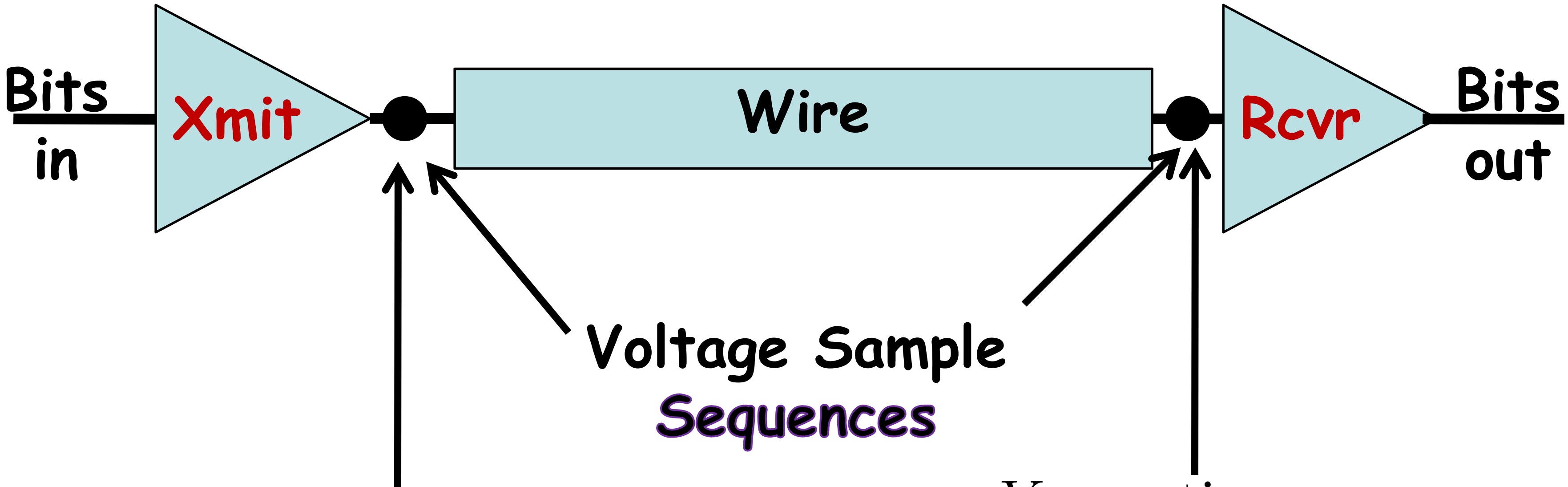
6.02 Lecture 4 – Signals and Noise

- Big Eye versus Small Eye
  - Noise problem worse for small eyes
  - Bit Error Rate
- Signals and Noise
  - Decompose into Noisefree plus Noise
  - Noise Metrics (Sample Mean and Variance)
- Probability Density Functions
  - Connection to Histograms
  - Use in estimating bit error rates

