6.02 Lecture 5 - Analyzing Noise

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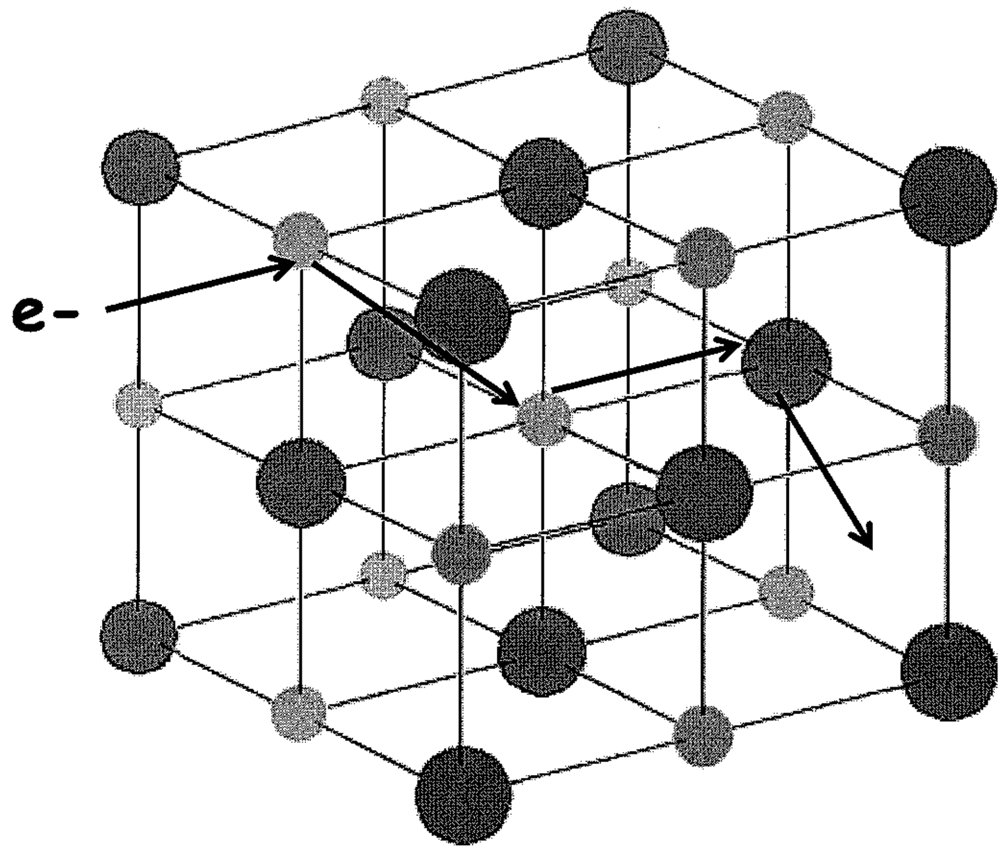
- Noise
  - Sources of Noise
  - Why Normal (Gaussian) PDF's?
- Normal Random Variables
  - Shift invariance - two views.
  - Joint and Conditional Probability
  - Cumulative Distribution Function
- Eye Slicing
  - Combining ISI and Noise
  - Evaluating BER

Noise Can Be Due to Fundamental Processes

2

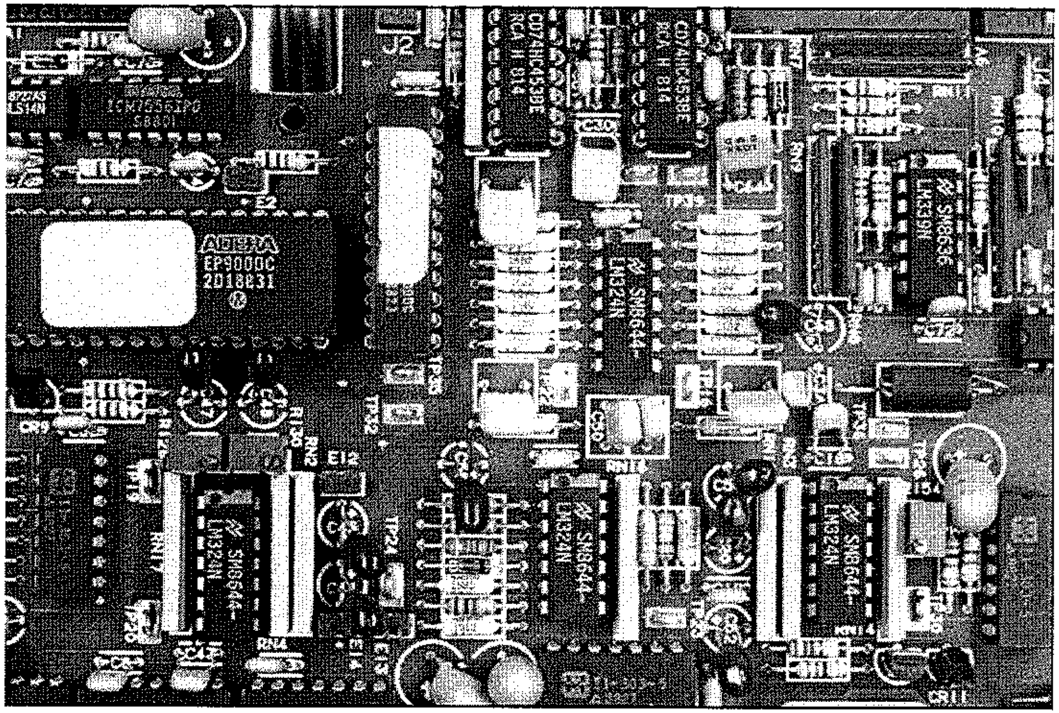
Electron Moving Through Crystal with Vibrating Atoms



