**6.02 Digital Communication Systems—Spring 2009**

**Quiz 1 - 7:30-9:30pm (Two Hours)**
**Thursday, March 5, 2009**

| Check your section | Section | Time | Room | Rec. Instr. |
|---|---|---|---|---|
| ☐ | 1 | 10-11 | 13-5101 | Chris Terman |
| ☐ | 2 | 11-12 | 13-5101 | Chris Terman |
| ☐ | 3 | 1-2 | 5-233 | Mythili Vutukuru |
| ☐ | 4 | 2-3 | 5-233 | Mythili Vutukuru |
| ☐ | 5 | 1-2 | 38-166 | Vladimir Stojanovic |
| ☐ | 6 | 2-3 | 38-166 | Vladimir Stojanovic |

Directions: The exam consists of 6 problems on 10 pages. Please make sure you have all the pages. **Enter all your work and your answers directly in the spaces provided on the printed pages of this exam. Please make sure your name is on all sheets. DO IT NOW!** All sketches must be adequately labeled. Unless indicated otherwise, **answers must be derived or explained in the space provided**, not just simply written down. This examination is closed book, but students may use one 8 1/2 $\times$ 11 sheet of paper for reference. Calculators may not be used.

The probability density function for a zero-mean unit standard deviation Normal(Gaussian) random variable is:

$$f_X(x) \equiv \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

The cumulative distribution function for a zero-mean unit standard deviation Normal(Gaussian) random variable is:

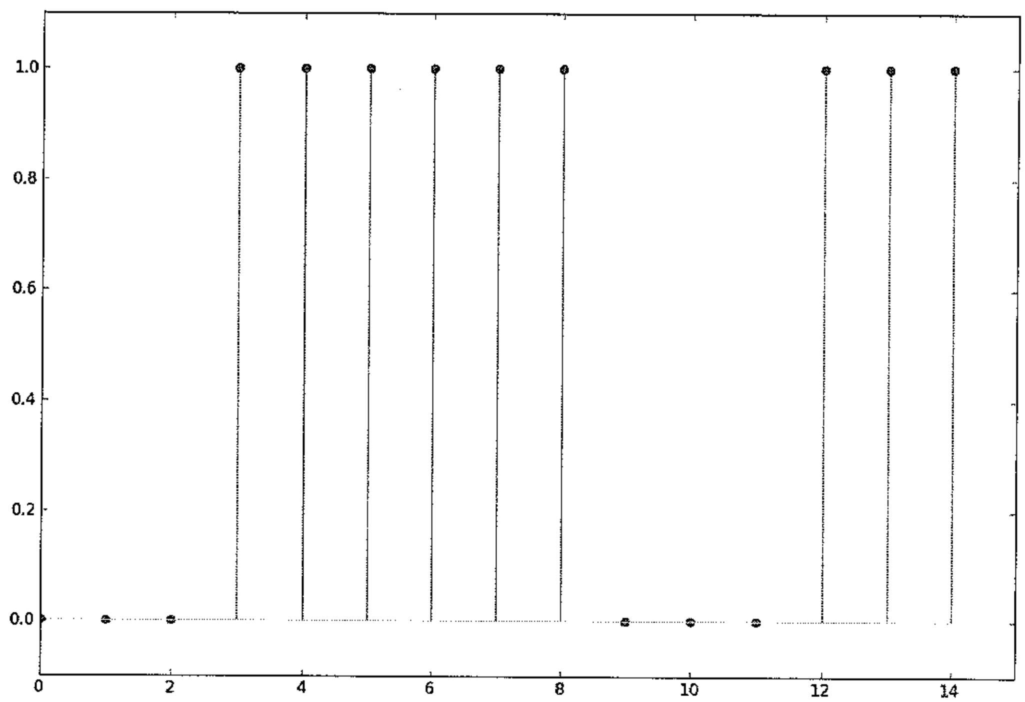
$$\Phi(x) \equiv \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-\hat{x}^2}{2}} d\hat{x}$$

**Please leave the rest of this page blank for use by the graders:**

| Problem | No. of points | Score | Grader |
|---|---|---|---|
| 1 | 5 | | |
| 2 | 10 | | |
| 3 | 17 | | |
| 4 | 24 | | |
| 5 | 24 | | |
| 6 | 20 | | |
| Total | 100 | | |

**Problem 1 (5 points)**

In the following plot of a voltage waveform from a transmitter, the transmitter sends 0 Volts for a zero bit and 1.0 Volts for a one bit, and is sending bits with with a certain number of samples per bit.