Transmission Setup and Notation



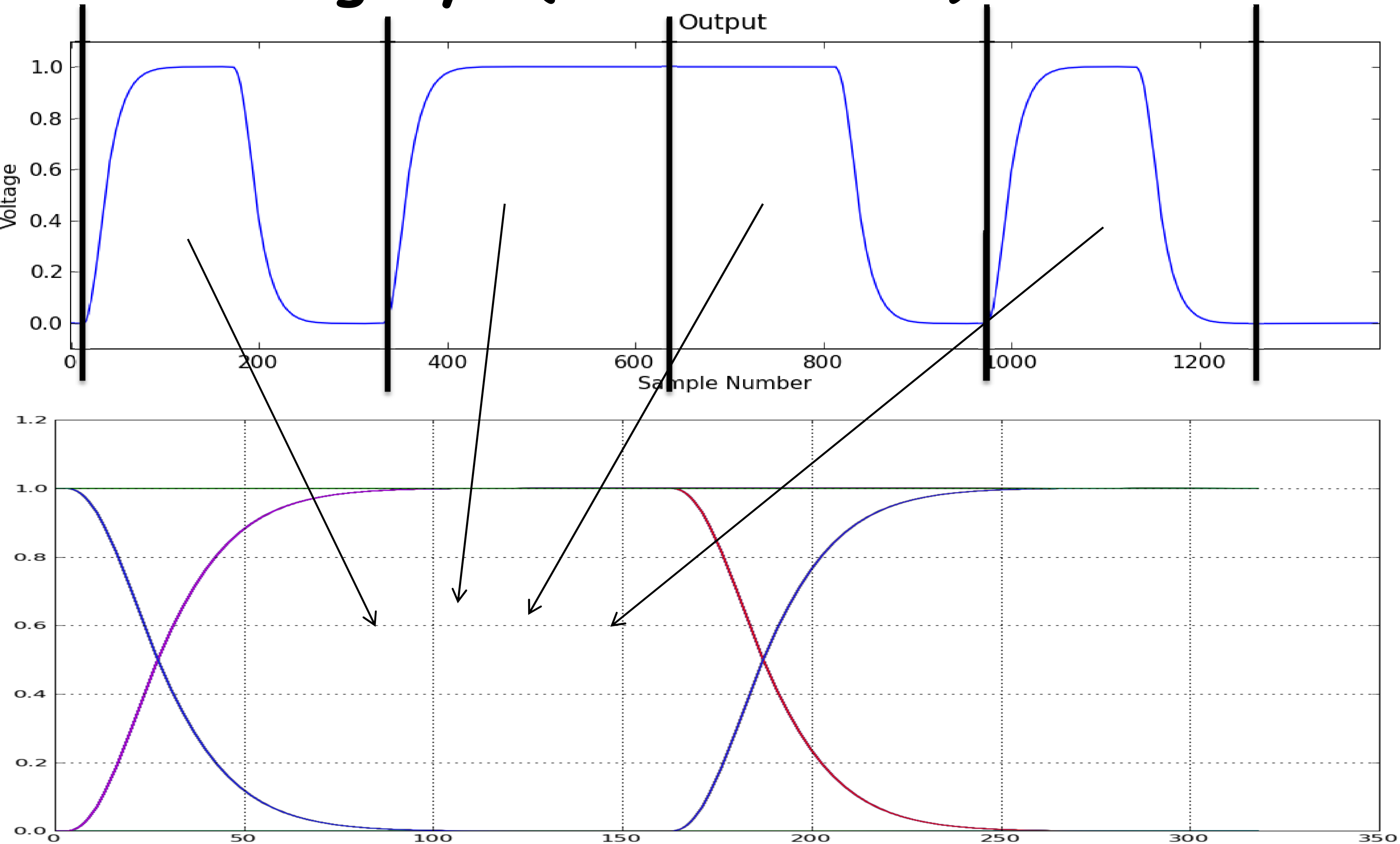
$X \equiv$ entire sequence

$x[n] \equiv n^{th}$ sample value

$Y \equiv$ entire sequence

