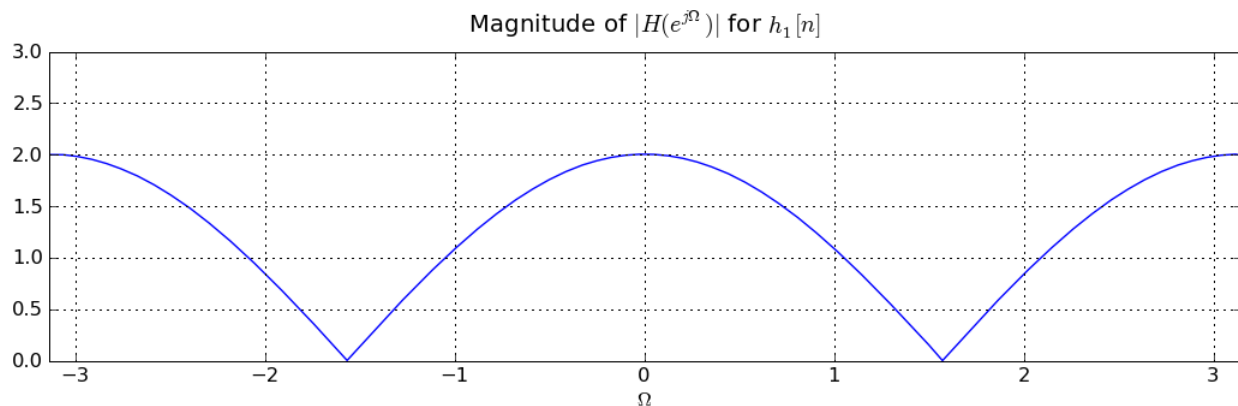
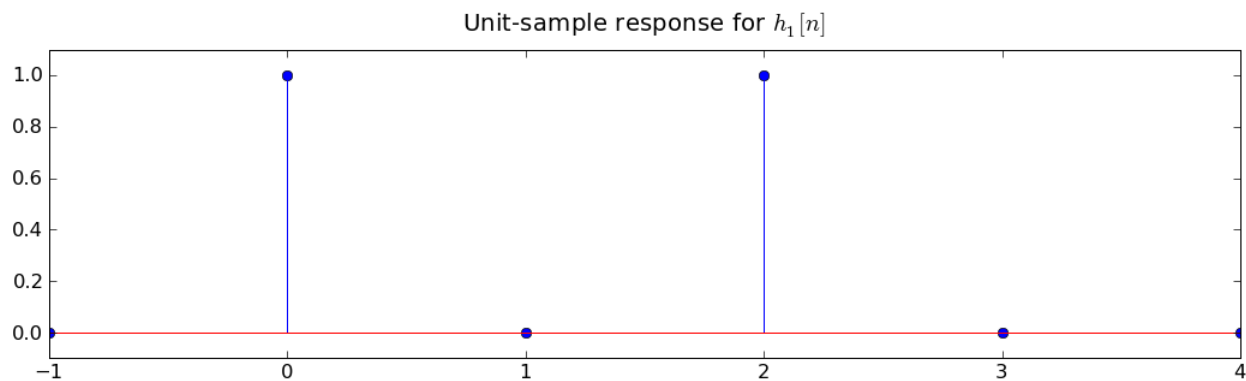


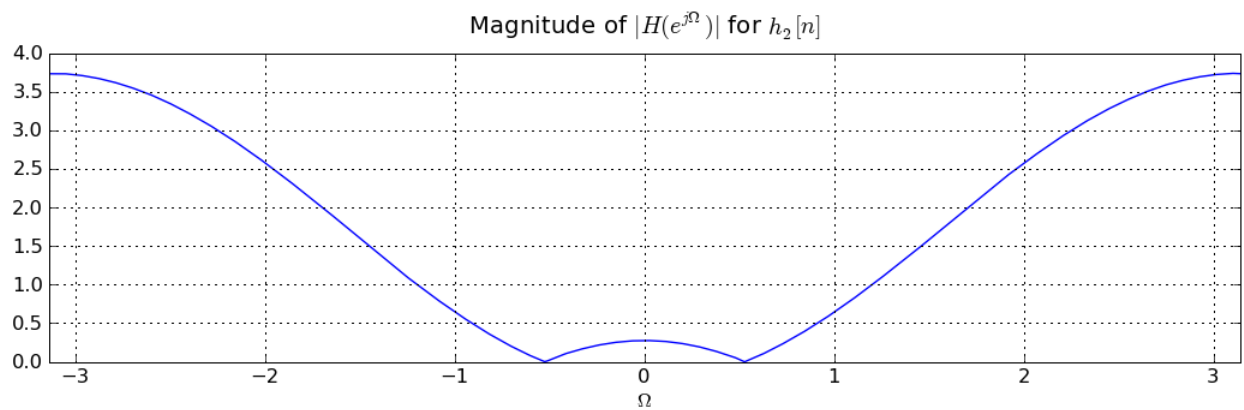
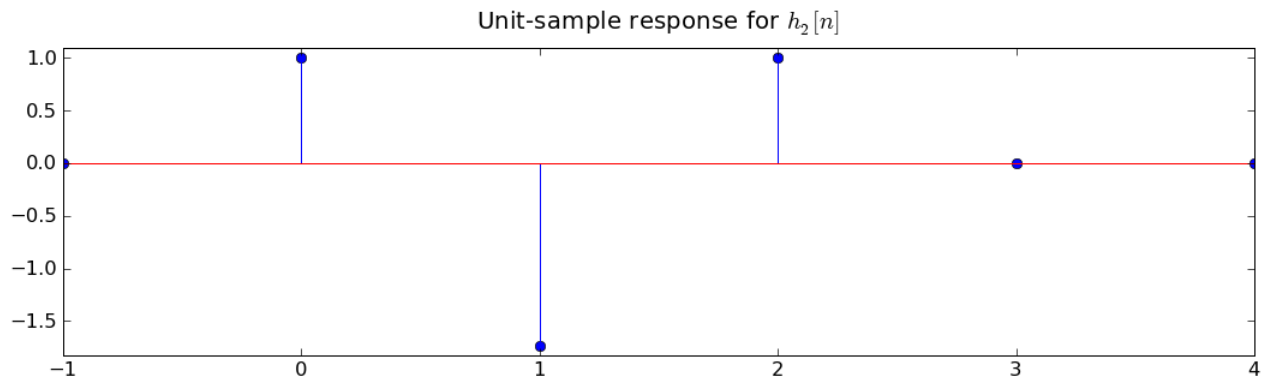
Review Problem 1

In answering the questions below, please consider the unit sample response and frequency response of two filters, H_1 and H_2 , plotted below.



Note, the only nonzero values of unit sample response for H_1 are :

$$h_1[0] = 1, h_1[1] = 0, h_1[2] = 1$$

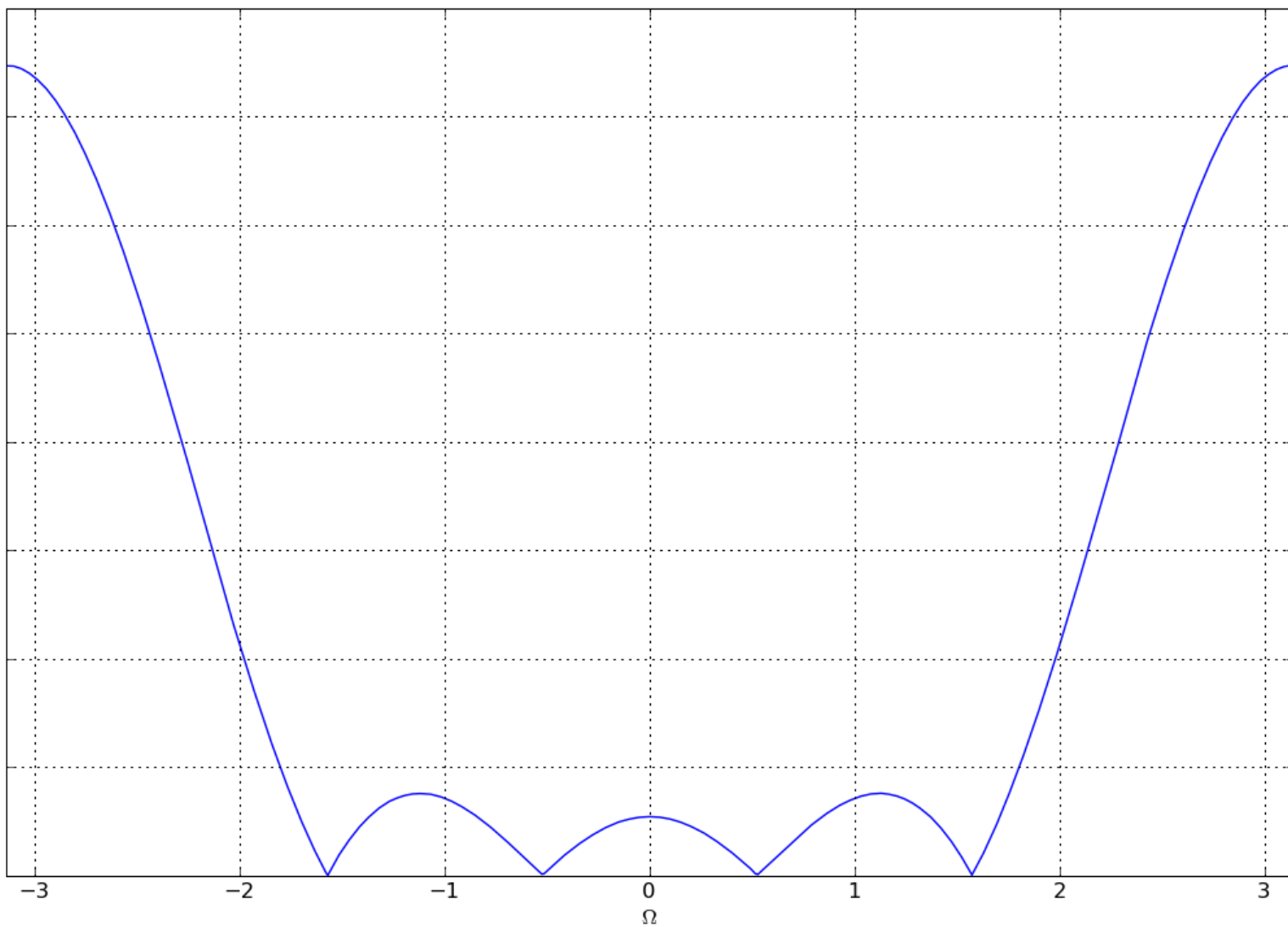


Note, the only nonzero values of unit sample response for H_2 are :