**1A.** (2 points) What is the largest number of samples per bit the transmitter could be using?

3 (Some students might say 1)

**1B.** (3 points) What is the sequence of bits being sent?

0 1 1 0 1 (If 1A is 1, then 000111111000111)

**Problem 2 (10 points)**

The input sequence to a linear time-invariant (LTI) system is given by

$x[0] = 0$,
$x[1] = 1$,
$x[2] = 1$ and
$x[n] = 0$ for all other values of $n$

and the output of the LTI system is given by

$y[0] = 1$,
$y[1] = 2$,
$y[2] = 1$ and
$y[n] = 0$ for all other values of $n$.

**2A.** (3 points) Is this system causal? Why or why not?

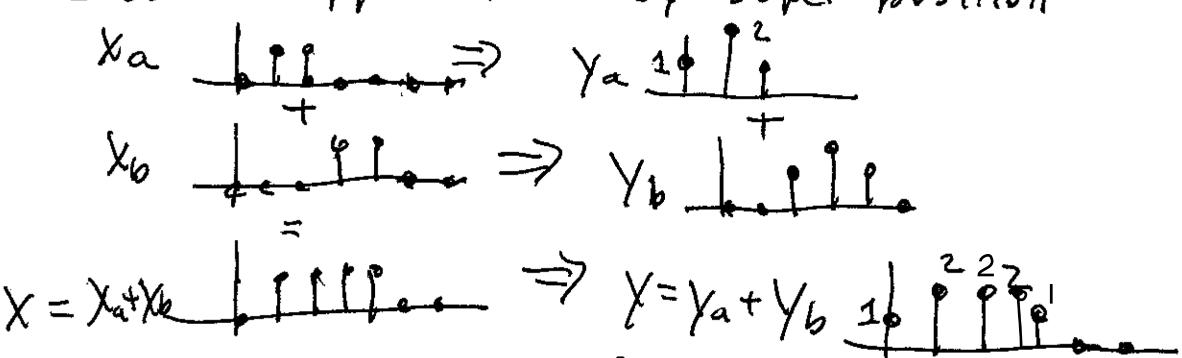
System is not causal $x[n] = 0$ $n < 1$, but
$y[0] = 1$ (y becomes nonzero before x does)

**2B.** (7 points) What are the nonzero values of the output of this LTI system when the input is

$x[0] = 0$,
$x[1] = 1$,
$x[2] = 1$,
$x[3] = 1$,
$x[4] = 1$ and
$x[n] = 0$ for all other values of $n$?

Note, can be done by determining $h[n]$ and then convolving, much harder

Easiest approach is by super position

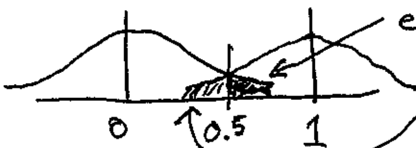


**Problem 3 (17 points)**

Suppose the voltage sampled by a receiver is $1.0 + noise$ volts when the transmitter sends a one bit, and $0.0 + noise$ volts when the transmitter sends a zero bit, where $noise$ is a zero-mean Normal(Gaussian) random variable with standard deviation $\sigma$.

**3A.** (4 points) If the transmitter is equally likely to send zeros or ones, and 0.5 volts is used as the threshold for deciding the bit value, give an expression for the bit-error rate (BER) in terms of the zero-mean unit standard deviation Normal cumulative distribution function, $\Phi$, and $\sigma$.