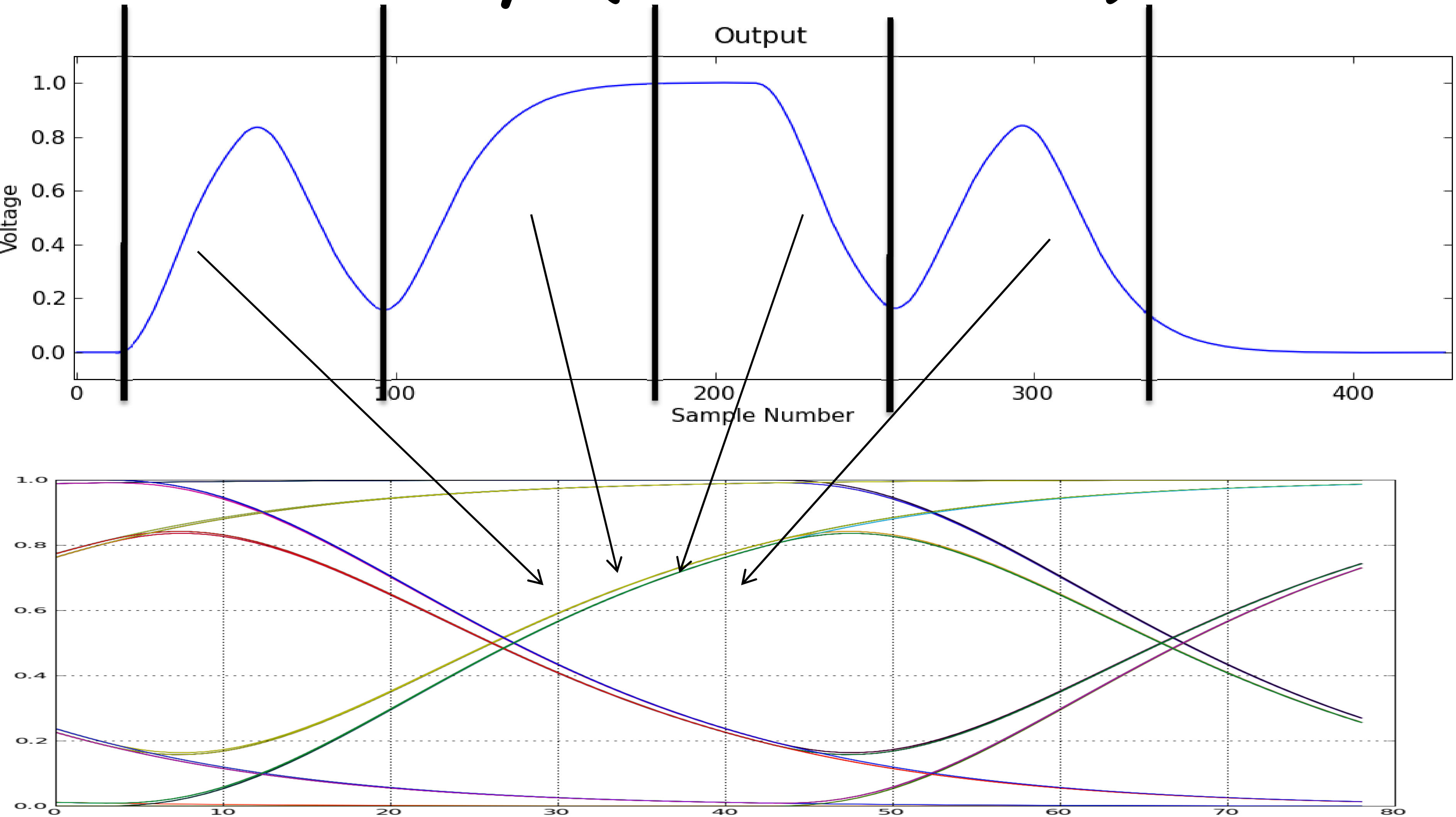
$y[n] \equiv n^{th}$ sample value

Big Eye (slow bit rate) case



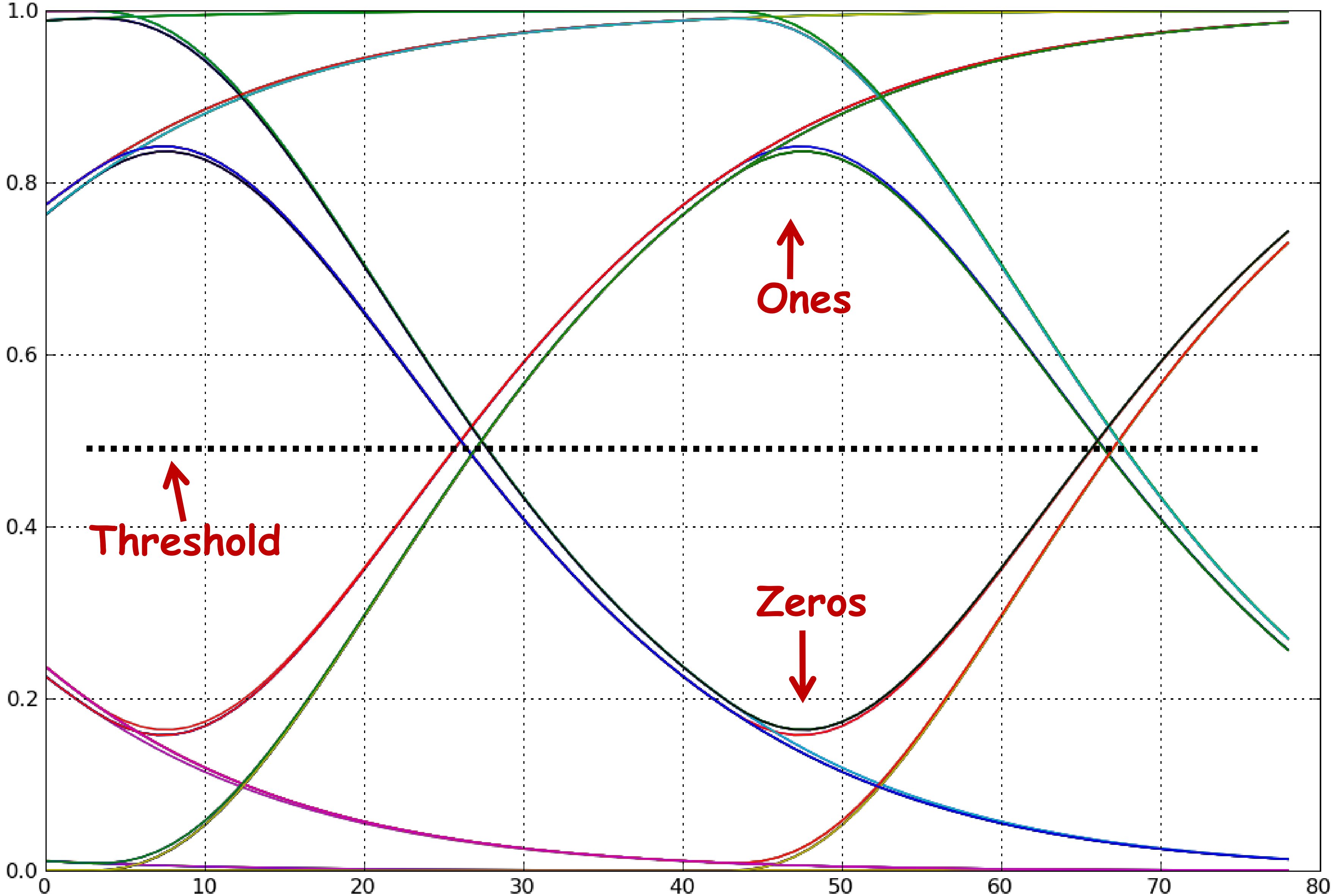
Eye Diagram Generated with 160 samples per bit