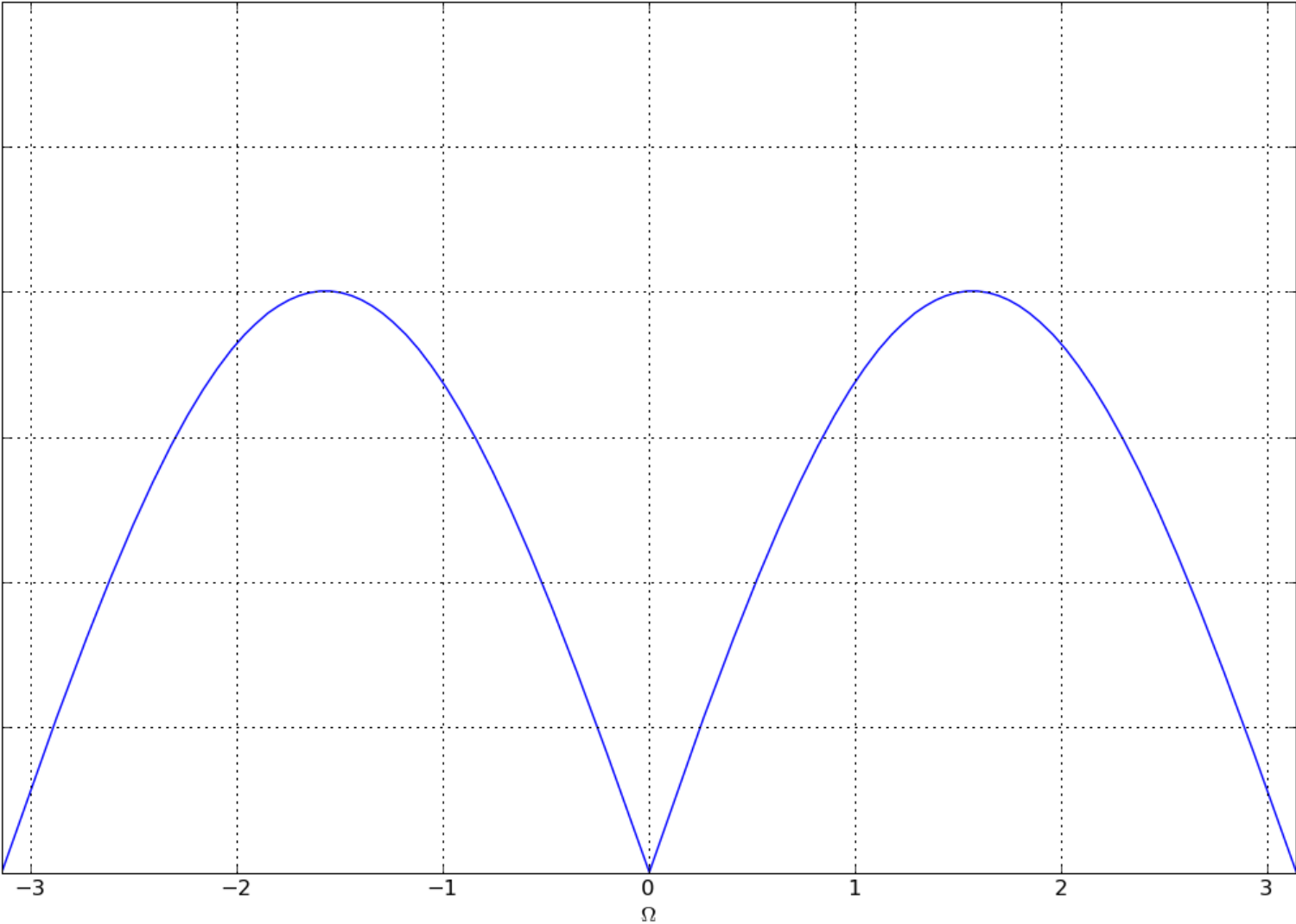
$$h_2[0] = 1, h_2[1] = -\sqrt{3}, h_2[2] = 1$$

In answering the several parts of this review question consider four linear time-invariant systems, denoted A, B, C, and D, each characterized by the magnitude of its frequency response, $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, $|H_C(e^{j\Omega})|$, and $|H_D(e^{j\Omega})|$ respectively, as given in the plots below. This is a review problem, not an actual exam question, so similar concepts are tested multiple times to give you practice

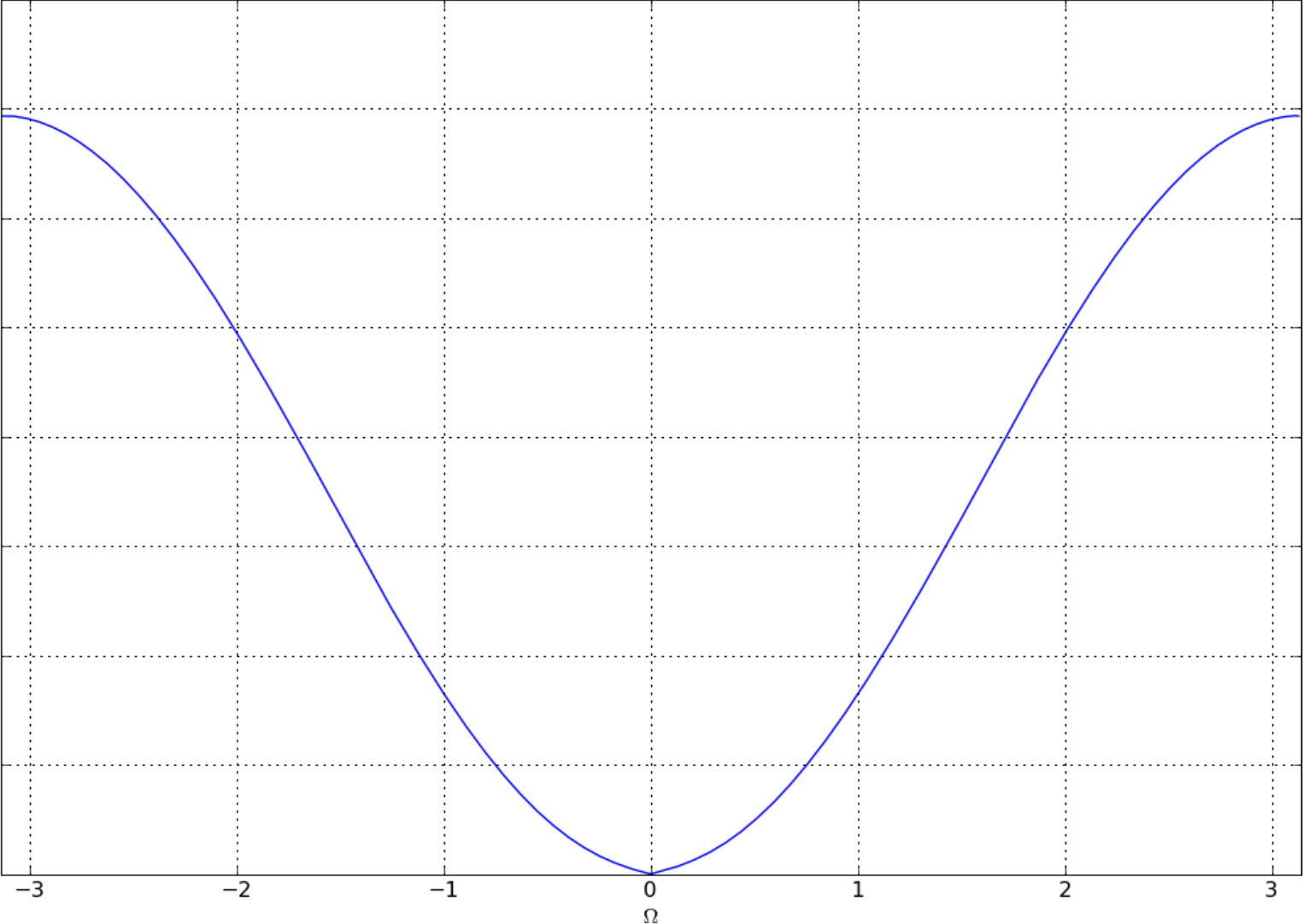
Magnitude of $|H(e^{j\Omega})|$ for H_A



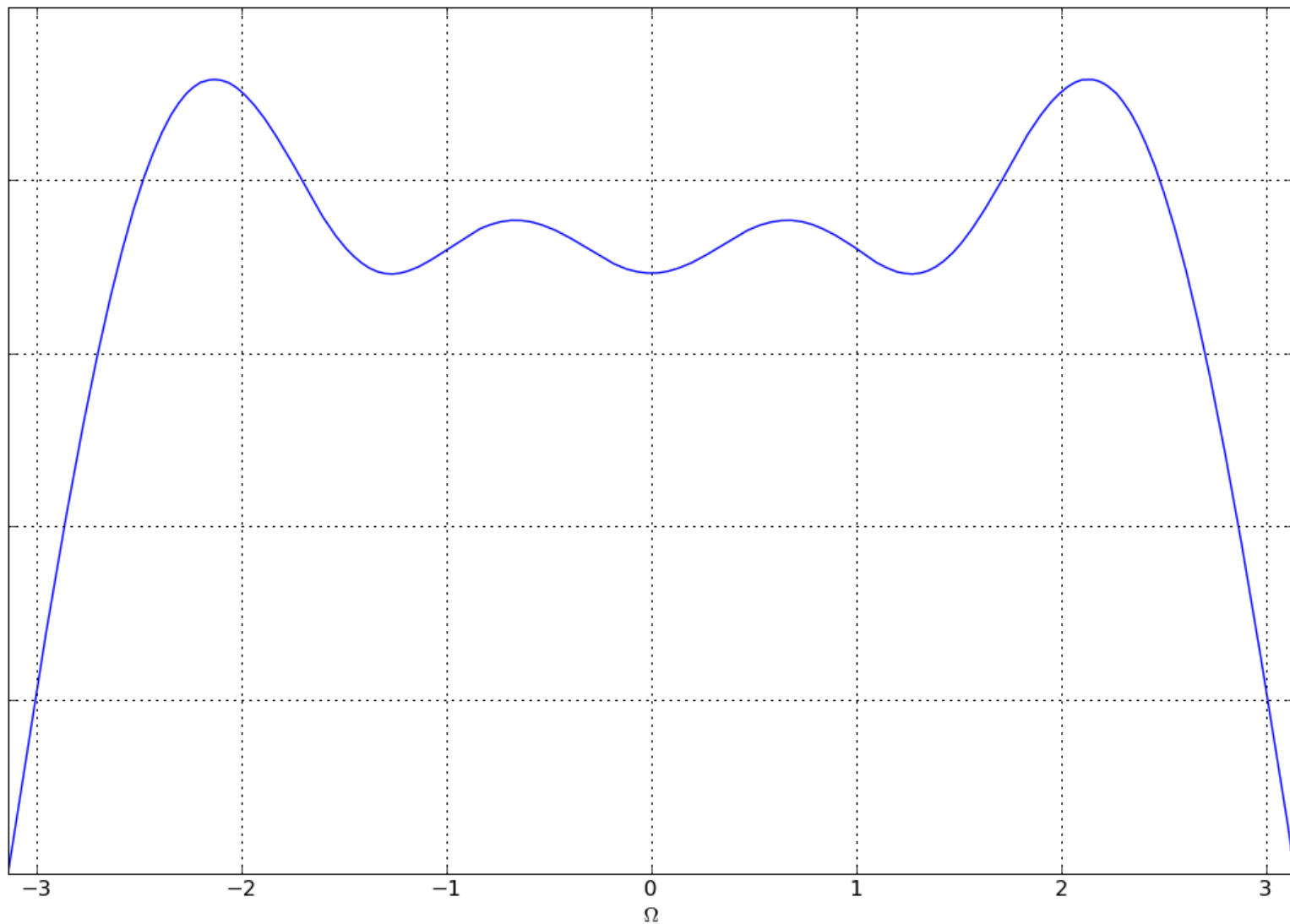
Magnitude of $|H(e^{j\Omega})|$ for H_B



Magnitude of $|H(e^{j\Omega})|$ for H_C



Magnitude of $|H(e^{j\Omega})|$ for H_D



- (A) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha\delta[n] - h_1[n]$$

and what is the numerical value of absolute value of α , $|\alpha|$.

- (B) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \sum_{m=0}^{m=n} h_1[m]h_2[n-m].$$

and what are the numerical values of $h[3]$ and $H(e^{j0})$?

- (C) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$$h[n] = \alpha\delta[n] - \sum_{m=0}^{m=n} h_1[m]h_2[n-m].$$

and what is the numerical value of absolute value of α , $|\alpha|$.