Note 2 possible acceptable answers




errors given bit = 0 $\quad 1-\Phi\left(\frac{0.5}{\sigma}\right)$

errors given bit = 1 $\quad \Phi\left(\frac{0.5-1}{\sigma}\right) = \Phi\left(\frac{-0.5}{\sigma}\right)$

So $\frac{1}{2}\left(1-\Phi\left(\frac{0.5}{\sigma}\right)\right) + \frac{1}{2}\Phi\left(\frac{-0.5}{\sigma}\right)$ or (by symmetry of gaussian) $\Phi\left(\frac{-0.5}{\sigma}\right)$

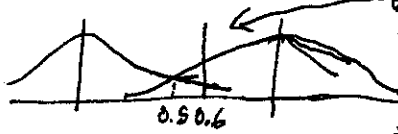
**3B.** (7 points) If the transmitter is equally likely to send zeros or ones, and 0.5 volts is used as the threshold for deciding the bit value, for what value of $\sigma$ is the probability of a bit error approximately equal to $\frac{1}{5}$? Note that $\Phi(0.85) \approx \frac{4}{5}$ (see cover page for definition of $\Phi$).

$$\left(1-\Phi\left(\frac{0.5}{\sigma}\right)\right) = \Phi\left(\frac{-0.5}{\sigma}\right) = BER$$

$$(1-\Phi(0.85)) = \frac{1}{5} \Rightarrow \frac{0.5}{\sigma} = 0.85 \Rightarrow \sigma = \frac{0.5}{0.85} = \frac{10}{17} = .588$$

**3C.** (3 points) Will your answer for $\sigma$ in part 3B change if the threshold is shifted to 0.6 volts? Do not try to determine $\sigma$, but justify your answer.

$$\frac{1}{2}\left(1-\Phi\left(\frac{0.6}{\sigma}\right)\right) + \frac{1}{2}\Phi\left(\frac{-0.4}{\sigma}\right) \Rightarrow$$




Error unbalanced will be higher than balanced case $\sigma \to$ change

or The errors when bit transmitted is a one will increase more than errors when bit transmitted is a zero will decrease if $\sigma$ = 3B answer, so $\sigma$ must change (to a lower) for $BER = \frac{1}{5}$

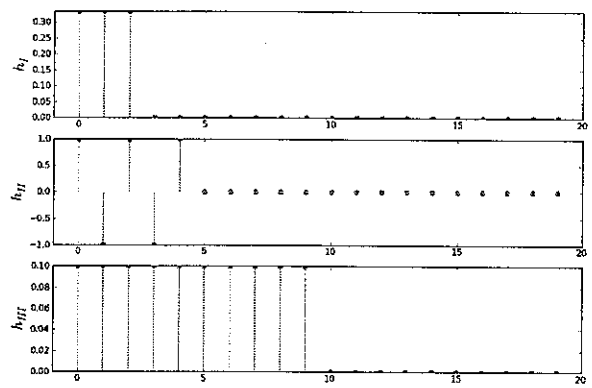
**3D.** (3 points) Will your answer for $\sigma$ in part 3B change if the transmitter is twice as likely to send ones as zeros, but the threshold for deciding bit value is still 0.5 volts? Do not try to determine $\sigma$, but justify your answer.

$$\frac{1}{3}\left(1-\Phi\left(\frac{0.5}{\sigma}\right)\right) + \frac{2}{3}\left(\Phi\left(\frac{-0.5}{\sigma}\right)\right) = \Phi\left(\frac{-0.5}{\sigma}\right)$$

No change

**Problem 4 (24 points)**

Consider three linear time-invariant systems, denoted I, II, and III, each characterized by their unit-sample responses:

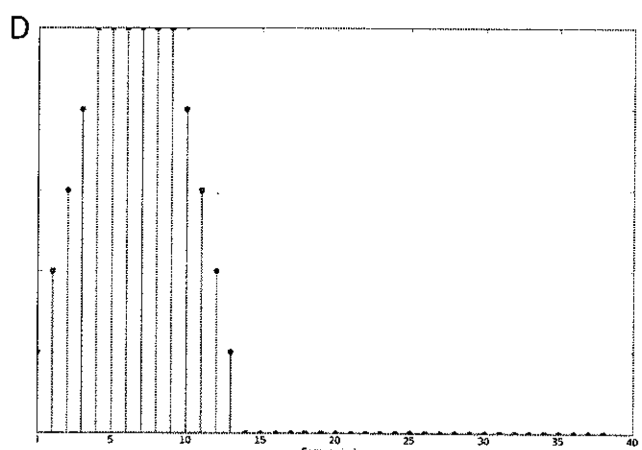



There are also three possible inputs to each of these linear time-invariant systems:

Sequence $X_1$:$x_1[n] = u[n]$
Sequence $X_2$:$x_2[n] = (-1)^n(u[n] - u[n-5])$
Sequence $X_3$:$x_3[n] = u[n] - u[n-5]$

Note that $u[n]$ denotes the unit step. That is $u[n] = 1$ for $n \geq 0$ and is zero otherwise.