Smaller Eye (faster bit rate) case

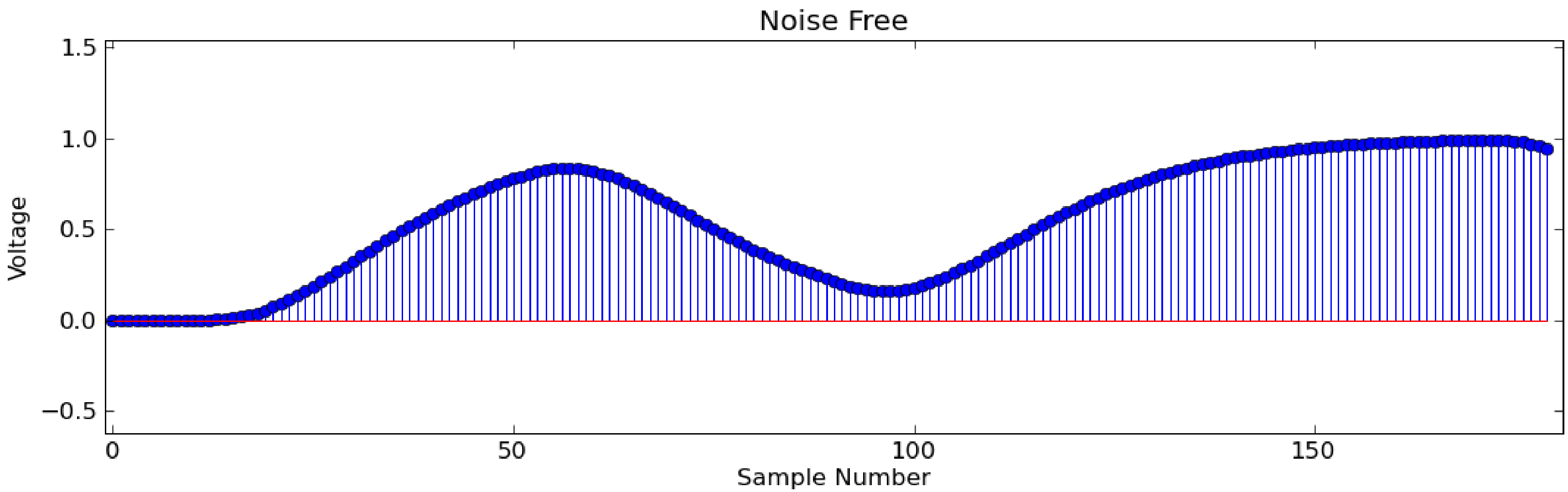


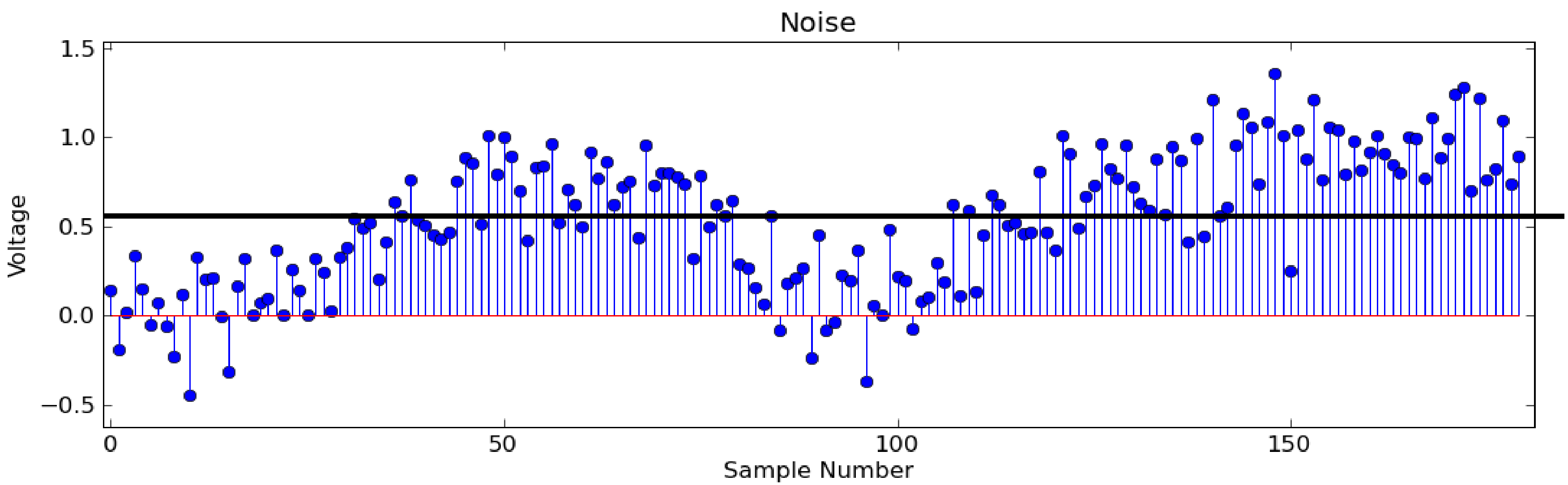
Eye diagram generated from 40 samples per bit and using a 200 bit long random sequence.

Why is a small eye bad?

