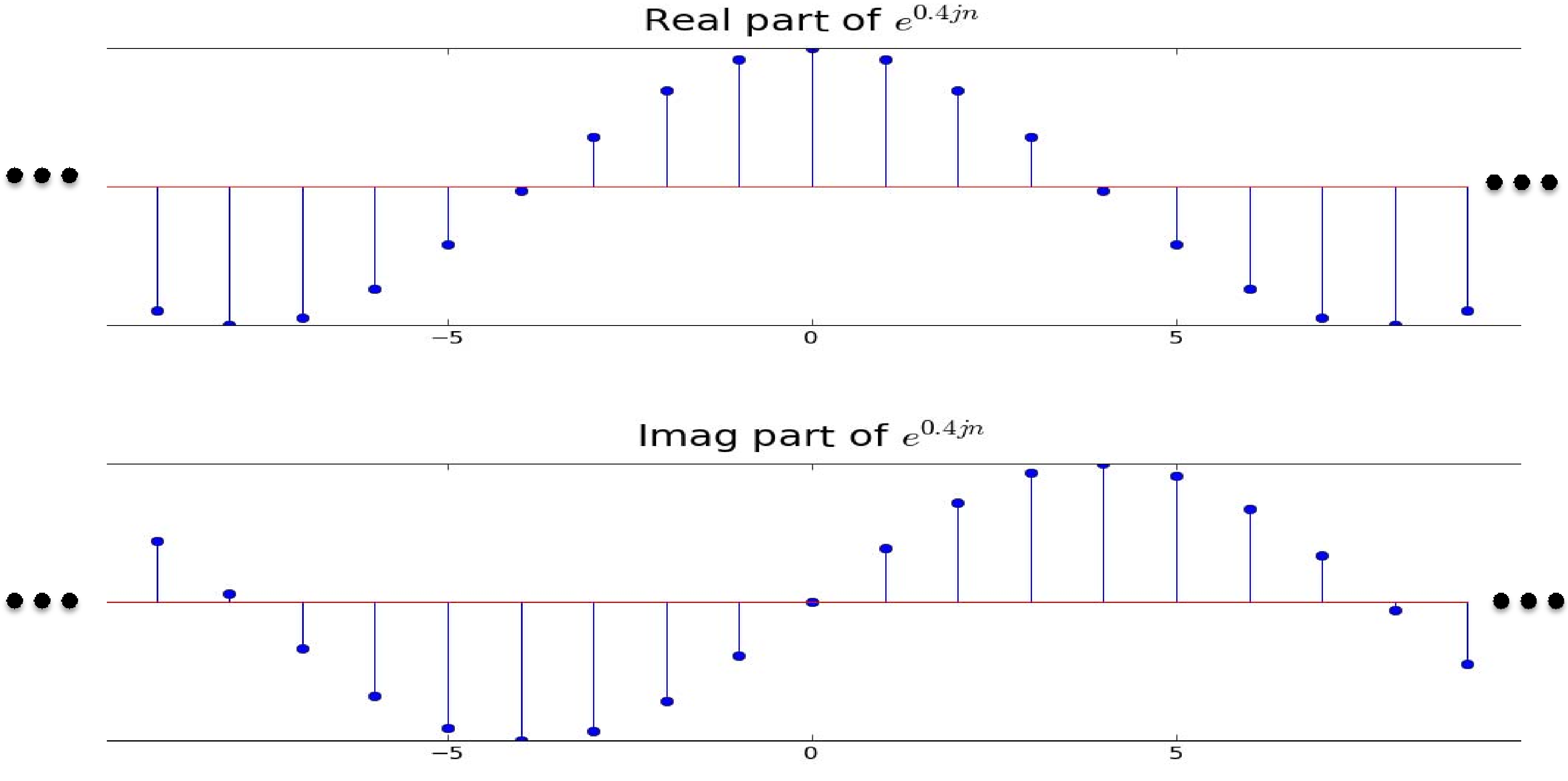
$$\Downarrow$$

$$y[n] = H(e^{j\Omega_1}) A_1 e^{j\Omega_1 n} + \ldots + H(e^{j\Omega_K}) A_K e^{j\Omega_K n}$$

  - Now need to separate the different frequencies
    - Use Filters to separate Y in to different channels
    - LTI systems with specific frequency responses

Eternal Complex Exponentials

$$x[n] = e^{0.4jn} = \cos 0.4n + j \sin 0.4n$$



Frequency Response

- From convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m] e^{j\Omega(n-m)}$$