**4A.** (12 points) Which system (I, II or III) and which input ($X_1, X_2$ or $X_3$) produced the following output, **AND** what is the numerical value of $D$? In addition to justifying your answer, for possible partial credit, briefly explain why you ruled out particular combinations of the given LTI systems and responses.



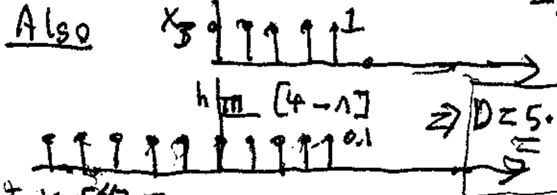

Response is 14 samples long

Not $X_1$ and any $h_I, h_{II}, h_{III}$ $y[n]$ nonzero forever

Not ~~$X_1$~~ $X_2, X_3$ and $h_I, h_{II}$ nonzero for fewer than 14 samples

$\boxed{X_3 \text{ and } h_{III}}$ 14 samples (nonzeros of $X_3$ + nz's $h_{III}$)

Also



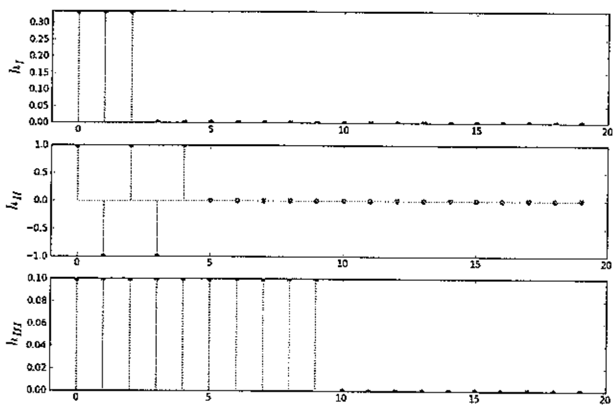

$\Rightarrow \boxed{D = 5 \cdot 0.1 = 1/2}$



Note Sliding $h_{III}[n]$ by $x_3[n]$ is only one to produce a sequence of 6 equal values starting at $y[4]$

**Problem 4 (continued)**

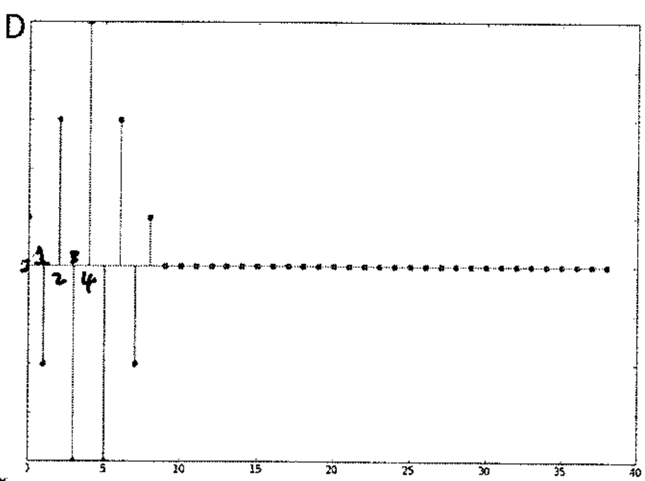
Consider the same three linear time-invariant systems as in 4A, denoted I, II, and III, each characterized by their unit sample responses (repeated here for your convenience):




There are also the same three possible inputs to each of these linear time-invariant systems (again repeated here for your convenience):

Sequence $X_1$:$x_1[n] = u[n]$
Sequence $X_2$:$x_2[n] = (-1)^n(u[n] - u[n-5])$
Sequence $X_3$:$x_3[n] = u[n] - u[n-5]$

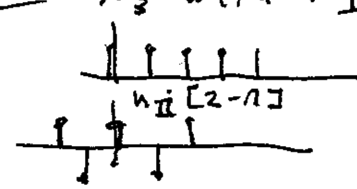
**4B.** (12 points) Which system (I, II or III) and which input ($X_1$, $X_2$ or $X_3$) produced the following output, **AND** what is the numerical value of $D$ in the plot below? In addition to justifying your answer, for possible partial credit, briefly explain why you ruled out particular combinations of the given LTI systems and responses.