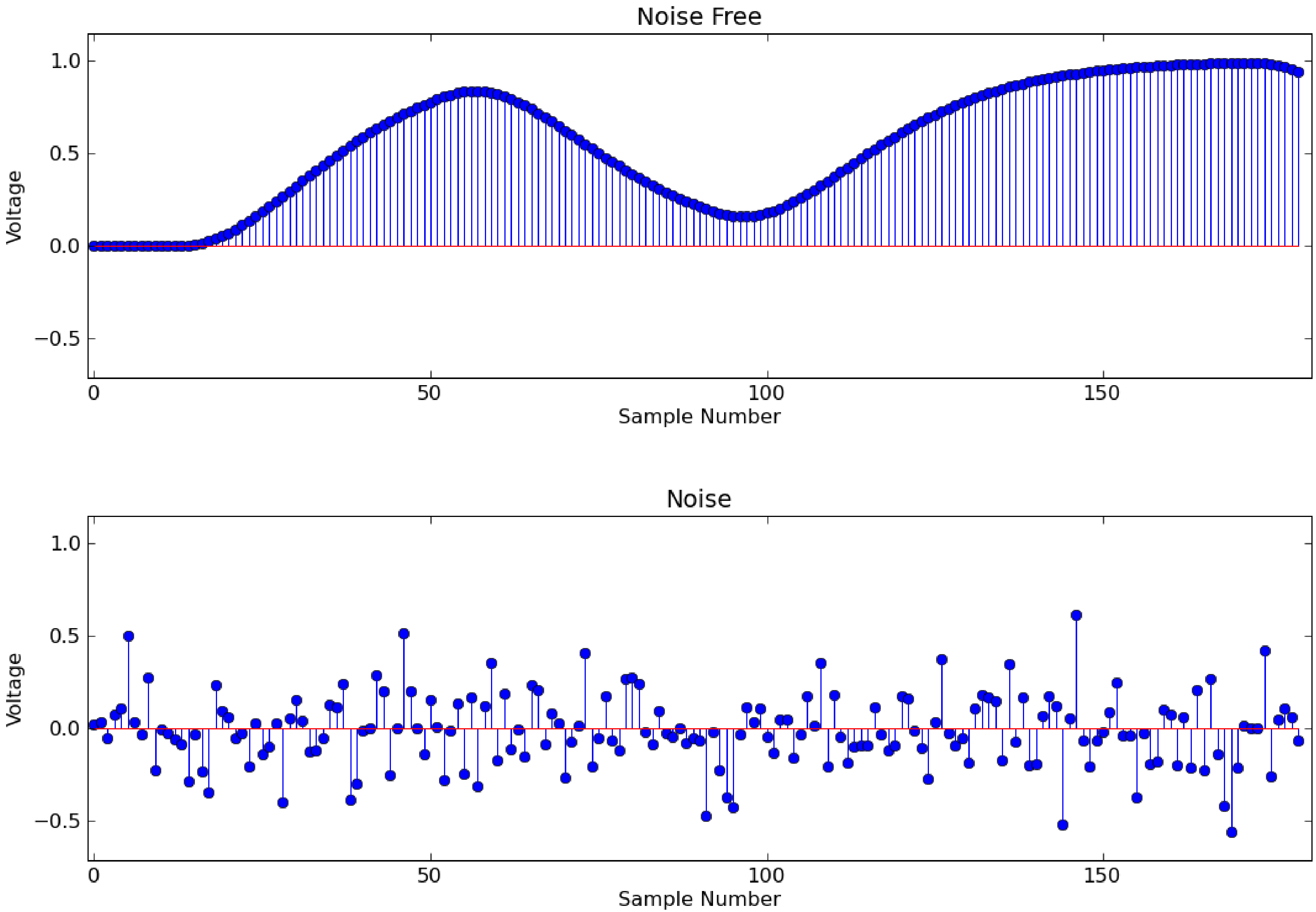


Noisy Signal Can Cause Bit Errors

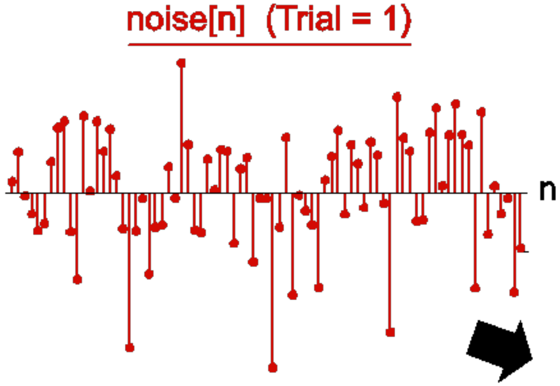


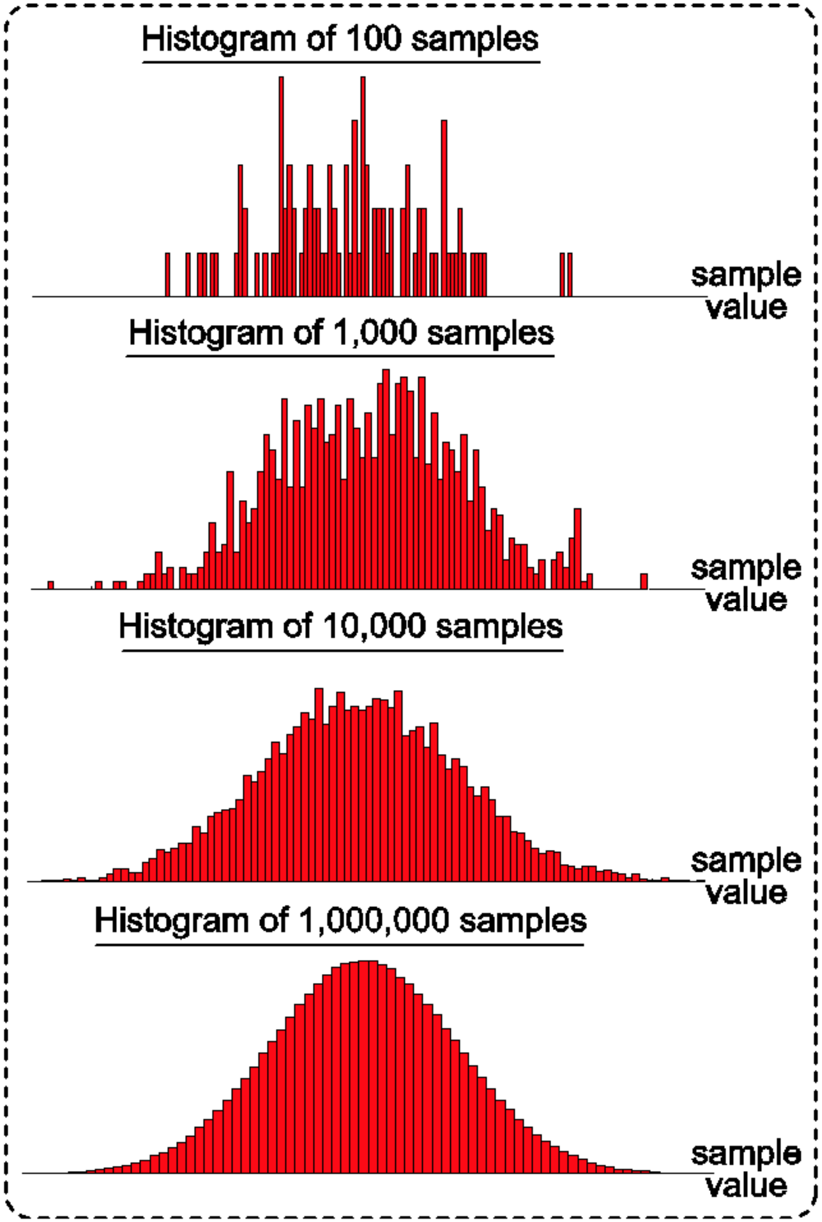


Decompose Into Noisefree + Noise



Experiment to see Statistical Distribution