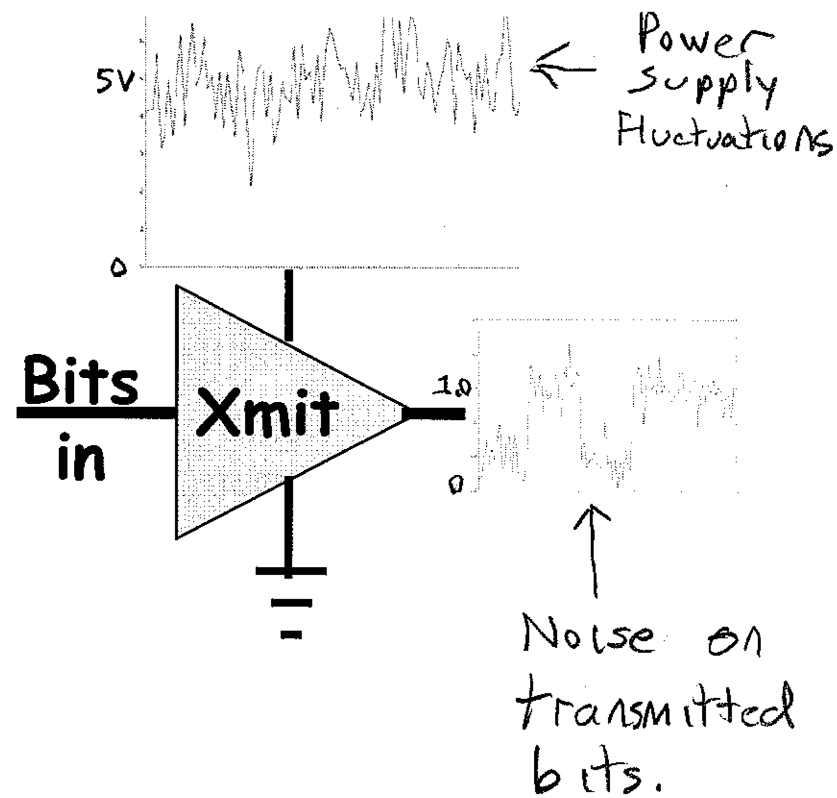
http://www4.nau.edu/meteorite/Meteorite/Images/Sodium_chloride_crystal.png

Randomized path leads to noisy current flow

Effect of Many Interactions Can Be Modeled as Noise



http://www.imageteck.net/PCB%20Board%203.JPG



Many components connected by thin wires (that have inductance and resistance) to single power supply – 1000's of devices switching on and off creates "noisy" power supply.