- (D) Which frequency response (A, B, C or D) corresponds to a unit sample response given by

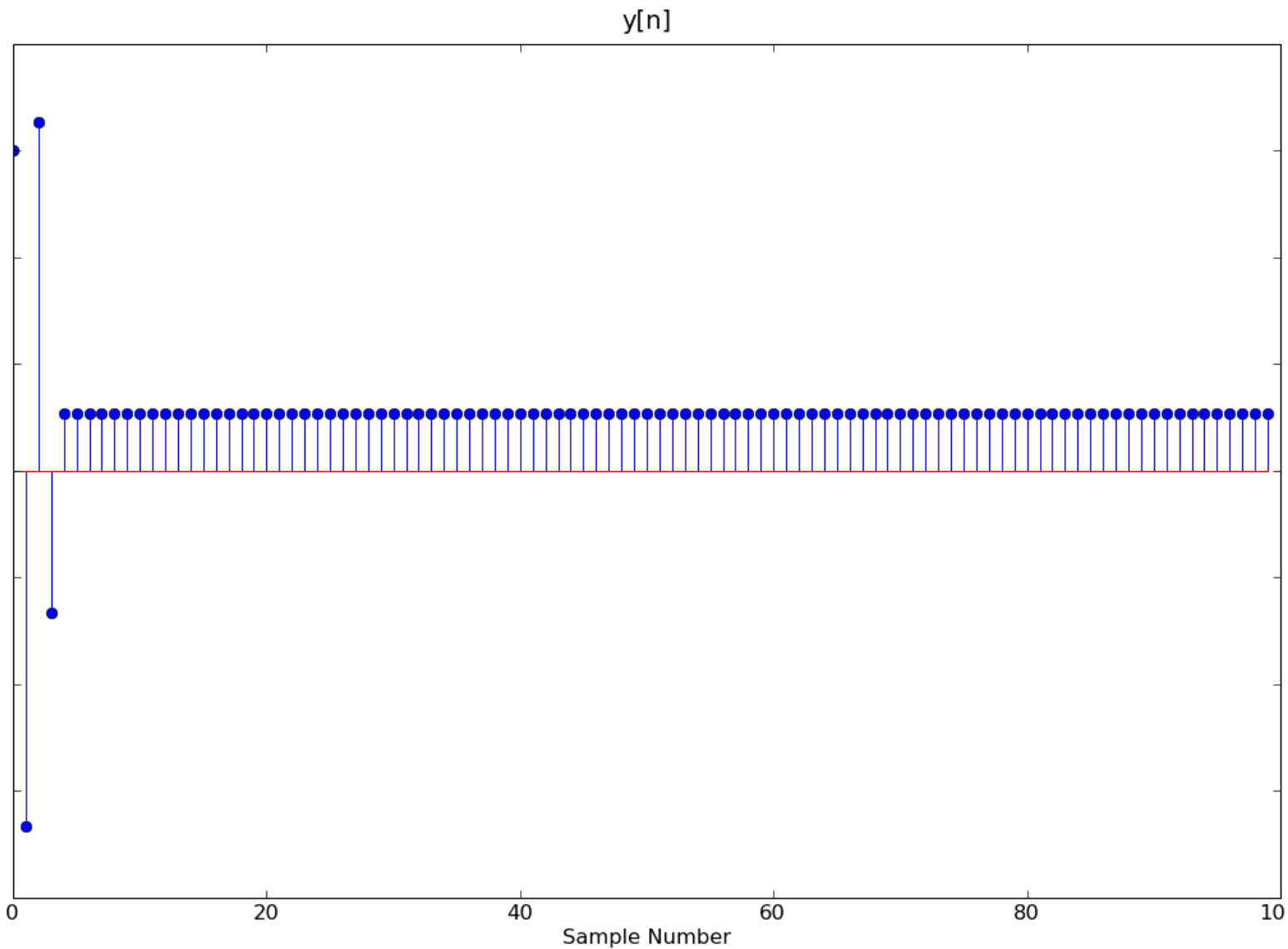
$$h[n] = \alpha\delta[n] - h_2[n]$$

and what is the numerical value of absolute value of α , $|\alpha|$.

(E) Suppose the input to each of the above four systems is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

$$x[n] = \cos \frac{\pi}{6.0}n + \cos \frac{\pi}{2.0}n + 1.0.$$

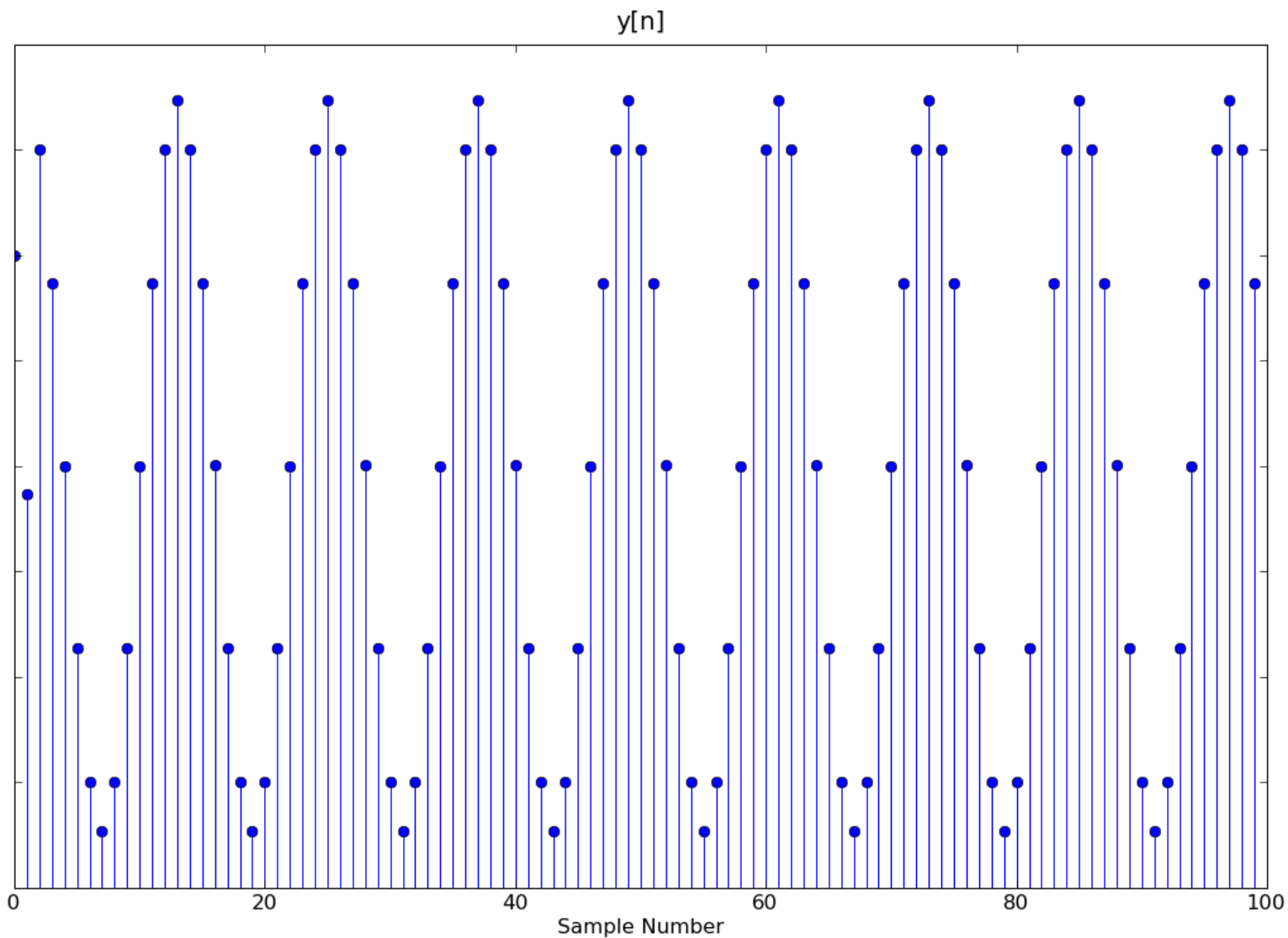
Which system (A, B, C or D) produced an output, $y[n]$ below, and what is the value of $y[n]$ for $n > 10$?



(F) Suppose the input to each of the above four systems is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

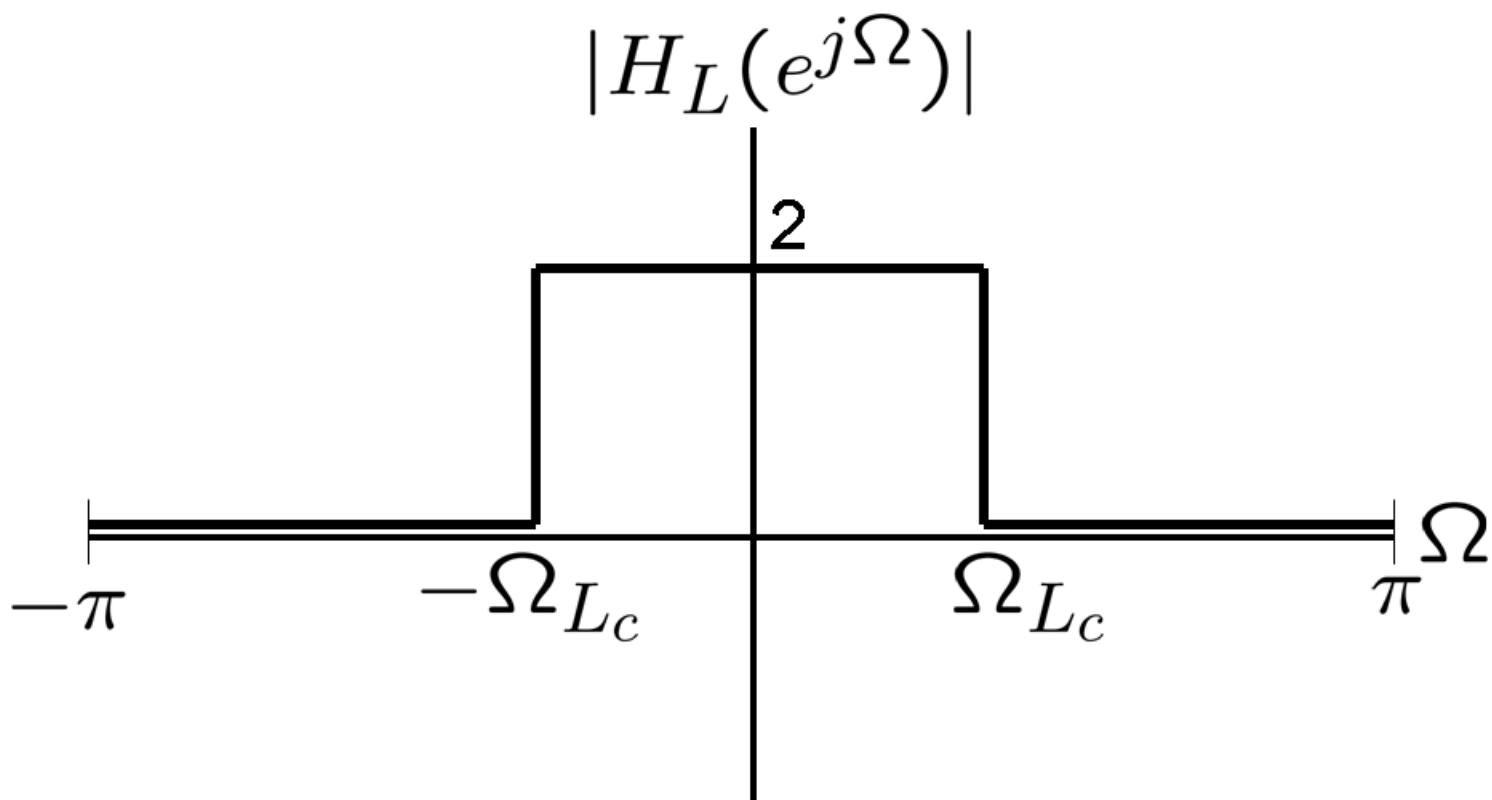
$$x[n] = \cos \frac{\pi}{6.0}n + \cos \frac{\pi}{2.0}n + 1.0.$$