Not $X_1$ with anything nonzero forever

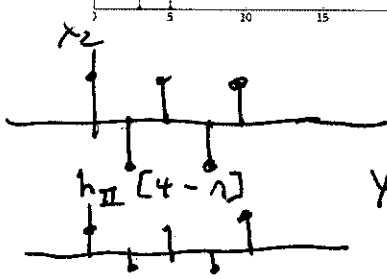
Not $X_3$ with $h_I$ or $h_{III}$ Always positive

Not $X_3$ with $h_{II}$




$y[n] = 1$ or $0$, never negative

$\boxed{X_2 \text{ with } h_{II}}$

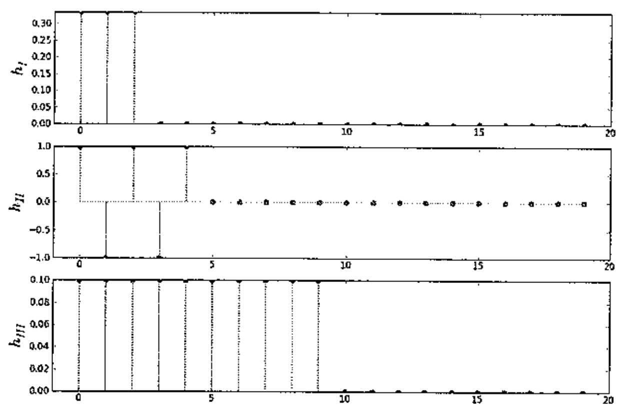



$y[4] = 1\cdot 1 + -1\cdot -1 + 1\cdot 1 + -1\cdot -1 + 1\cdot 1 = \boxed{5 = D}$

 **NAME:** Sol Key

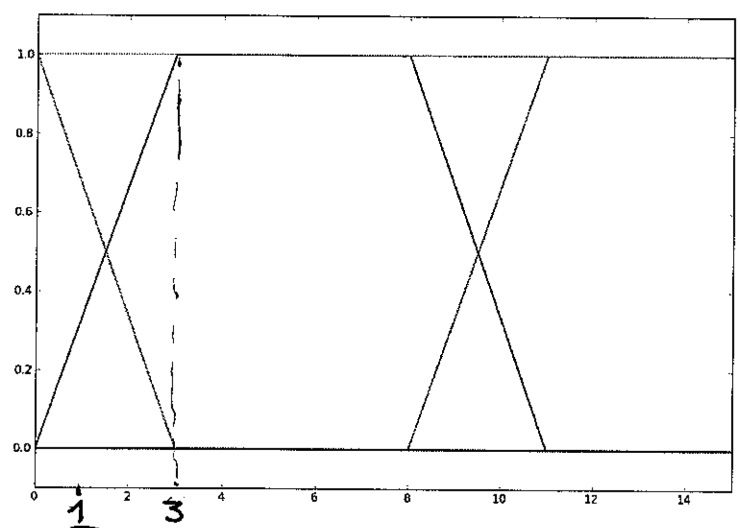
**Problem 5 (24 points)**

This question refers to the LTI systems, I, II and III, whose unit sample responses were given in questions 4 (again repeated here for your convenience):




In this question, the input to these systems are bit streams with eight voltage samples per bit, with eight one-volt samples representing a one bit and eight zero-volt samples representing a zero bit.

**5A.** (12 points) Which system (I, II or III) generated the following eye diagram? To ensure at least partial credit for your answer, explain what led you to rule out the systems you did not select.