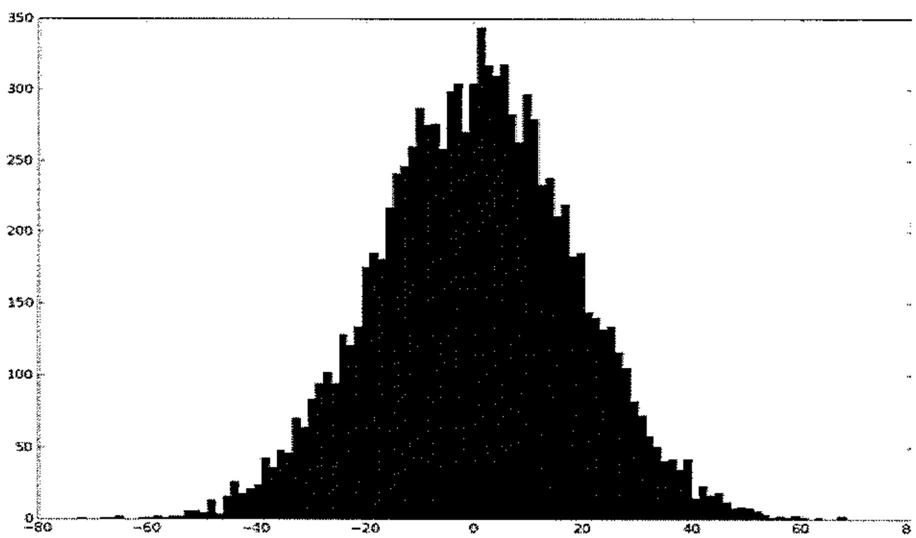
4

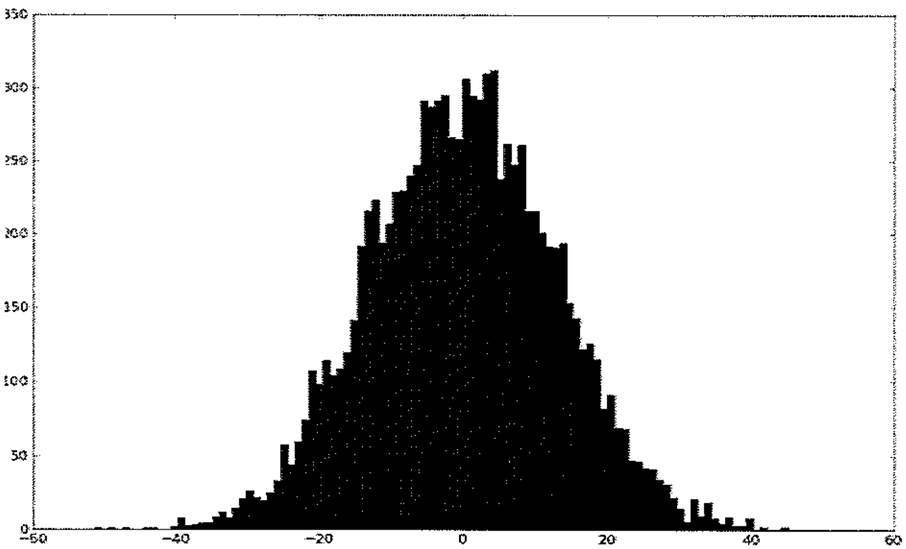
Why Use Normal (Gaussian) PDF

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ = mean or expected value

σ = standard deviation

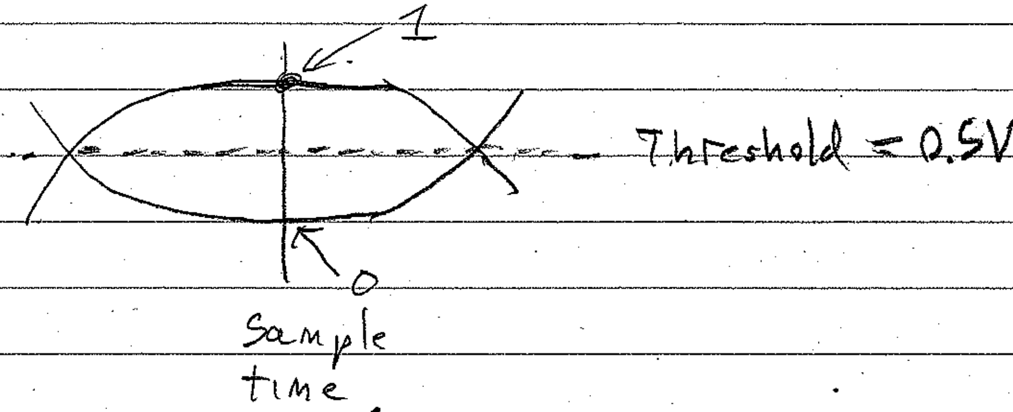




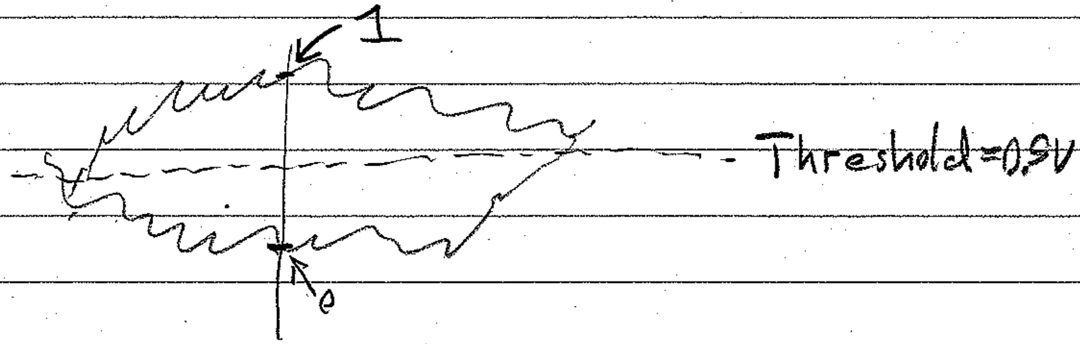
Histogram for 10000 trials of sums of 1000 uniformly (right) or triangularly (left) distributed [-1,1] random variables

Good Channel

No Noise



Noisy



If the current bit is one

$$V_{sample} = 1.0 + \text{noise}$$

$$f_{Noise}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x)^2/2\sigma^2}$$

or

mean value

$$f^{one}_{Vsample}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-1)^2/2\sigma^2}$$

Normal Distribution is shift invariant.

In General

$$Y = aX + b \quad f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \Rightarrow f_Y(y) = \frac{1}{\sqrt{2\pi}\,a} e^{-\frac{(y-b)^2}{2a^2}}$$

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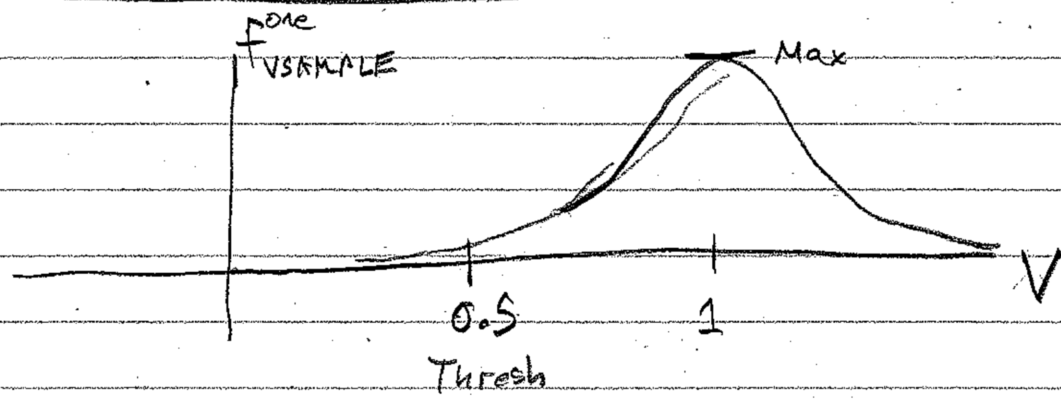
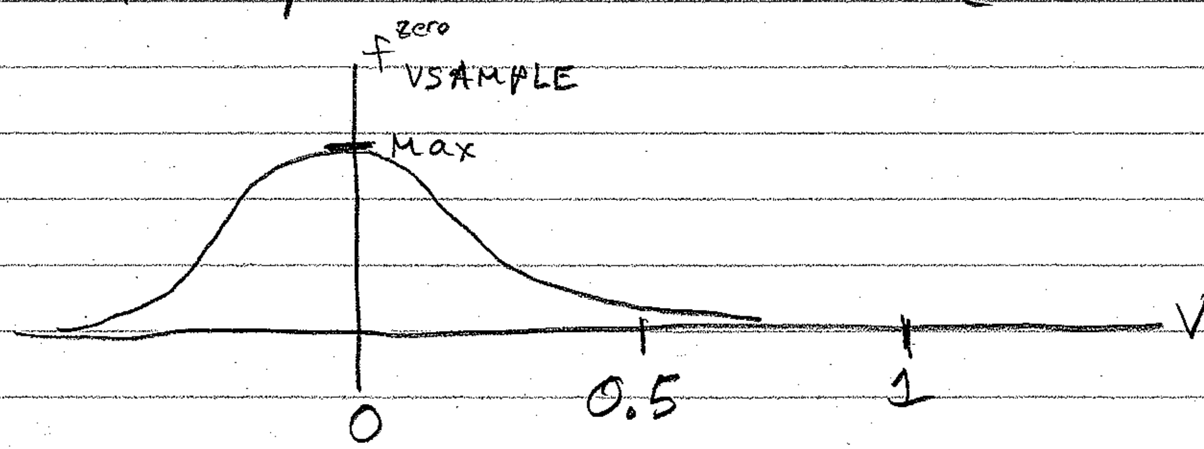
If current bit is a one



If current bit is a zero

$$V_{sample} = 0.0 + noise$$



What is the pdf of Vsample

$$f_{VSAMPLE}(x) = f_{Vsample}(V \mid bit=0) \cdot P(bit=0)$$

given

$$+ f_{Vsample}(V \mid bit=1) \cdot P(bit=1)$$

$$= f^{one}_{Vsample}(V) \cdot \frac{1}{2} + f^{zero}_{Vsample}(V) \cdot \frac{1}{2}$$