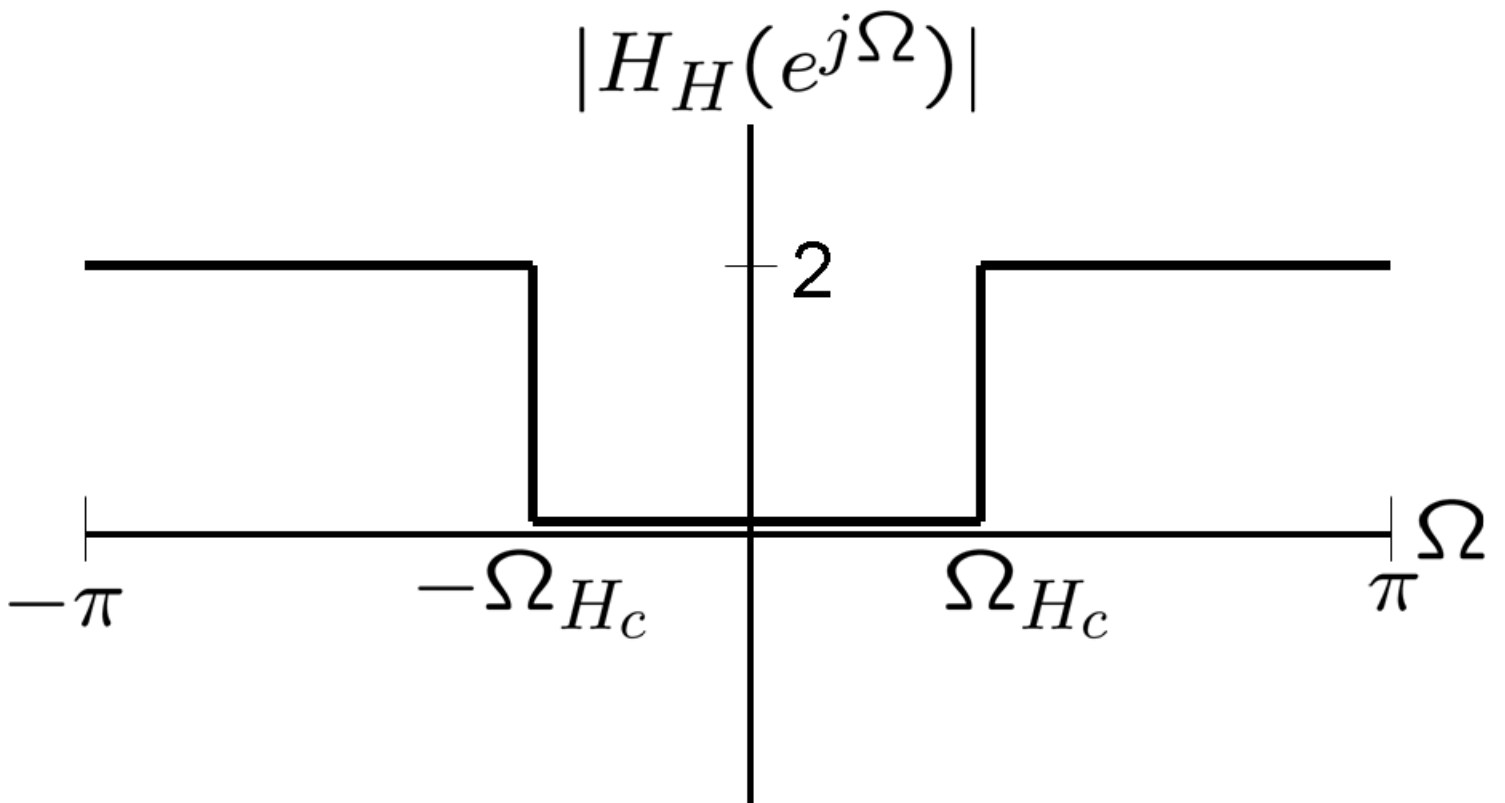
Which system (H1 or H2) produced an output, $y[n]$ below, and what is the value of $y[22]$?



Review Problem 2

The questions below refer to two linear time-invariant filters, a low-pass filter, H_L , and a high-pass filter, H_H , whose frequency response magnitudes are plotted below. Please note that in the pass band, each filter has a gain of **TWO**.



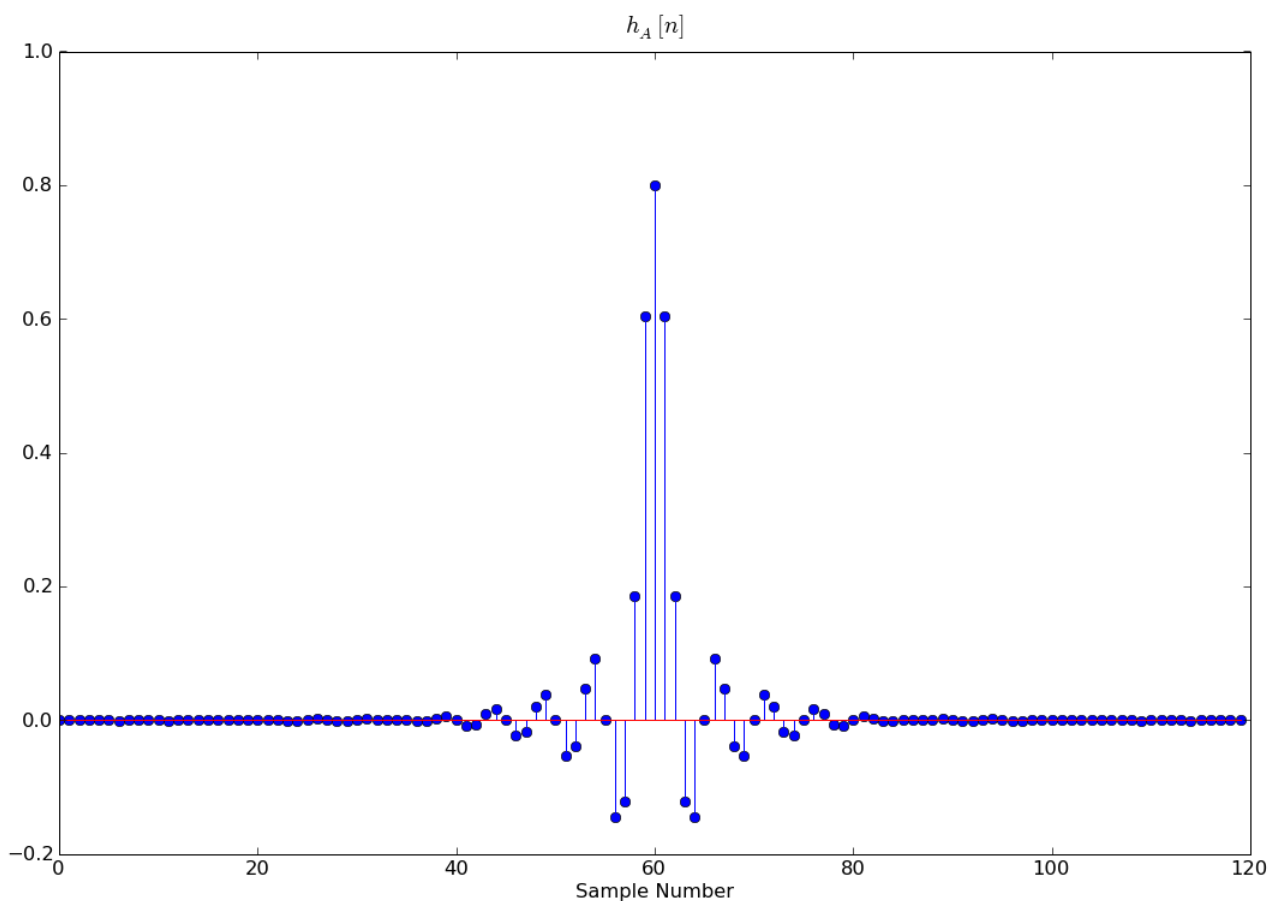


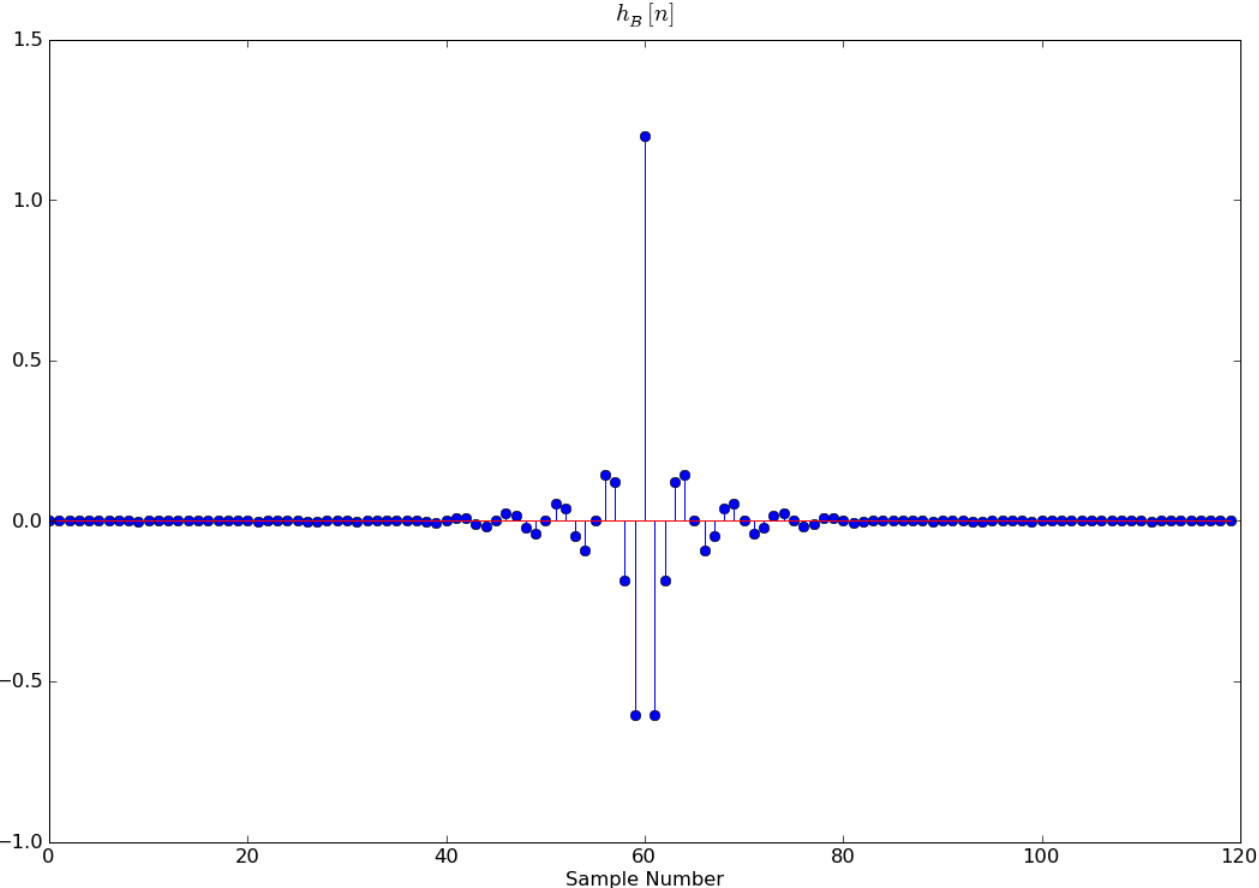
(A) For two unit sample responses with sample values $h_A[n]$ and $h_B[n]$, plotted below, which one could be a high-pass filter and which one could be a low-pass filter? In addition, for what is the value of