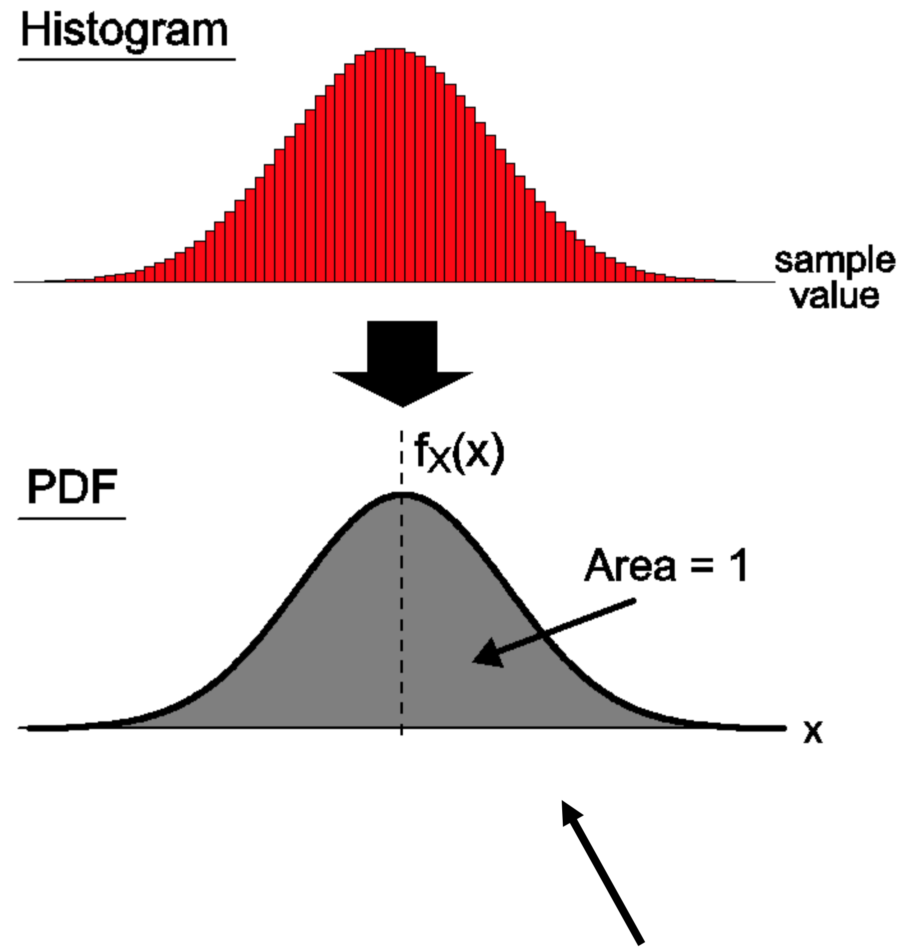
- Create histograms of sample values from trials of increasing lengths
- Assumption of independence and stationarity implies histogram should converge to a shape known as a probability density function (PDF)