A note about Definitions

Joint pdf

discrete

$f_{Vsample, Bit}(V, b)$

continuous

Marginal pdf (Summed over all cases of b)

$f_{Vsample}(V)$

v and b=0

and

$= f_{Vsample, bit}(V, b=0)$

$+$

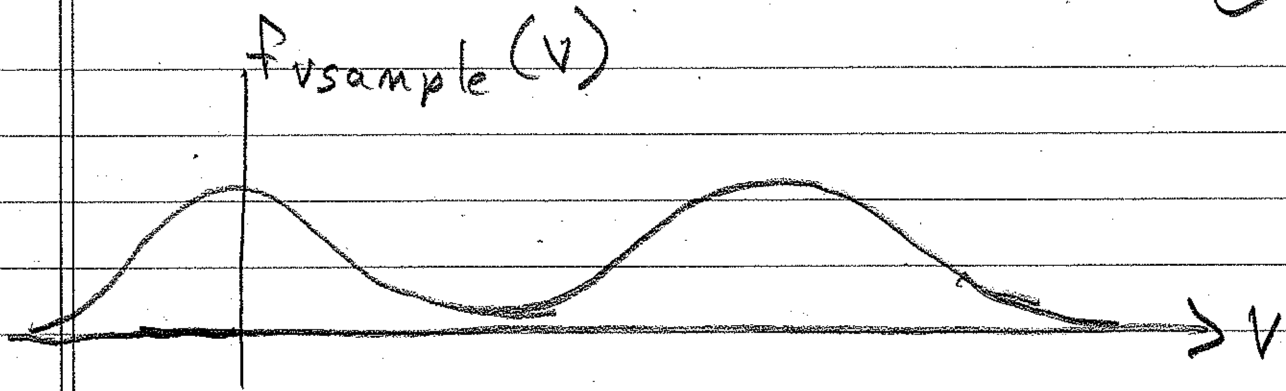
$f_{Vsample, bit}(V, b=1)$

$\underbrace{f_{Vsample}(V \mid bit=0)}_{f^{one}_{sample}(V)} \cdot P(bit=0)$

Conditional pdf

$\underbrace{f_{Vsample}(V \mid bit=1)}_{f^{zero}_{sample}(V)} P(bit=1)$

Conditional pdf


$f_{v\,sample}(v)$
v

Cumulative Distribution Function

Def

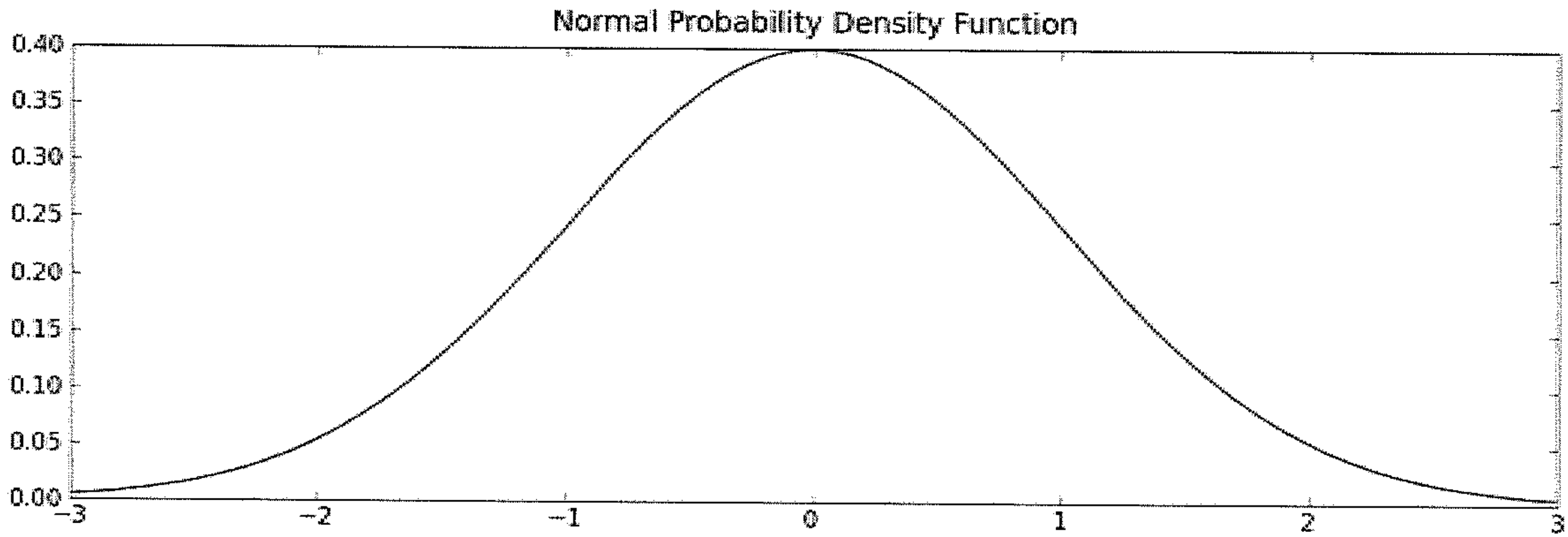
$$\int_{-\infty}^{\hat{x}} f_X(x)\,dx = P(x < \hat{x})$$