$$\sum_{m=0}^{m=\infty} h_A[m](-1)^m$$

and

$$\sum_{m=0}^{m=\infty} h_B[m](-1)^m?$$

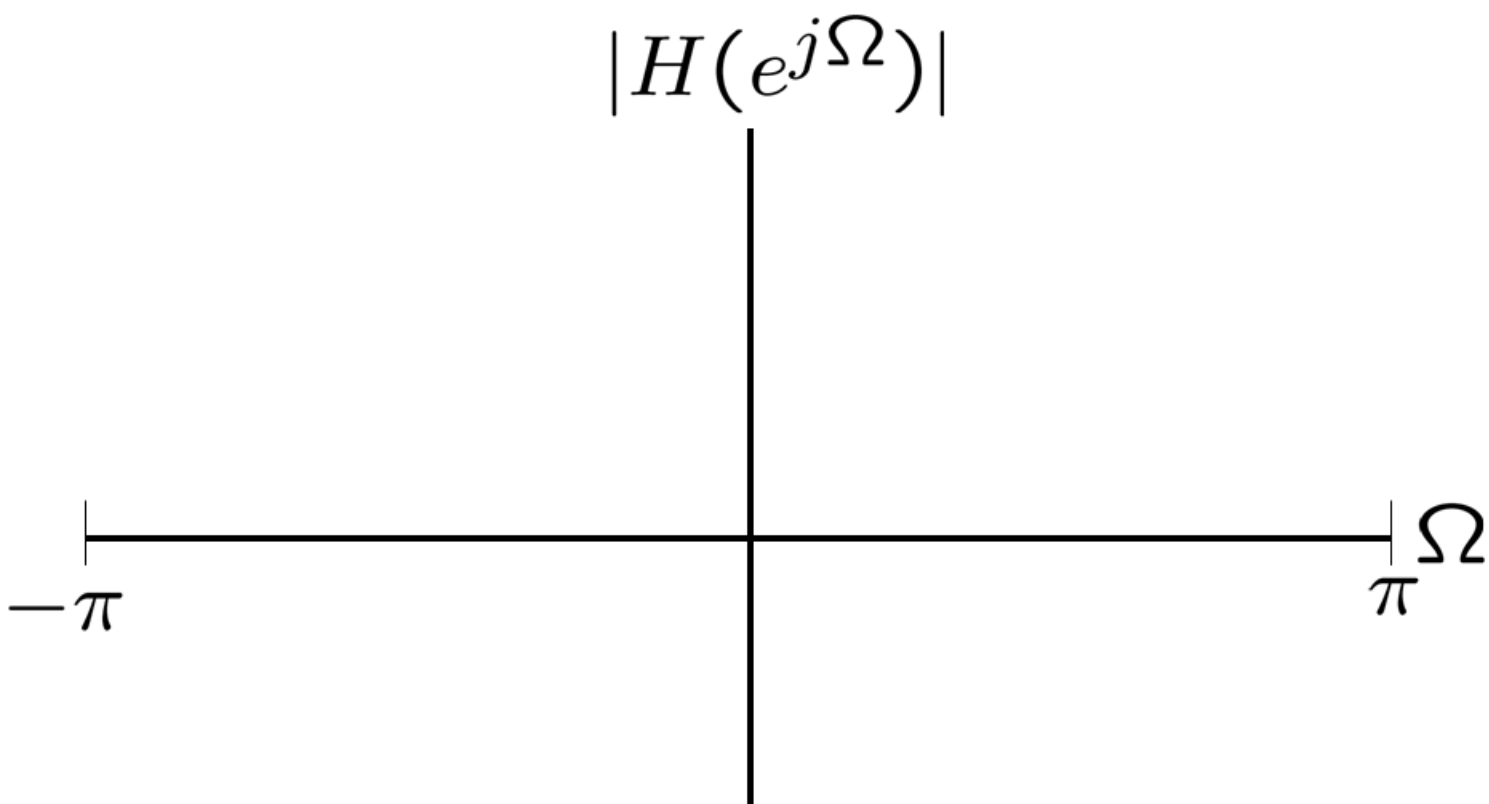




- (B) On the axes below, please plot the magnitude of the frequency response of the system H , whose unit sample response is given by the convolution of the low-pass and high-pass filter's unit sample responses, as in

$$h[n] = \sum_{m=0}^{m=n} h_L[m]h_H[n-m].$$

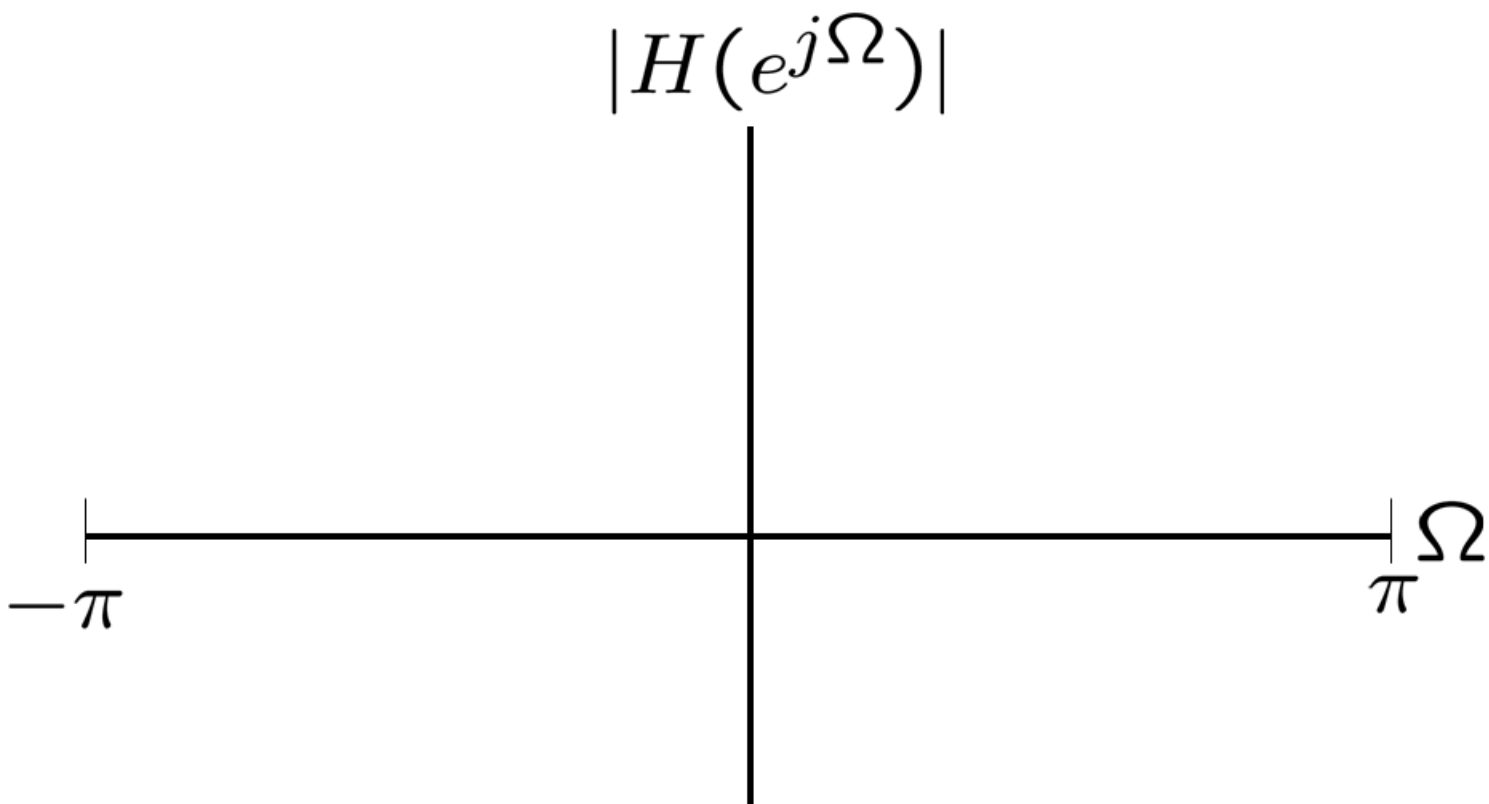
For your plot, please assume $\Omega_{cL} > \Omega_{cH}$, and clearly label important magnitude values and key frequency points.



- (C) On the axes below, please plot the magnitude of the frequency response of the system H , whose unit sample response is given by the sum of the low-pass and high-pass filter's unit sample responses, as in

$$h[n] = h_L[n] + h_H[n].$$

For your plot, please assume $\Omega_{cL} < \Omega_{cH}$ (the opposite case from part B) and clearly label important magnitude values and key frequency points.



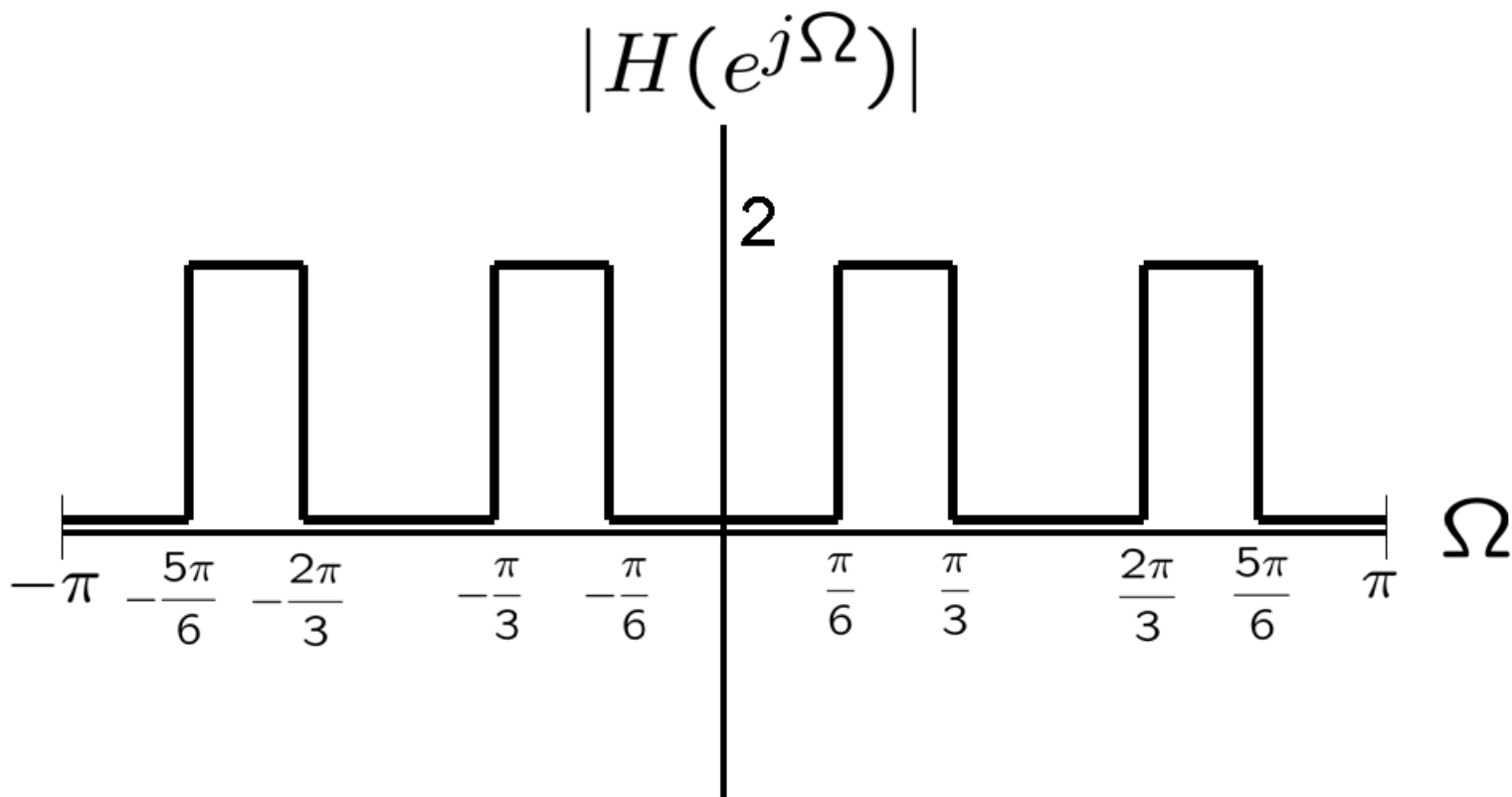
- (D) Will your plot in part C change if H 's unit sample response is the difference of the low-pass and high-pass filter's unit sample responses, as in