Reorganizing

$$y[n] = \left(\sum_{m=-\infty}^{\infty} h[m] e^{-j\Omega m} \right) e^{j\Omega n}$$

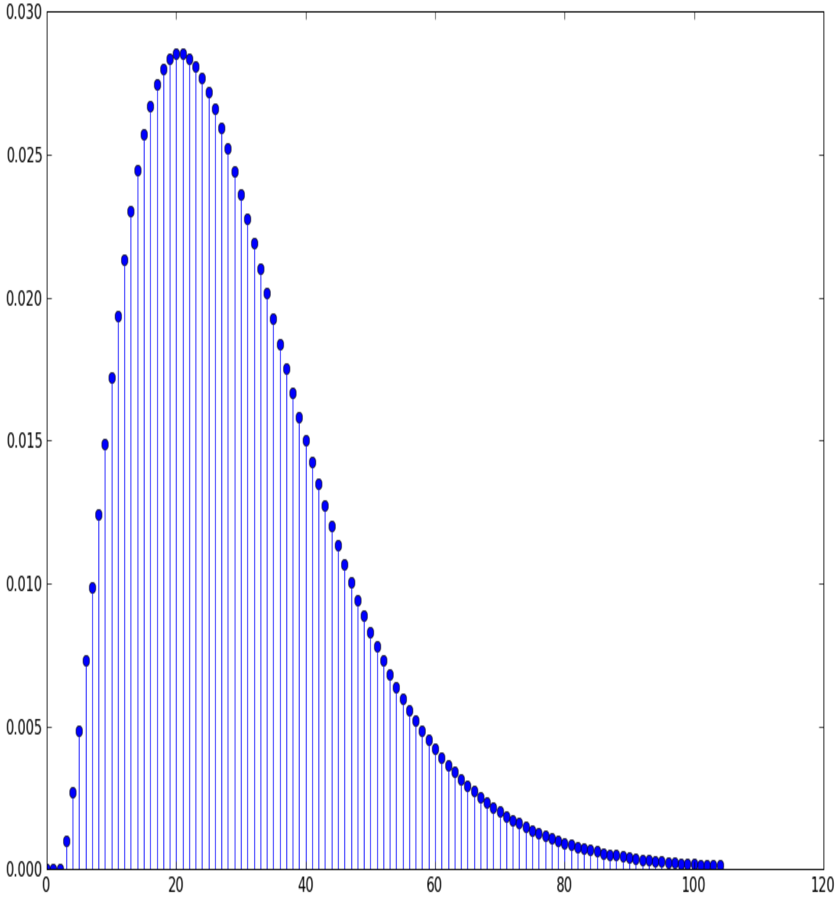
A *complex* number if the sum converges

$$y[n] = H\left(e^{j\Omega}\right) e^{j\Omega n}$$

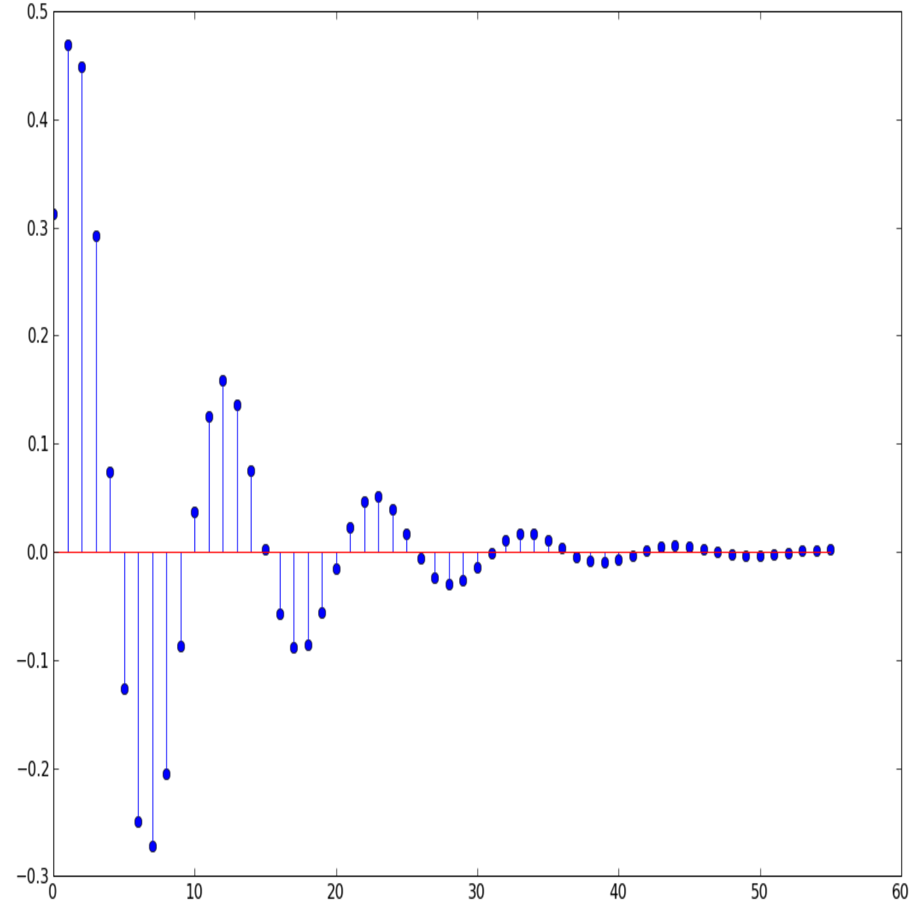
$H(e^{j\Omega})$, $-\pi < \Omega \leq \pi$, is the frequency response