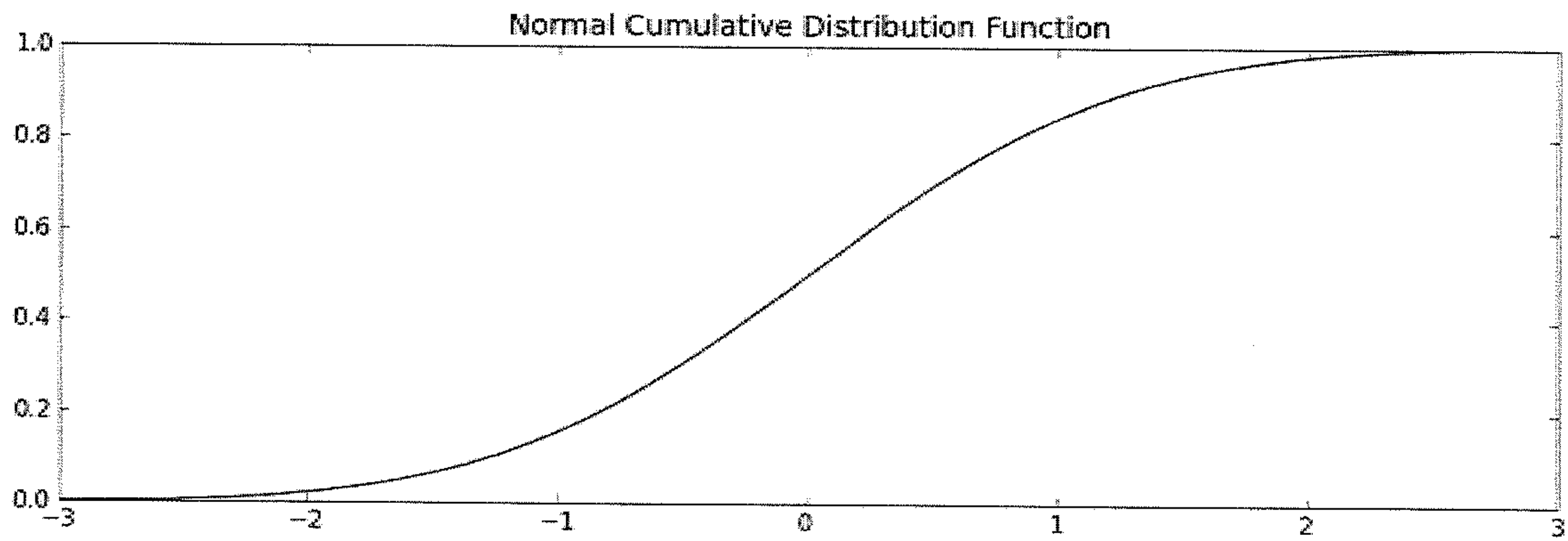
For Normal R.V.

$$\int_{-\infty}^{\hat{x}} \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}\,dx = \underbrace{CDF(\hat{x})}_{\text{This is a probability}}$$

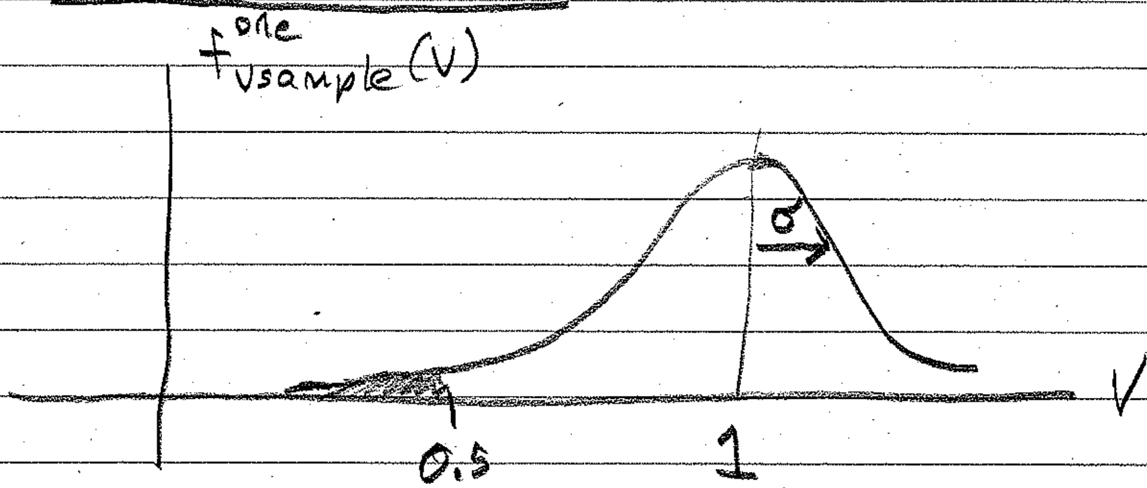
Use CDF for bit errors

Cumulative Distribution Function





bit = 1 case



$CDF^{one}_{vsample}(0.5)$

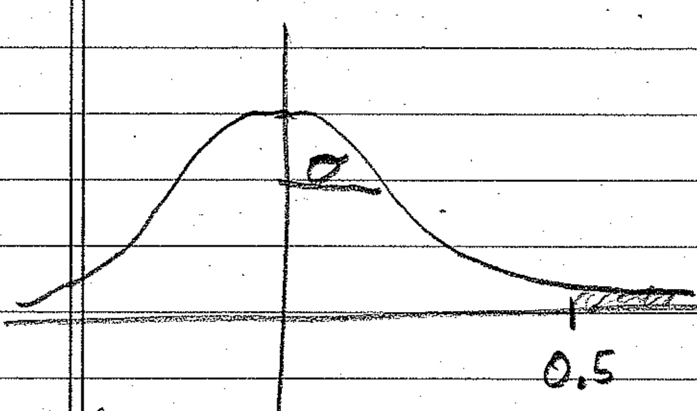
mean

$$\underbrace{CDF_{\substack{Normal \\ \sigma=1\ \mu=0}}}_{\text{function in python}}\left(\frac{0.5-1}{\sigma}\right) = \underbrace{E_{10}}_{\substack{\text{supposed to} \\ \text{be one,} \\ \text{got zero}}}$$