Rise takes 3 samples can only be $h_I$

$h_{II}$ would have a rise 1,0,1,0,1

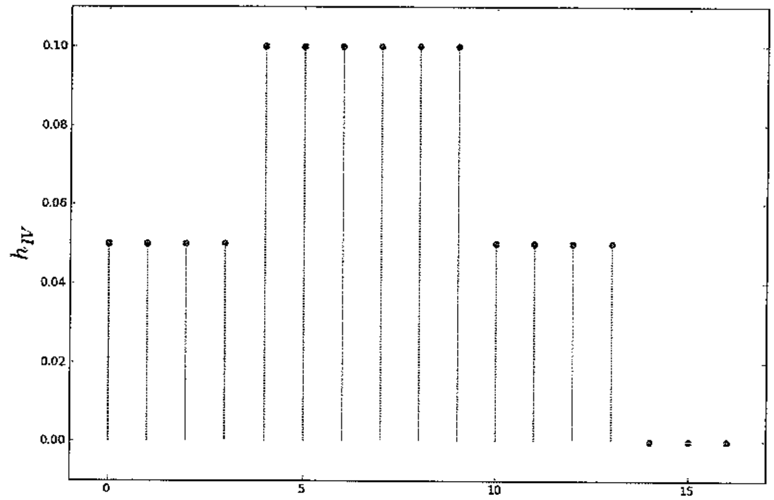
$h_{III}$ would have a ten sample rise

Also Note

All sequences produce some ISI but only $h_I$ has an extremely limited ISI

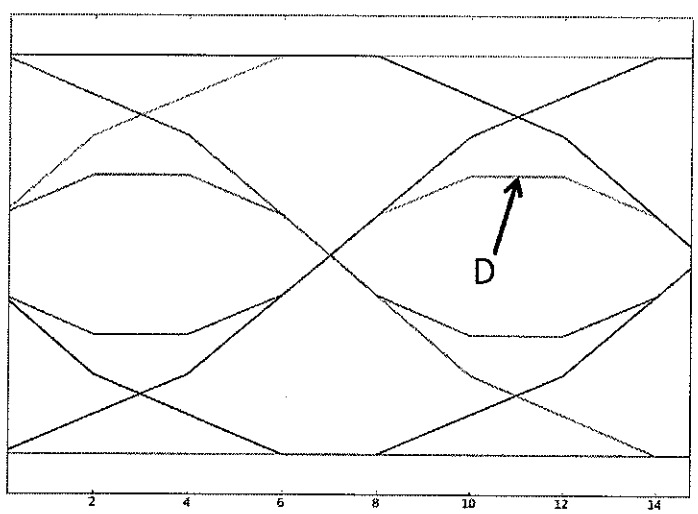
**Problem 5 (continued)**

This question refers to a fourth LTI system whose unit sample response, $h_{IV}[n]$ is given below:



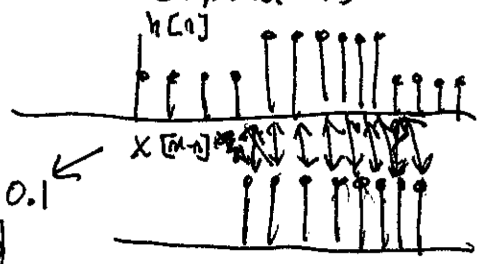

where, just like in problem 5A, the input to this system is a bit stream with eight voltage samples per bit, with eight one-volt samples representing a one bit and eight zero-volt samples representing a zero bit.

**5B.** (12 points) Determine the voltage level denoted by D in the eye diagram generated from the system with unit sample response $h_{IV}[n]$.




Lowest curve above threshold must be due to and isolate 1 bit (eight ones surrounded by zeros)

Since D is at the maximum of the isolated 1, must correspond to




$2 \cdot 0.05 + 6 \cdot 0.1$

$= \boxed{0.7 = D}$

**NAME:** Sol Key

**Problem 6 (20 points)**

Consider a transmitter that encodes pairs of bits using four voltage values. Specifically:

00 is encoded as zero volts,
01 is encoded as $^{1}/_{3}V_{high}$ volts,
10 is encoded as $^{2}/_{3}V_{high}$ volts and
11 is encoded as $V_{high}$ volts.