Recall Channel Unit Sample Responses

Slow Channel

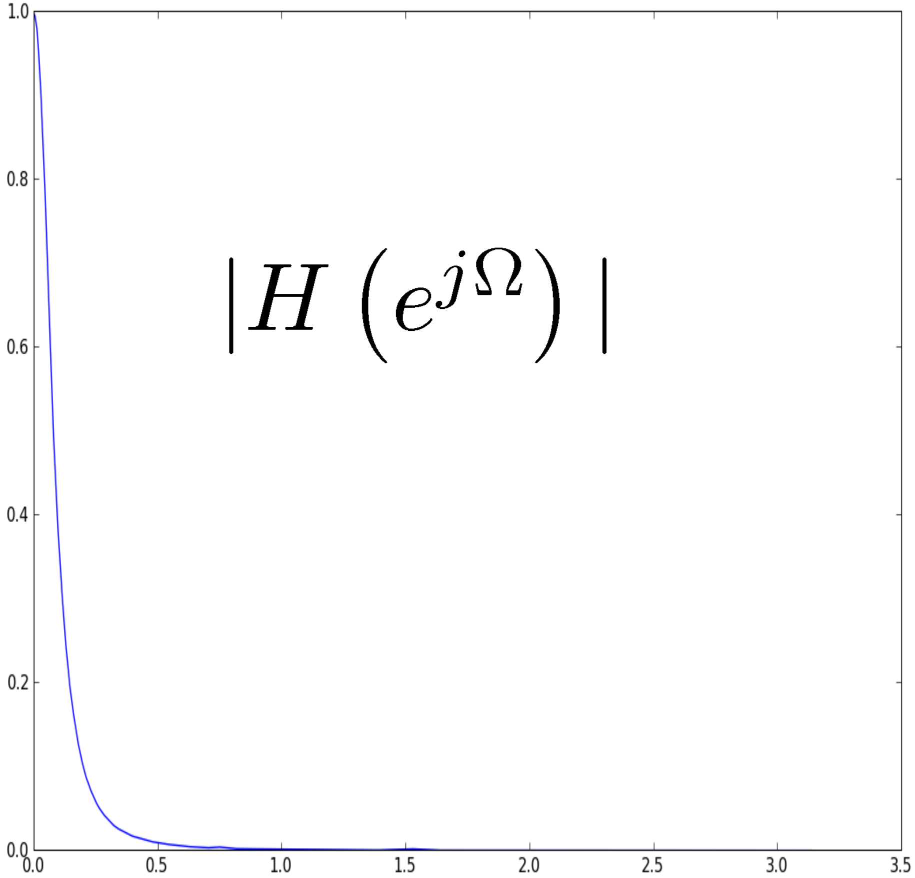


Fast Channel

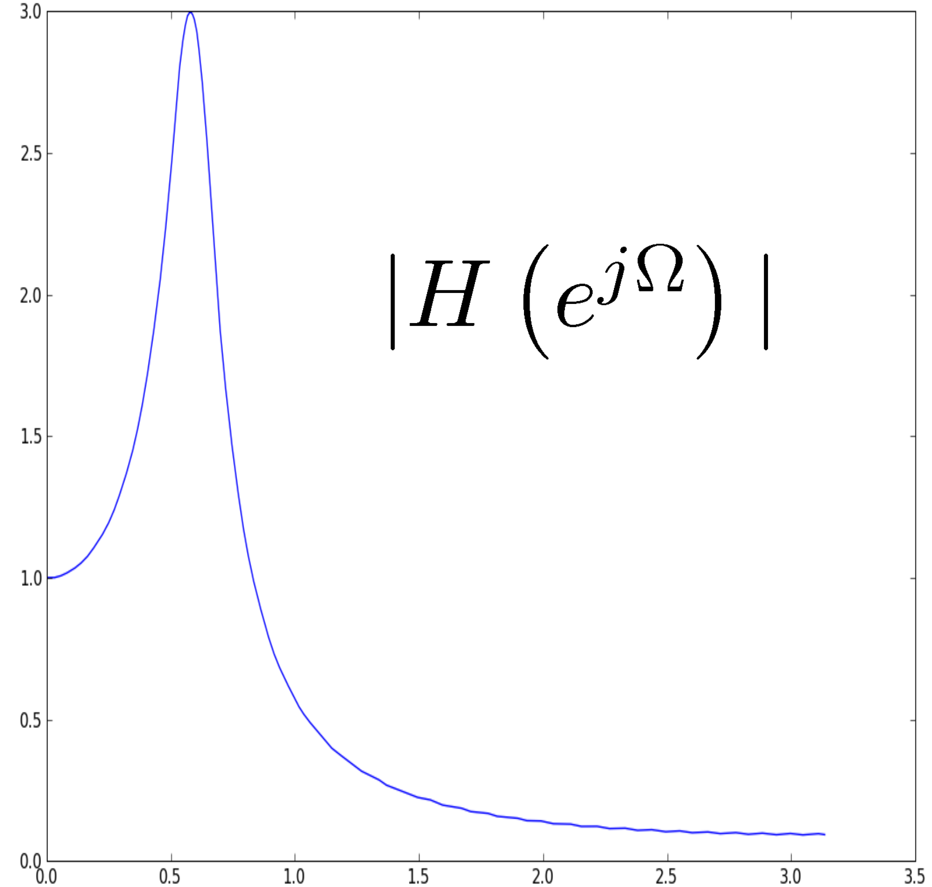


Magnitude of Frequency Response

Slow Channel



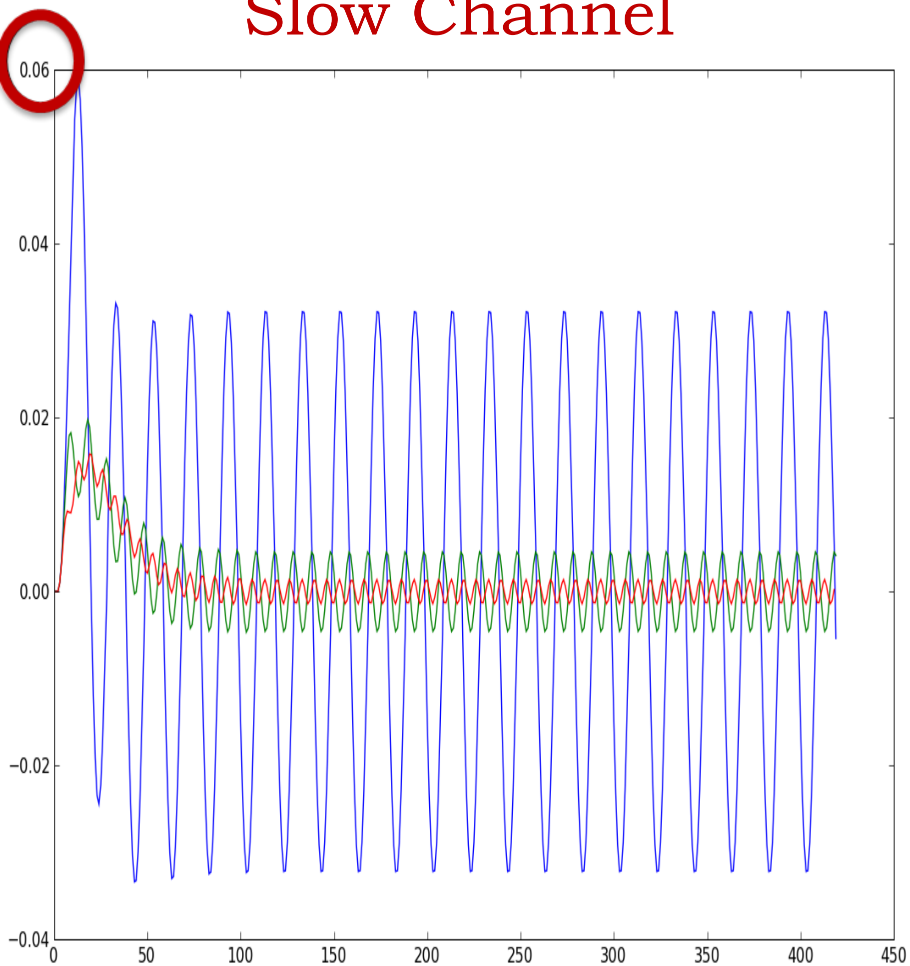
Fast Channel



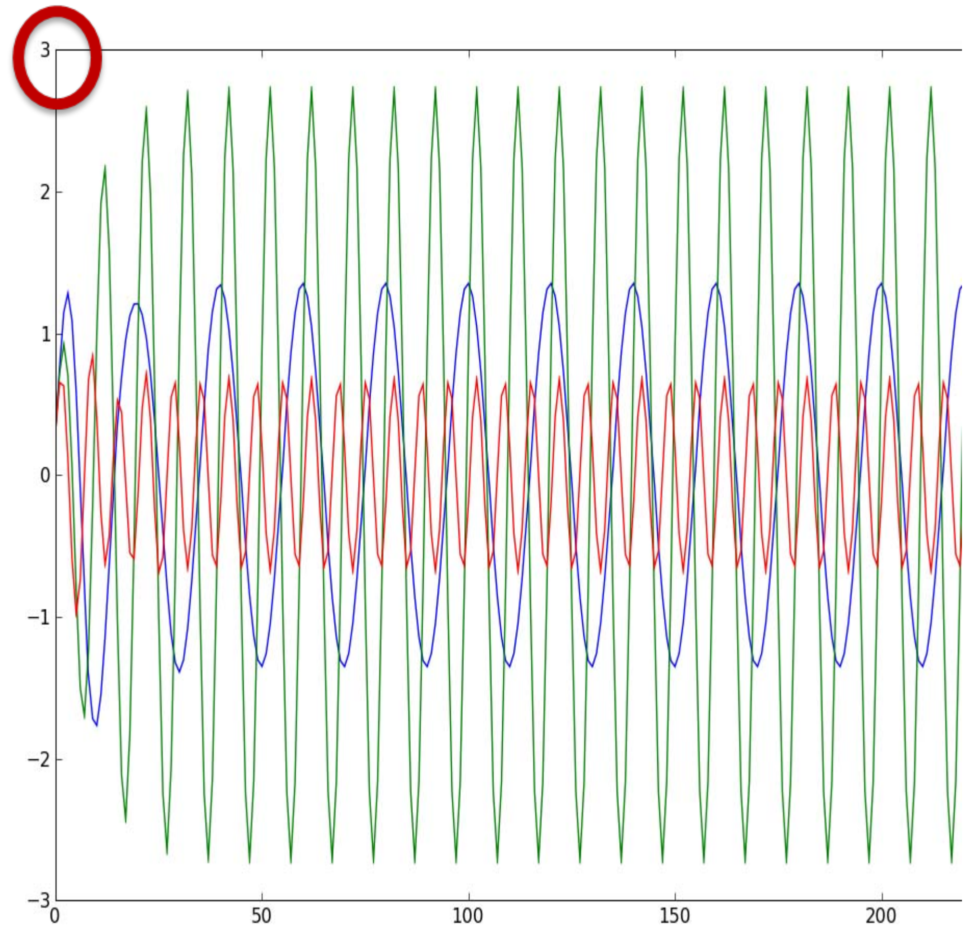
Response to Cosine starting at zero

$$x[n] = \cos(\Omega n)u[n] \;\; \Omega = \{\frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10}\}$$

Slow Channel



Fast Channel



Summary and Larger Picture

- Frequency Division Multiplexing
  - K channel's each using a different frequency

$$z_k = e^{j\Omega_k}, \quad 0 \leq \Omega_k \leq \pi$$

- Filtering
  - Design $\left|H\left(e^{j\Omega}\right)\right|$
  - Use Zeros to eliminate undesired frequencies
    - Must be in complex-conjugate pairs $e^{j\Omega_k}, e^{-j\Omega_k}$
  - Use Poles to magnify desired frequency
    - Magnitude < 1, and conjugate pairs $re^{j\Omega_k}, re^{-j\Omega_k} \quad 0 < r < 1$
- After Spring Break
  - Encoding Information using different frequencies
    - What happens when we use

$$x[n] = A_1[n]e^{j\Omega_1 n} + A_2[n]e^{j\Omega_2 n}$$

    - Do the modulated complex exponentials still stay separated?