Slide thanks to C. Sodini and M. Perrott

The Probability Density Function PDF

- Define x as a random variable whose PDF has the same shape as the histogram we just obtained
- Denote PDF of x as $f_X(x)$
  - Scale $f_X(x)$ such that its overall area is 1

$$\Rightarrow \int_{-\infty}^{\infty} f_X(x) = 1$$



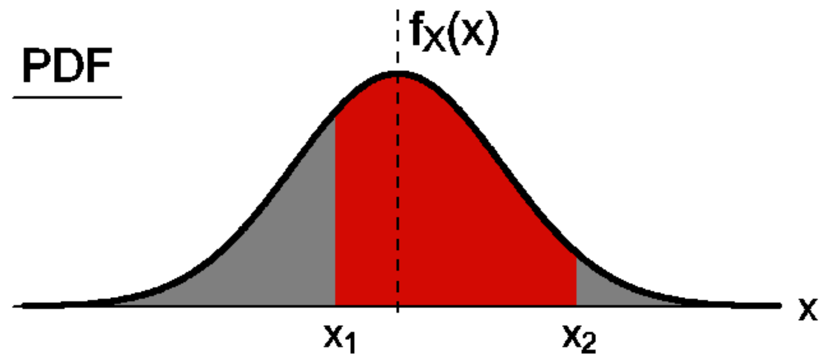
This shape is referred to as a Gaussian PDF

Slide thanks to C. Sodini and M. Perrott

Formalizing Probability

- The *probability* that random variable *x* takes on a value in the range of x_1 to x_2 is calculated from the PDF of *x* as:

$$\text{Prob}(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f_X(x)dx$$



- Note that probability values are always in the range of 0 to 1
  - Higher probability values imply greater likelihood that the event will occur

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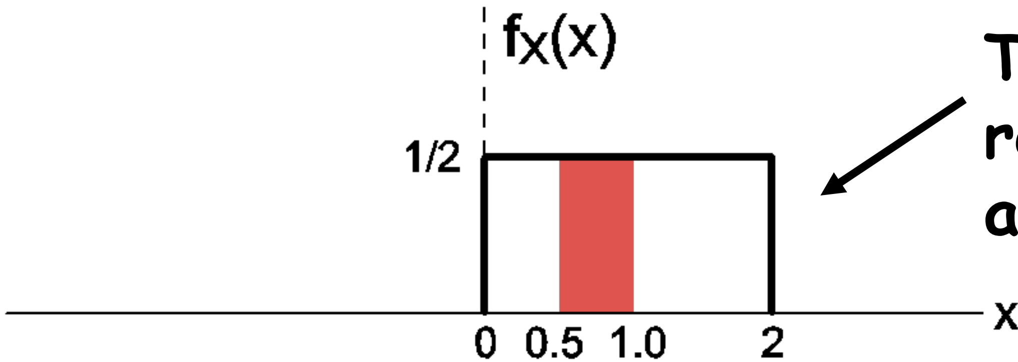
Example Probability Calculation



This shape is referred to as a uniform PDF

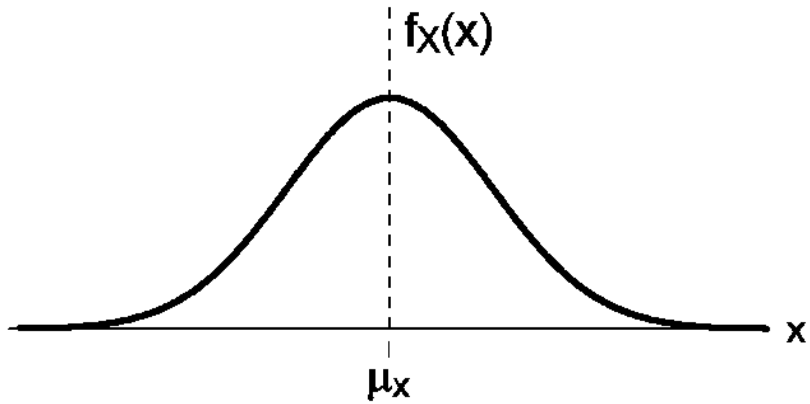
- **Verify that overall area is 1:**

$$\int_{-\infty}^{\infty} f_X(x)dx = \int_0^2 0.5\,dx \ = \boxed{1}$$

- **Probability that x takes on a value between 0.5 and 1.0:**

$$\mathrm{Prob}(0.5 \leq x \leq 1.0) = \int_{0.5}^{1.0} 0.5\,dx \ = \boxed{0.25}$$

Mean and Variance



- **The mean of random variable *x*, μ_x, corresponds to its average value**
  - **Computed as**

$$\mu_x = \int_{-\infty}^{\infty} x f_X(x) dx$$

- **The variance of random variable *x*, σ_x^2, gives an indication of its variability**
  - **Computed as**