For this problem we will assume a wire that only adds noise. That is,

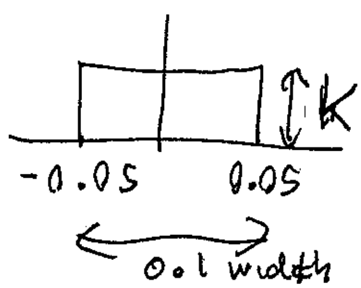
$$y[n] = x[n] + noise[n]$$

where $y[n]$ is the received sample, $x[n]$ the transmitted sample whose value is one of the above four voltages, and $noise[n]$ is a random variable.

Please assume all bit patterns are **equally likely** to be transmitted.

Suppose the probability density function for $noise[n]$ is a constant, $K$, from $-0.05$ volts to $0.05$ volts and zero elsewhere.

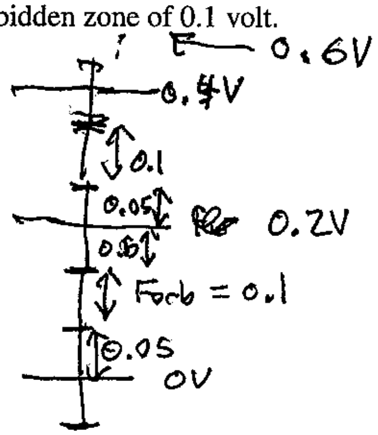
**6A.** (3 points) What is the value of $K$?




$0.1 \cdot K = \text{area} = 1$ (for pdf)

$K = 10$

**6B.** (3 points) What is the smallest numerical value for $V_{high}$ such that a threshold detector can determine the transmitted bit pair from $y[n]$ WITHOUT ERROR. Assume the threshold detector needs a forbidden zone of 0.1 volt.

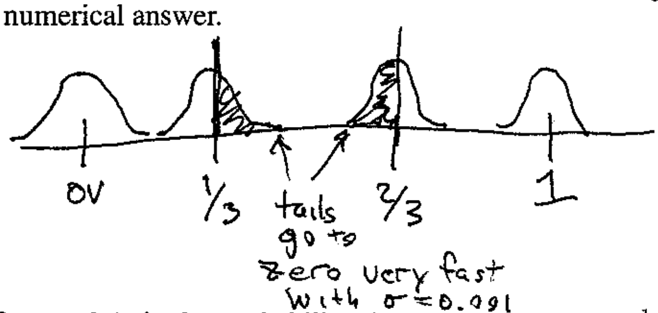



$0.6V = V_{high}$

**Problem 6 (continued)**

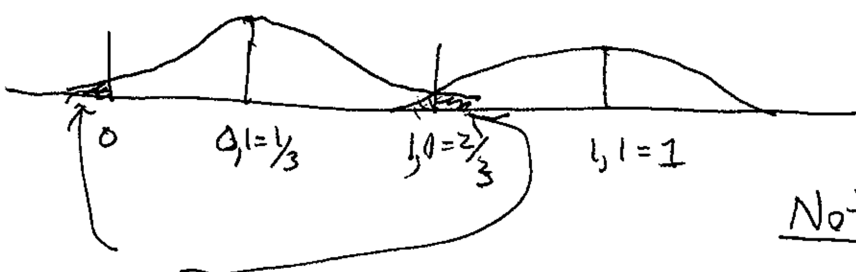
Suppose now $V_{high} = 1.0$ volts and the probability density function for $noise[n]$ is a zero-mean Normal with standard deviation $\sigma$.

**6C.** (7 points) If $\sigma = 0.001$, what is the **approximate** probability that $\frac{1}{3} < y[n] < \frac{2}{3}$? You should be able to give a numerical answer.




$$\frac{1}{2} \cdot \text{prob}\left(V_{noiseless} = \tfrac{1}{3}\right) + \frac{1}{2}\,\text{prob}\left(V_{noiseless} = \tfrac{2}{3}\right) = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{4}$$

**6D.** (7 points) If $\sigma = 0.1$, is the probability that a transmitted 01 ($\frac{1}{3}$ volts) will be incorrectly received the same as the probability that a transmitted 11 (1.0 volts) will be incorrectly received? Explain your answer.




Not the same

At 01 two ways to make an error (Mistaken for 00 or mistaken for 1,0). In the 11 case only one way to make an error (Mistaken for a 1,0). So an error given 1,1 is sent has 1/2 the probability of an error given a 0,1 is sent.

**End of Quiz 1!**