$$h[n] = h_L[n] - h_H[n]?$$

Please still assume $\Omega_{cL} < \Omega_{cH}$.

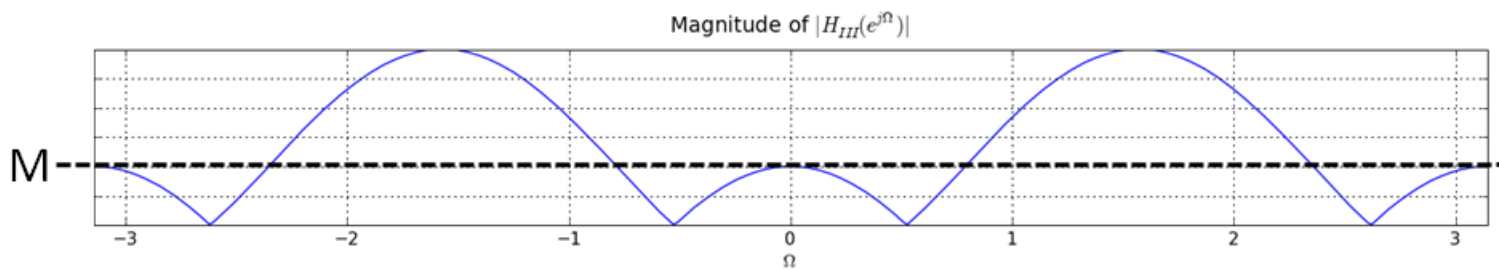
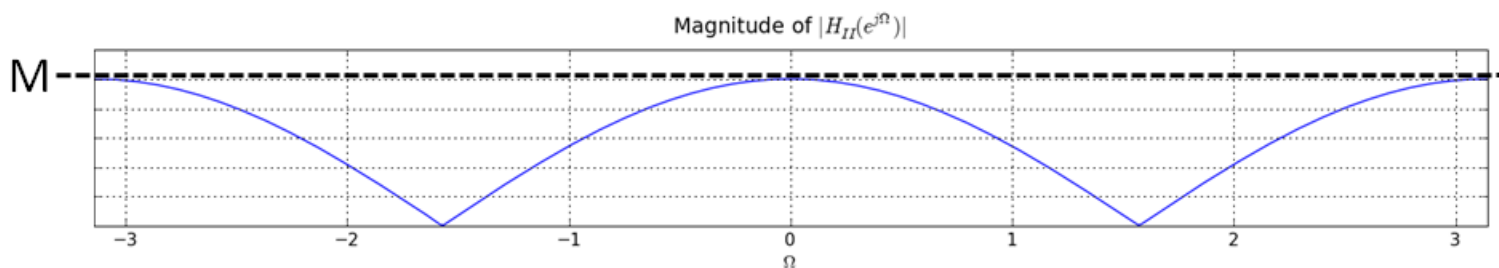
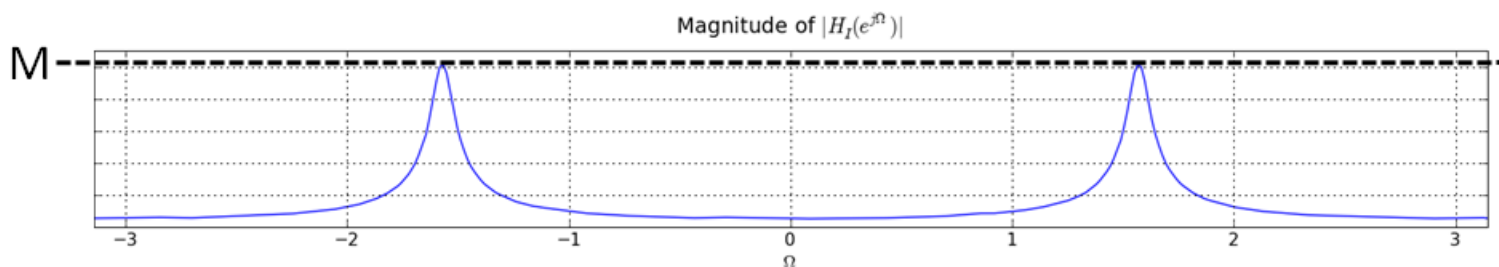
- (E) If all you know are the magnitudes of the frequency responses for H_L and H_H , do you have enough information to answer part C if $\Omega_{cL} > \Omega_{cH}$? Why or why not?

(F) Suppose you have two low-pass filters, one with cut-off frequency Ω_{cL}^A and a second with cut-off frequency Ω_{cL}^B , and two high-pass filters, one with cut-off frequency Ω_{cH}^A and a second with cut-off frequency Ω_{cH}^B . Draw a diagram that shows how you would combine these four filters, and give values for Ω_{cL}^A , Ω_{cL}^B , Ω_{cH}^A , and Ω_{cH}^B , to generate a filter with the frequency response given below.



Review Problems 3

In answering the following questions, please refer to the following three plots of the magnitude of three frequency responses, $|H_I(e^{j\Omega})|$, $|H_{II}(e^{j\Omega})|$, and $|H_{III}(e^{j\Omega})|$, given below.



(A) Suppose the input to a linear time invariant system is the sequence

$$x[n] = 2 + \cos \frac{5\pi}{6}n + \cos \frac{\pi}{6}n + 3(-1)^n$$

What is the maximum value of the sequence x , and what is the smallest positive value of n for which x achieves its maximum?

$$\max_m x[m] = \text{-----}$$

$$\text{Smallest } n > 0 \text{ for which } x[n] = \max_m x[m] \text{ -----}$$

(B) Suppose the sequence X from part A is the input to a linear time invariant system described by one of the three frequency response plots above (I, II or III). If y is the resulting output and is given by

$$y[n] = 8 + 12(-1)^n,$$

which frequency response plot describes the system, and what is the value of M in the plot you selected? Be sure to justify your selection.

$$\text{Frequency response plot (I, II, or III)} = \text{-----}$$

$$\text{Numerical value of } M = \text{-----}$$

- (C) Suppose the unit sample response of a linear time-invariant system has only three nonzero *real* values, $h[0]$, $h[1]$, and $h[2]$. In addition, suppose these three *real* values satisfy the three equations:

$$\begin{aligned} h[0] + h[1] + h[2] &= 5 \\ h[0] + e^{-j\frac{\pi}{2}\cdot 1}h[1] + e^{-j\frac{\pi}{2}\cdot 2}h[2] &= 0 \\ h[0] + e^{j\frac{\pi}{2}\cdot 1}h[1] + e^{j\frac{\pi}{2}\cdot 2}h[2] &= 0. \end{aligned}$$

Which of the above plots, I, II or III, is a plot of the magnitude of the frequency response of this system, and what is the value of M in the plot you selected? Be sure to justify your selection.

frequency response plot (I, II, or III) = _____

Numerical value of M = _____

- (D) For the system given in part C, if $y[n] = \sum_{m=0}^2 h[m]x[n-m]$ and $x[n] = e^{j\frac{\pi}{6}n}$ for all n , please determine the complex numerical value for

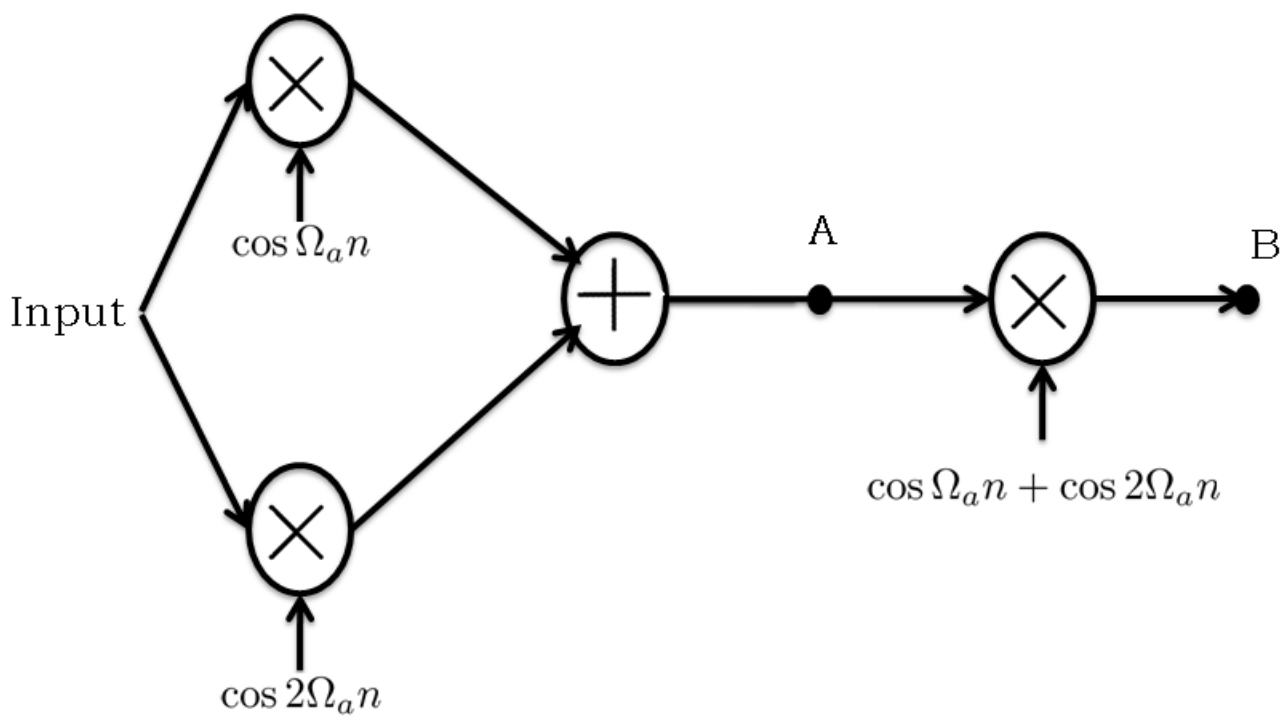
$$\frac{y[n]}{e^{j\frac{\pi}{6}n}}.$$

It might be helpful to know that the numerical values of $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin \frac{\pi}{6} = 0.5$, $\cos \frac{\pi}{3} = 0.5$ and $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