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx$$

- **The standard deviation of a random variable x, is denoted σ_x**

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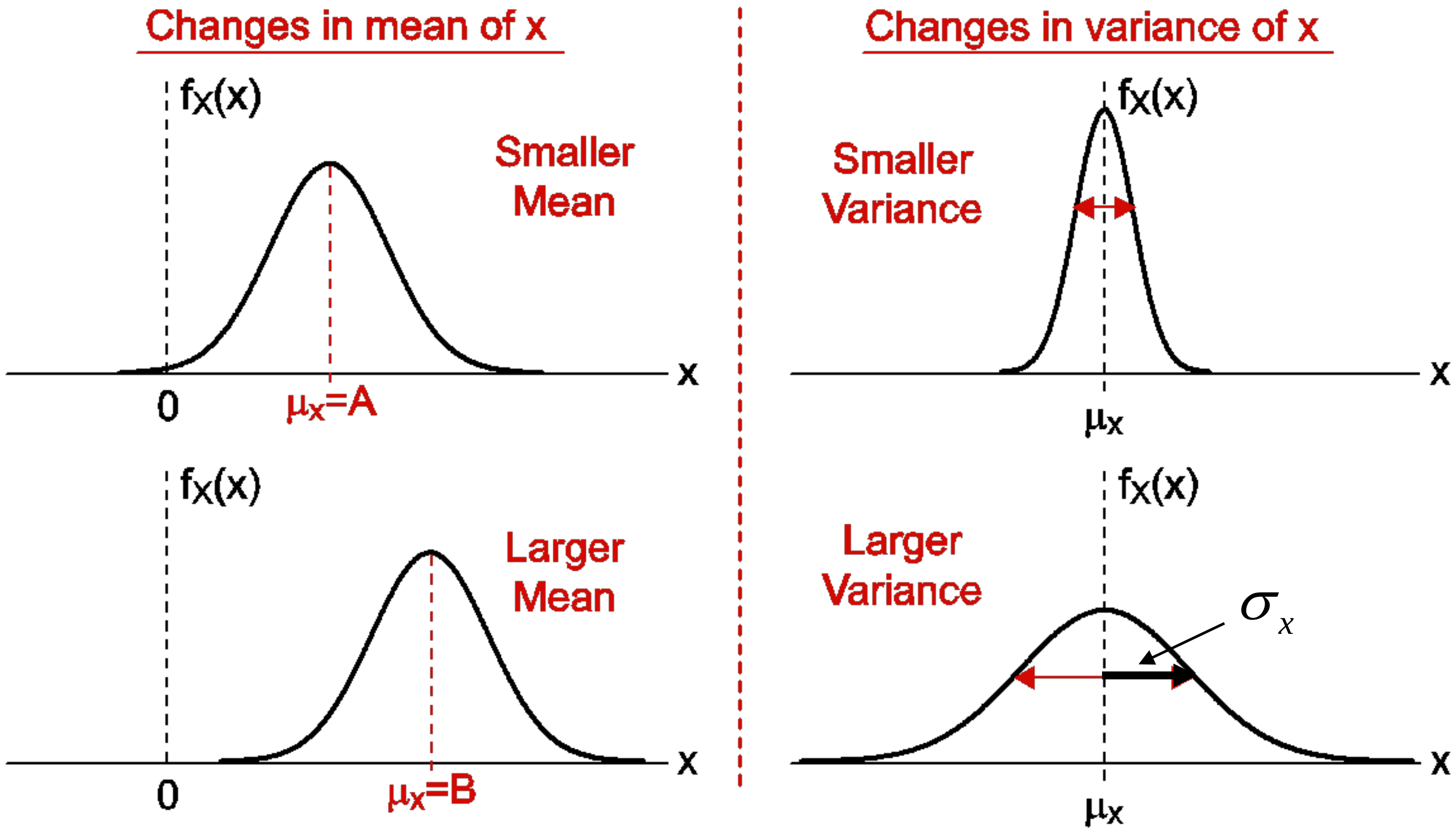
Visualizing Mean and Variance from PDF



- Changes in mean shift the *center of mass* of PDF
- Changes in variance narrow or broaden the PDF
  - Note that area of PDF must always remain equal to one

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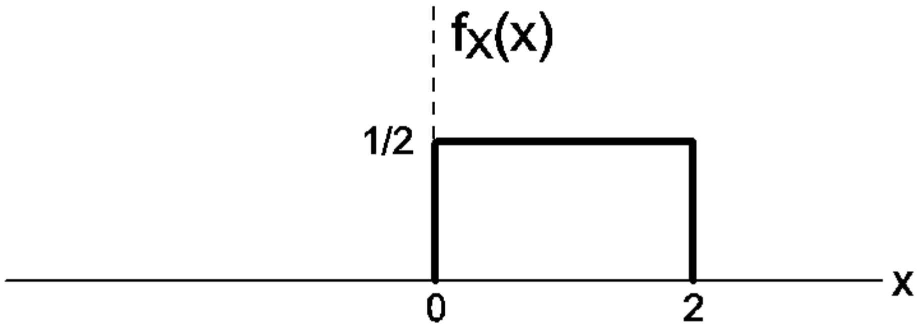
Example Mean and Variance Calculation



- **Mean:**

$$\mu_x = \int_{-\infty}^{\infty} x f_X(x)dx = \int_0^2 x\frac{1}{2}dx = \frac{1}{4}x^2\Big|_0^2 = \boxed{1}$$

- **Variance:**

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x-\mu_x)^2 f_X(x)dx = \int_0^2 (x-1)^2\frac{1}{2}dx$$

$$= \frac{1}{6}(x-1)^3\Big|_0^2 = \frac{1}{6}+\frac{1}{6} = \boxed{\frac{1}{3}}$$

Slide thanks to C. Sodini and M. Perrott