standard dev

$= Prob(Rcv=0 \mid Sent=1)$

bit = 0 case

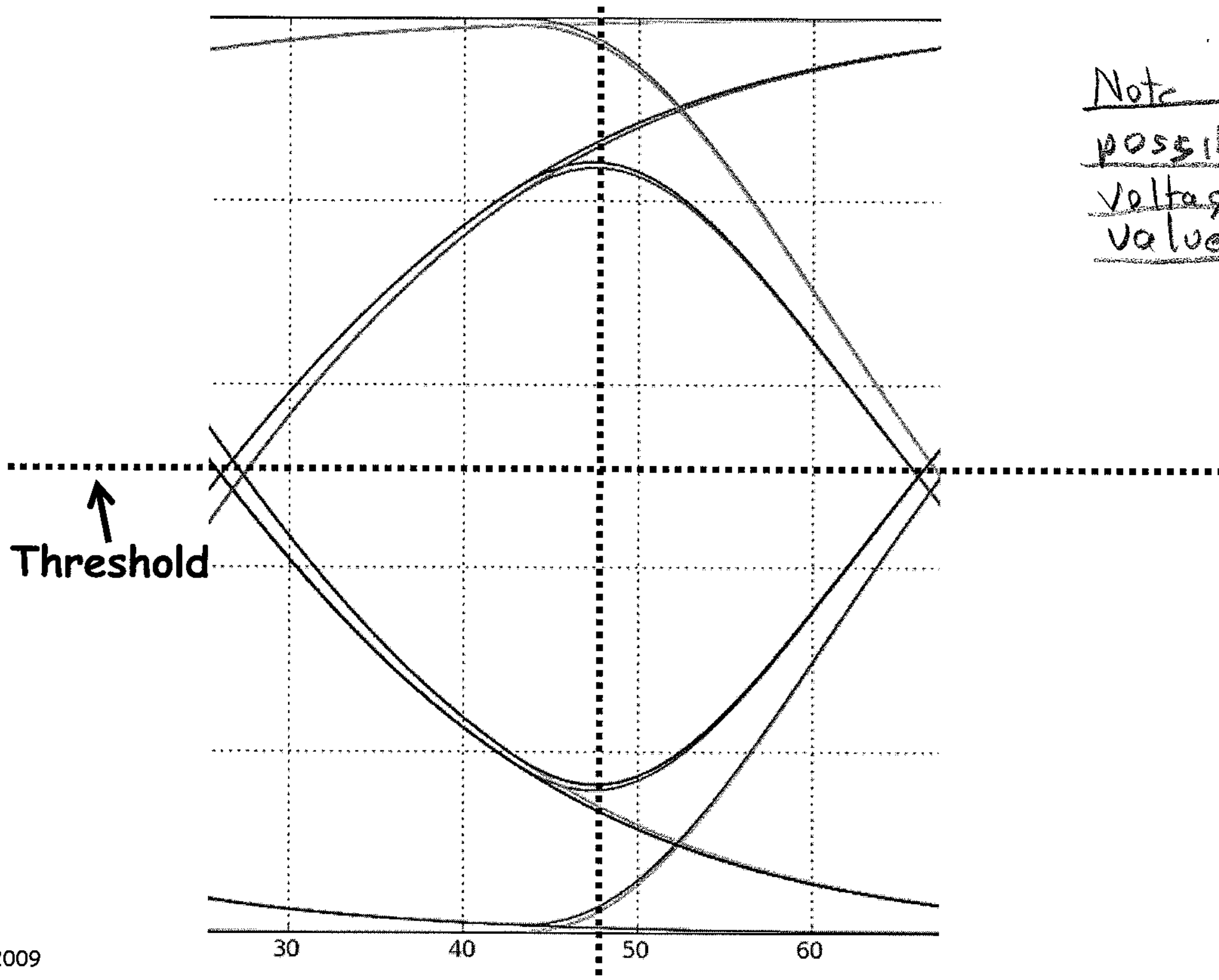


BE Probability

$Prob(Rcv=0 \mid Sent=1) \cdot Prob(Sent=1)$
$+$
$Prob(Rcv=1 \mid Sent=0) \cdot Prob(Sent=0)$

$$\left(1 - CDF_{\substack{Normal \\ \sigma=1\ \mu=0}}\left(\frac{0.5}{\sigma}\right)\right) = Prob(Rcv=1 \mid Sent=0)$$

Eye diagram slice



Note
possible
voltage
values

16 possible noise free voltages

Eight associated with current bit = 1

bit pattern								
Voltages →	1	0.99	0.93	0.92	0.86	0.85	0.83	0.82
Next	1	1	0	0	1	1	0	0
Current	1	1	1	1	1	1	1	1
prev	1	1	1	1	0	0	0	0
prevprev	1	0	1	0	1	0	1	0

To evaluate B.E.R (bit=0 and bit=1 equally likely)

Sum 16 CDF's

probability that next = current = prev = prevprev = 1

Thresh   mean

Noise Standard Deviation

$$\frac{1}{16}\,CDF\left(\frac{0.5-1}{\sigma}\right) + \frac{1}{16}\,CDF\left(\frac{0.5-0.99}{\sigma}\right)$$