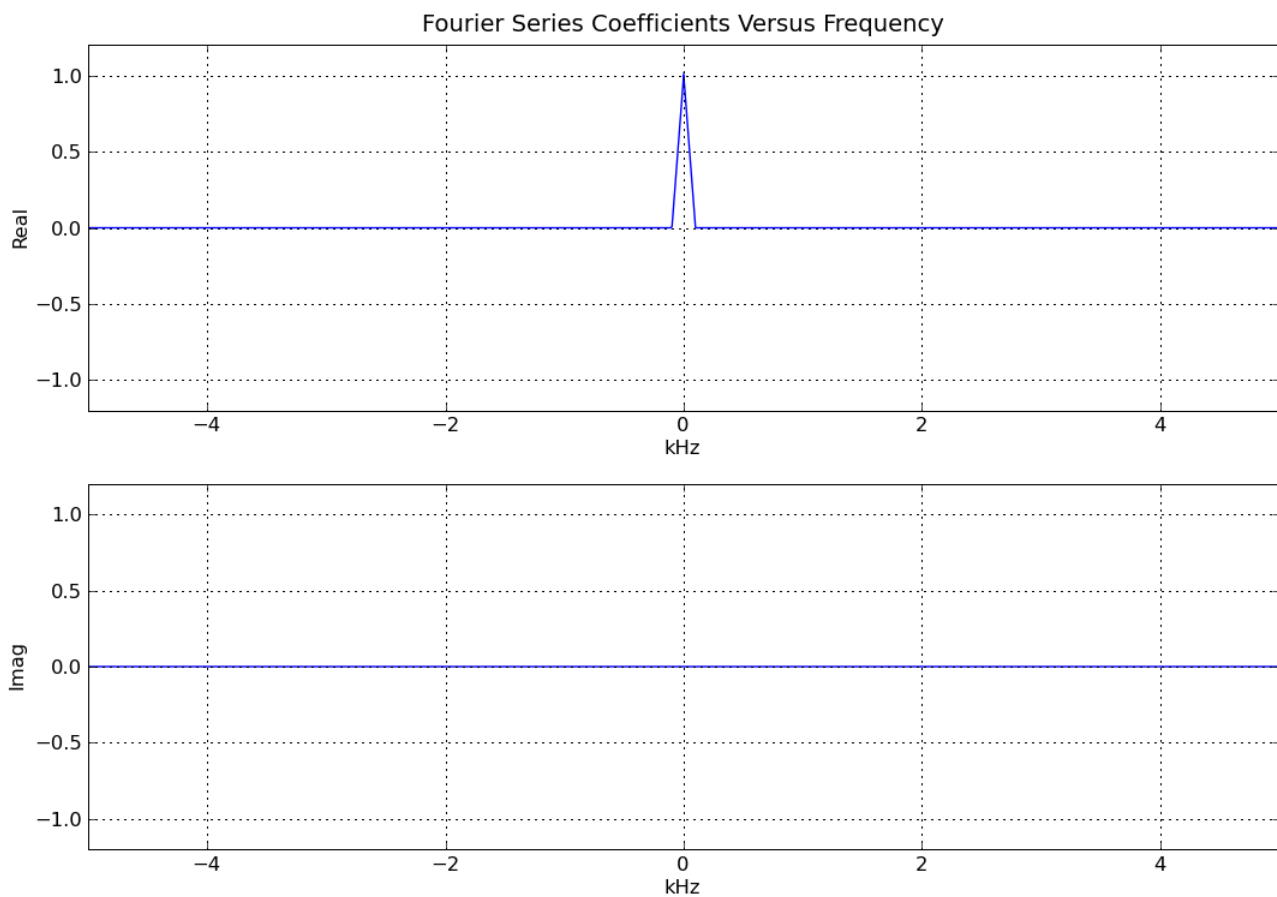
$$\frac{y[n]}{e^{j\frac{\pi}{6}n}} = \text{_____}$$

Review Problem 4

Consider the simple modulation-demodulation system below, where all signals are assumed periodic with period $N = 10000$, and the sampling frequency, f_s , is 10000 samples per second. In addition, $\Omega_a = 2\pi \frac{f_a}{f_s} = \frac{1000 \cdot 2\pi}{10000}$.

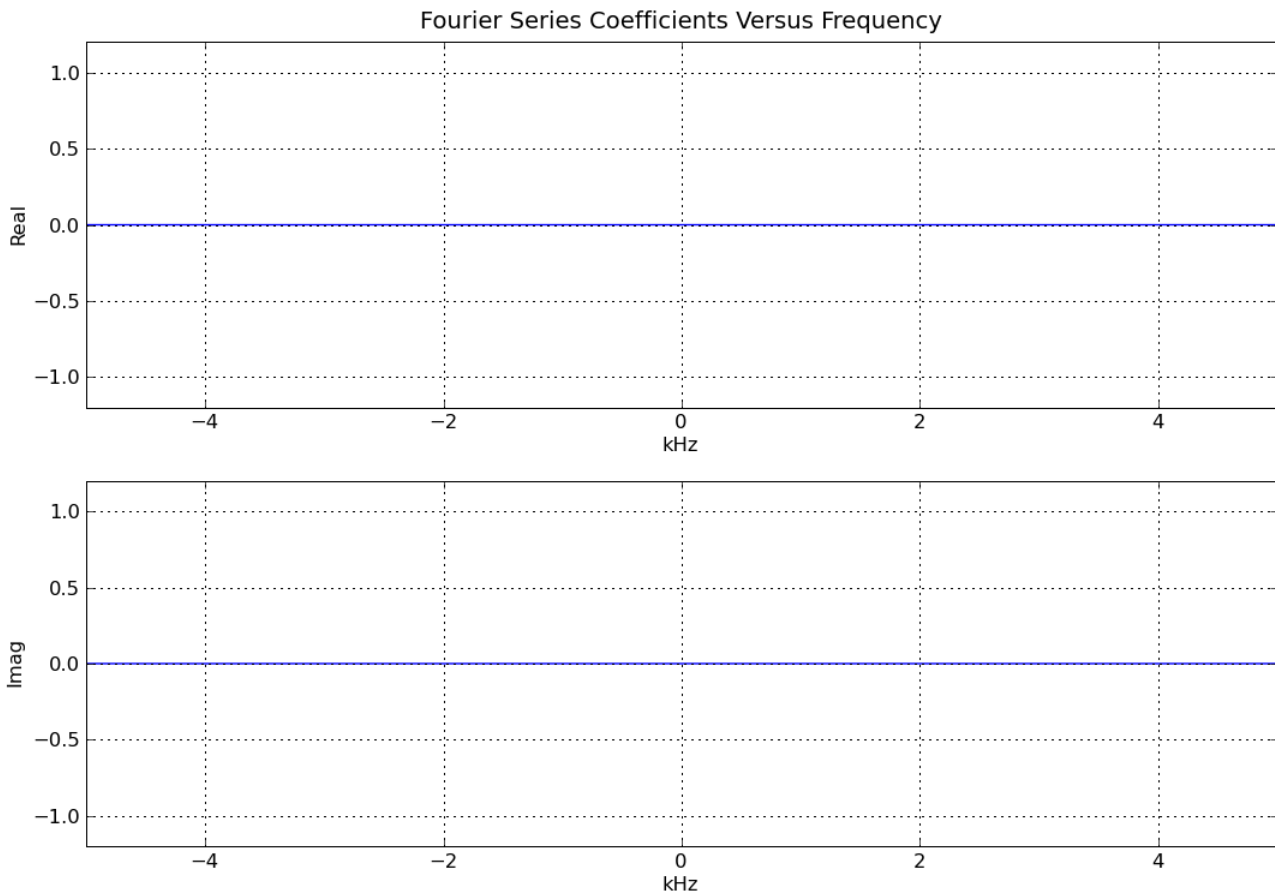


The Fourier Series coefficients versus frequency for the input to the modulation-demodulation system are plotted below for the case $N = 10000$ and $f_s = 10000$. Note that the Fourier coefficients are nonzero only for $-100 \leq f_k \leq 100$.

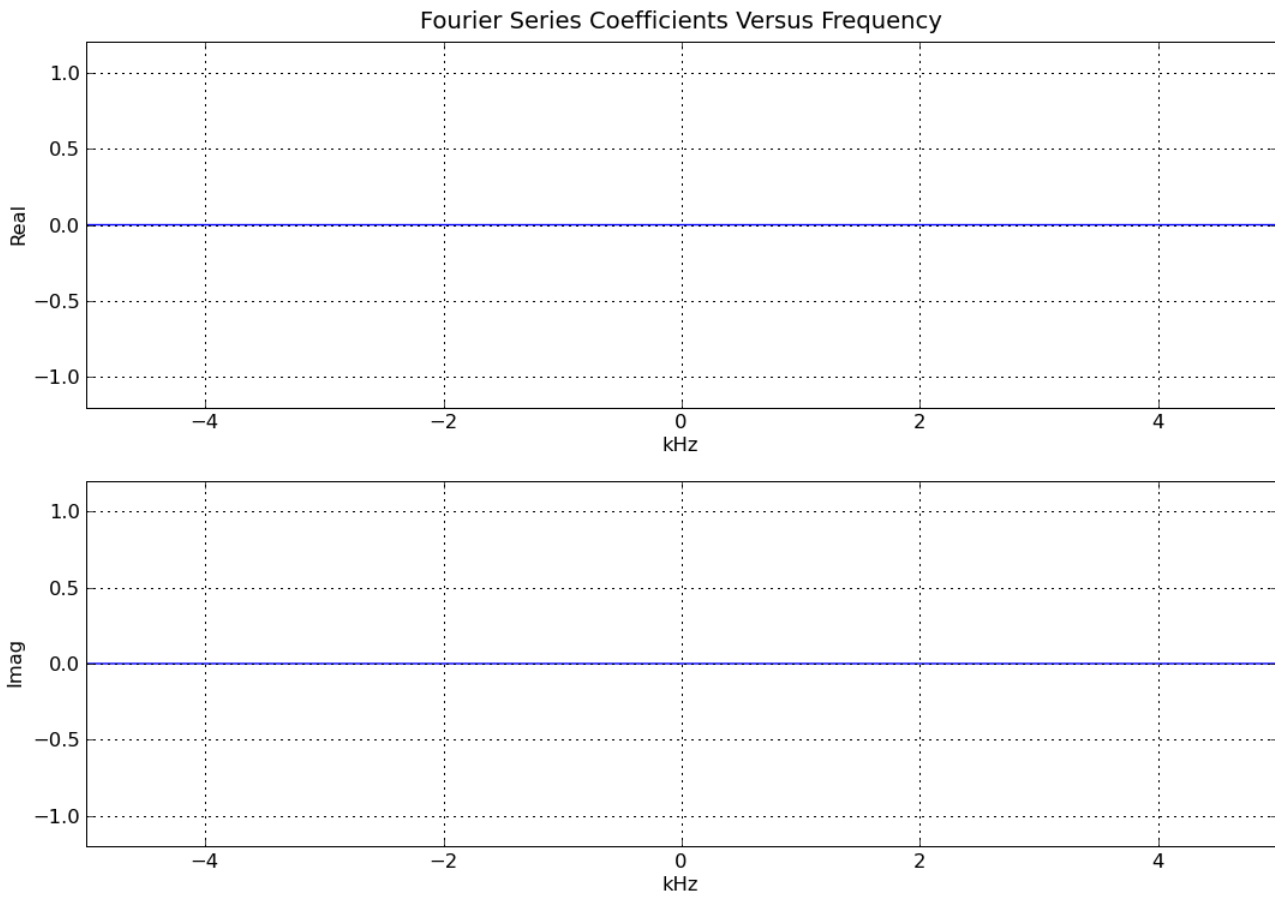


On the two sets of axes below, please plot the Fourier series coefficients versus Ω for the signals at location **A** and **B** in the above diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of Fourier Coefficients of signal at Point A