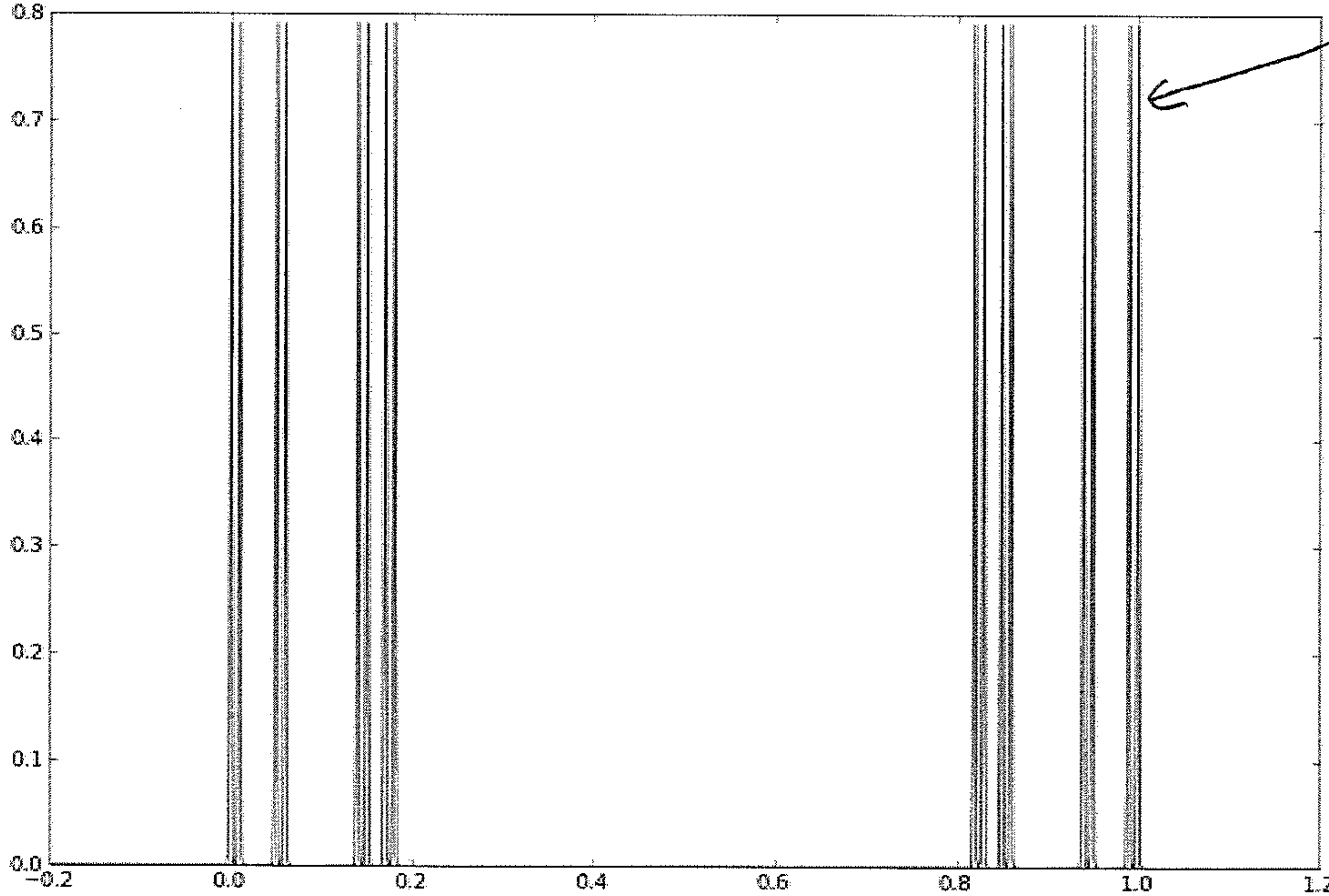
$$+ \frac{1}{16}\,CDF\left(\frac{0.5-0.93}{\sigma}\right) + \cdots$$

$$+ \frac{1}{16}\left(1-CDF\left(\frac{0.5-0.07}{\sigma}\right)\right) +$$

$$\frac{1}{16}\left(1-CDF\left(\frac{0.5-.01}{\sigma}\right)\right) + \frac{1}{16}\left(1-CDF\left(\frac{0.5-0.0}{\sigma}\right)\right)$$

Slice PDF (std=0.001)

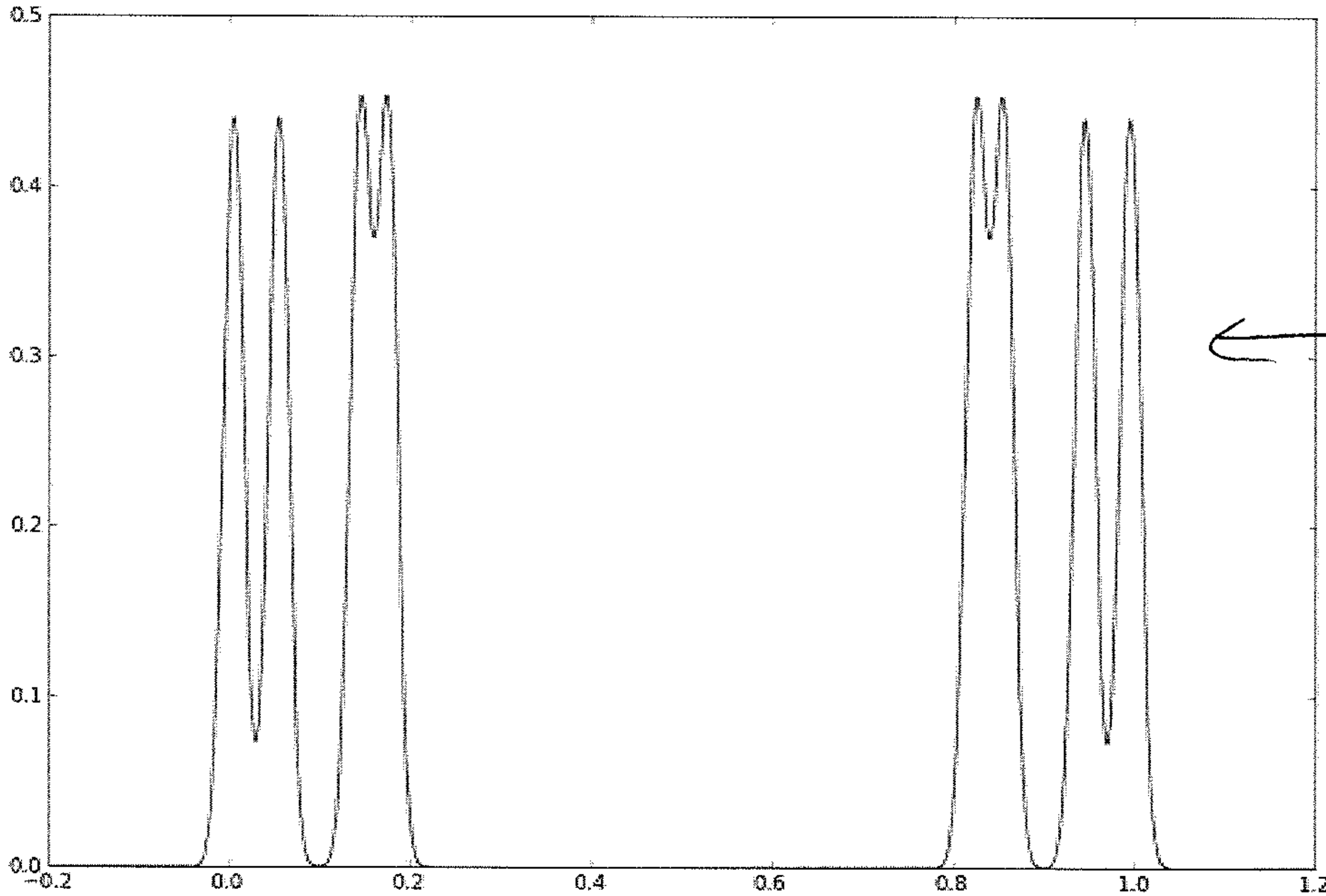
14



Note: can see all 16 peaks!

Slice PDF (std=0.01)

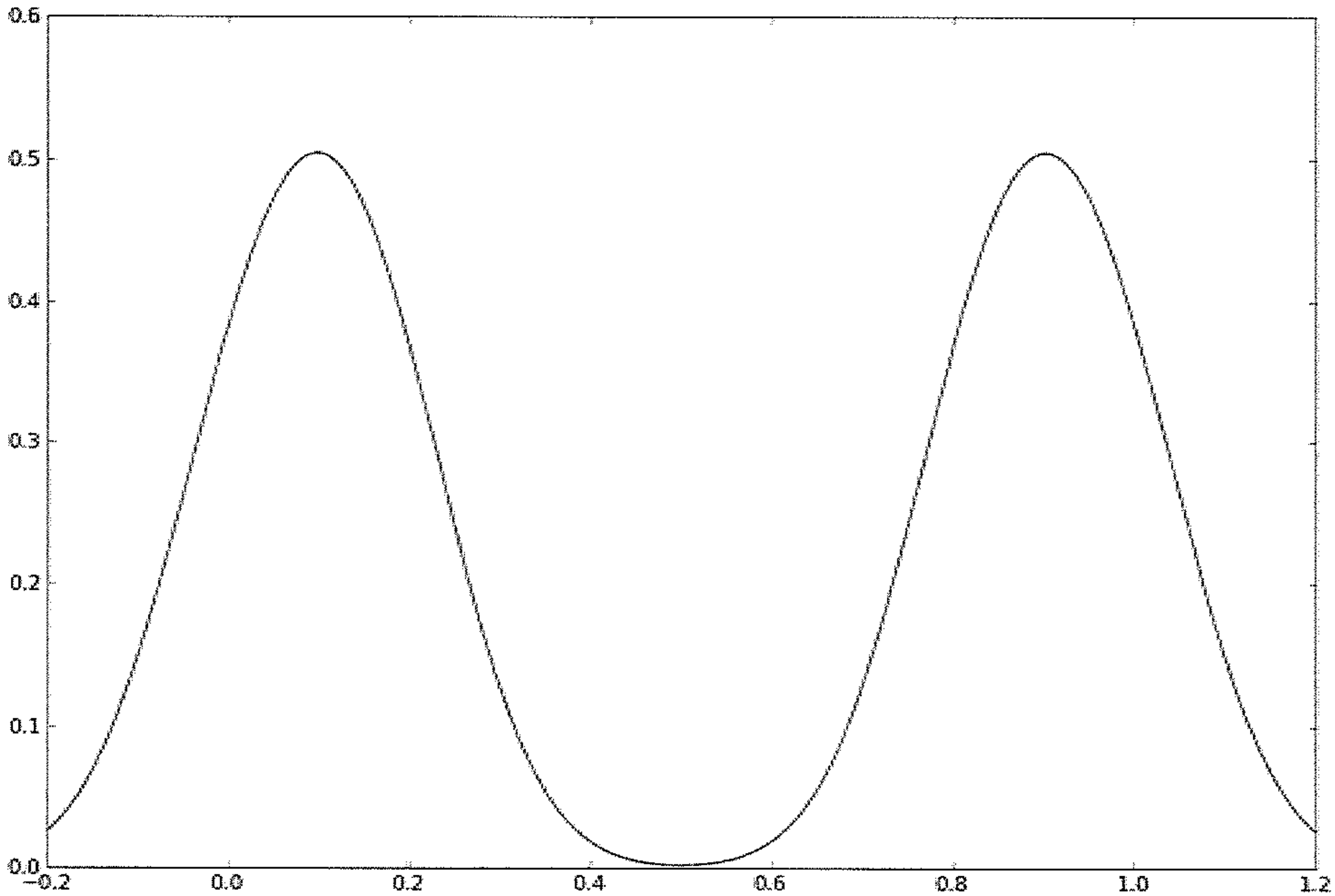
15



can not see individual peaks

Slice PDF (std=0.1)

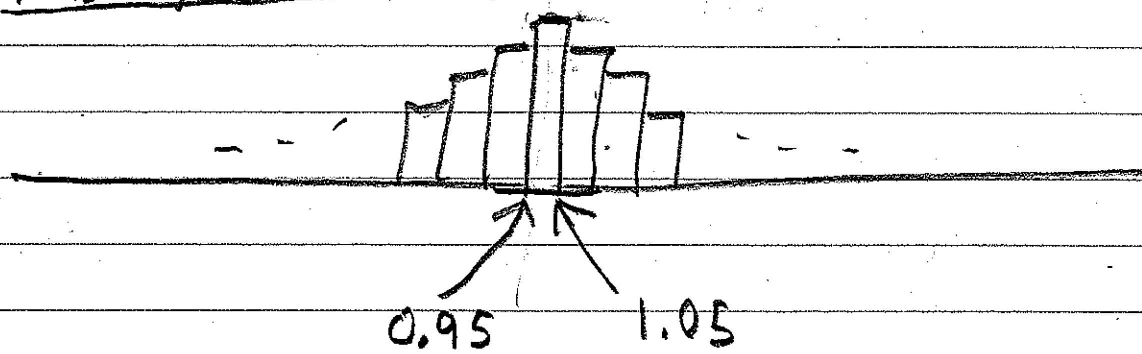
16



CDF's and Histogram Example

Suppose Gaussian with $\mu = 1$ and $\sigma = 3$ is used to generate noise

Histogram has bins



Probability of being in $0.95 \to 1.05$ bin

$$\int_{0.95}^{1.05} \frac{1}{\sqrt{2\pi}\, 3} e^{-\frac{(x-1)^2}{2(3)^2}} dx$$

$$= \Phi\left(\frac{1.05 - \overset{\text{Mean}}{1}}{\underset{\sigma}{3}}\right) - \Phi\left(\frac{0.95 - 1}{3}\right)$$

cumulative distribution function for a zero mean, unit variance Gaussian (Normal) random variable