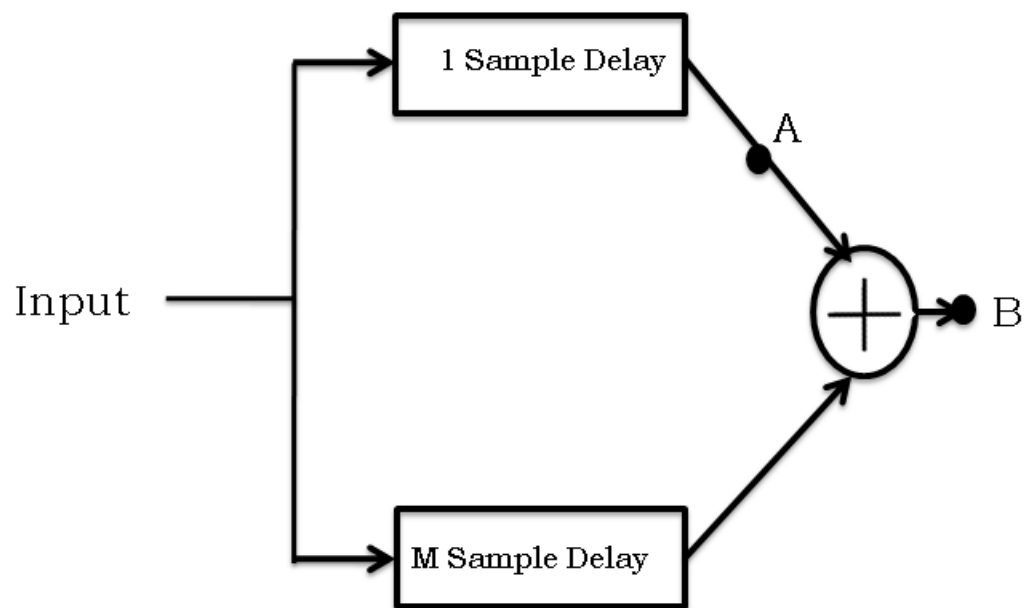


Plot of Fourier Coefficients of signal at Point B

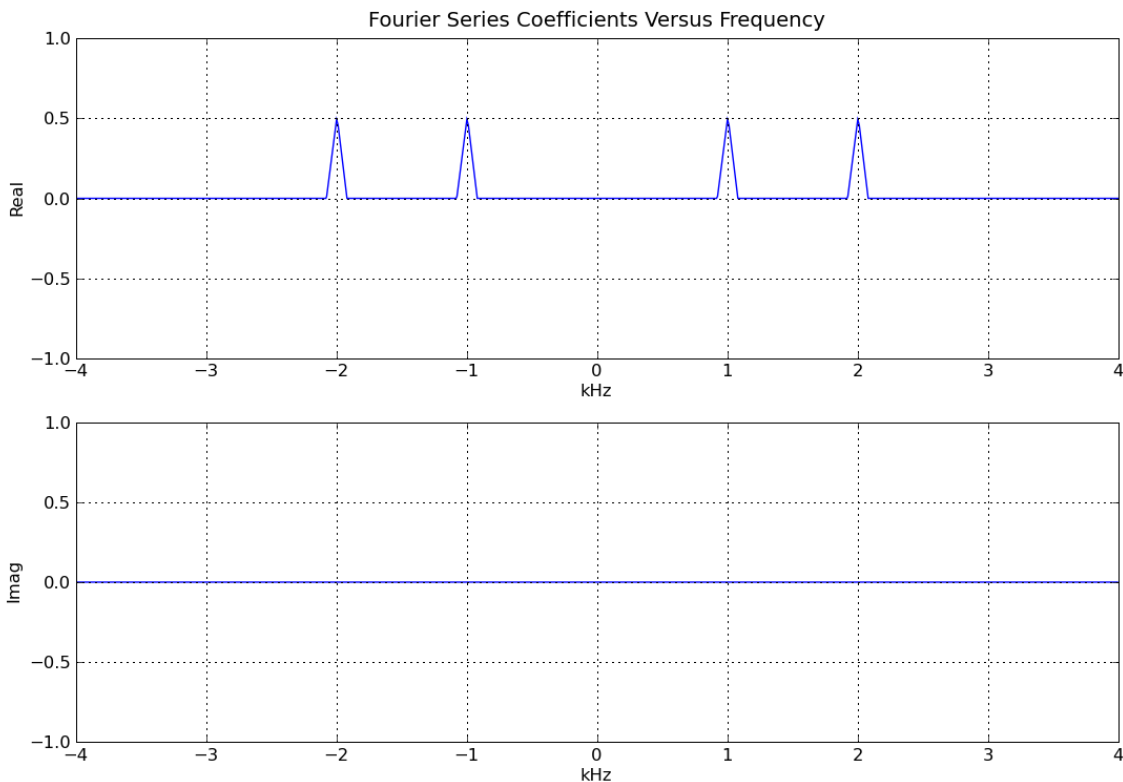


Review Problem 5

Consider the multiple delay system diagrammed below.

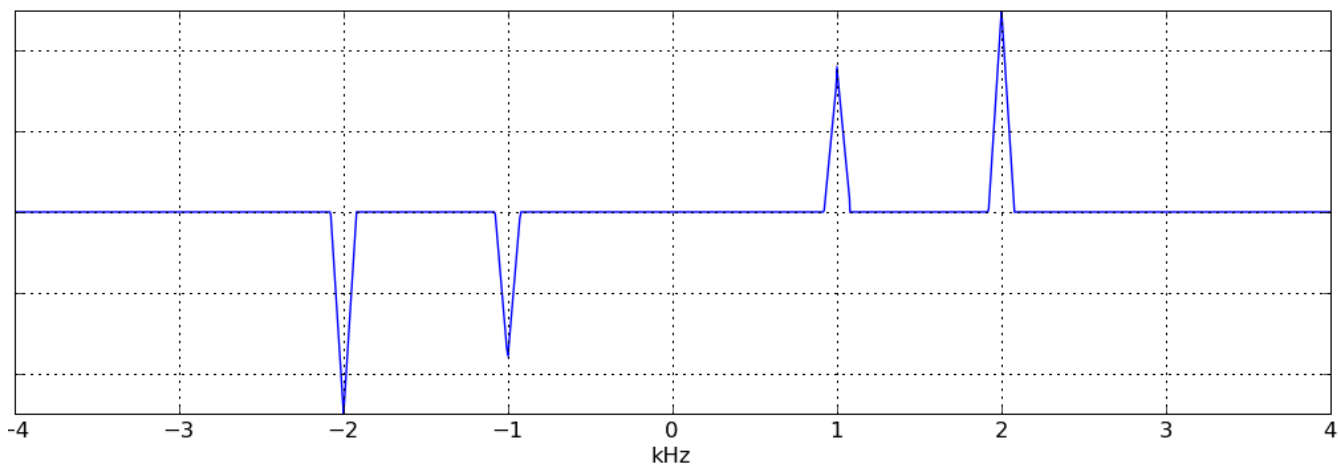
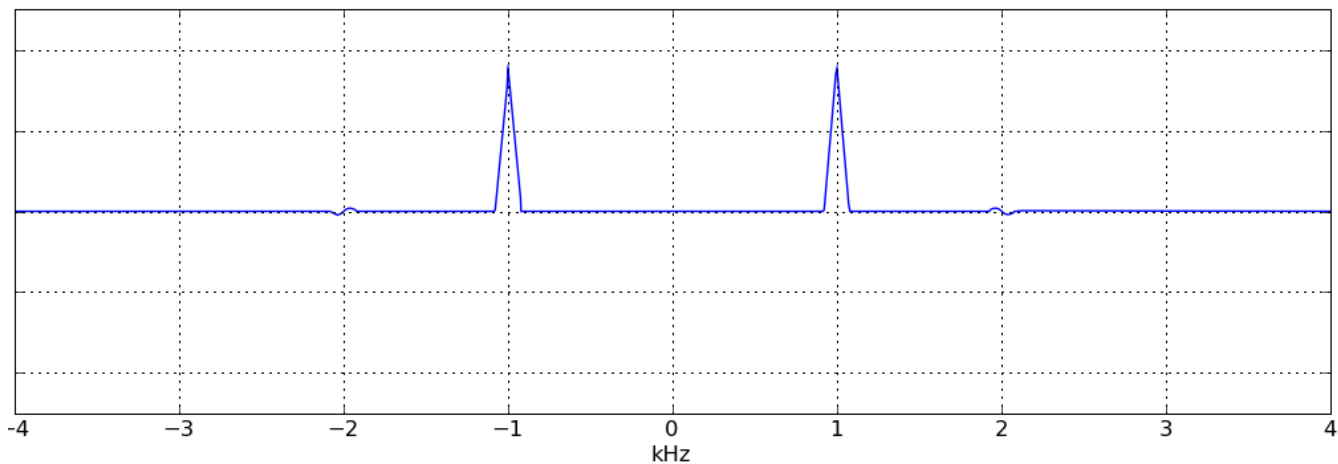


The input to the multiple delay system is a modulated signal that is periodic with period $N = 8000$ and sampling rate $f_s = 8000$. The Fourier Series coefficients versus frequency for this modulated signal are plotted below.

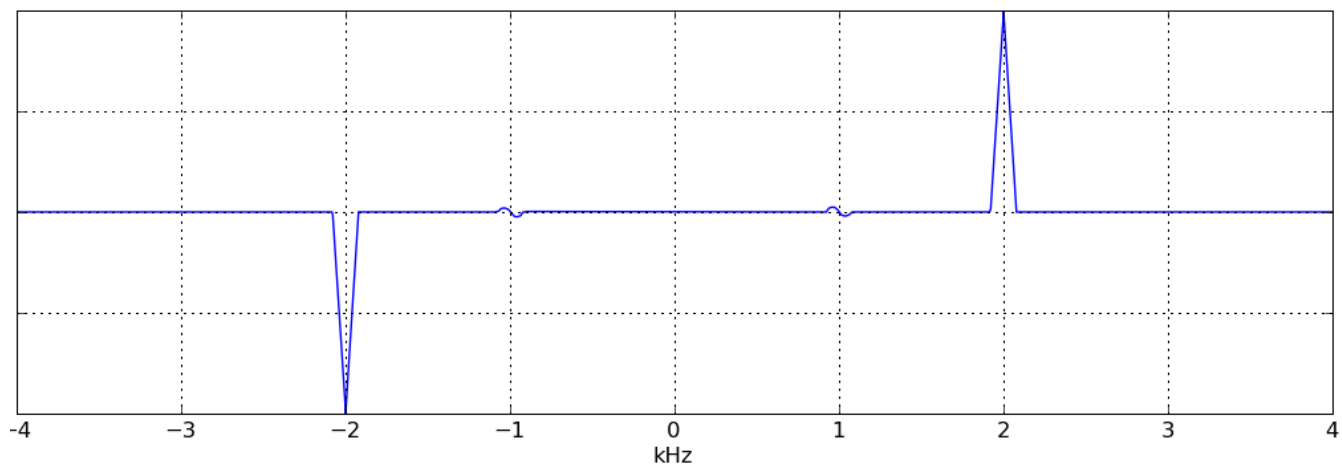
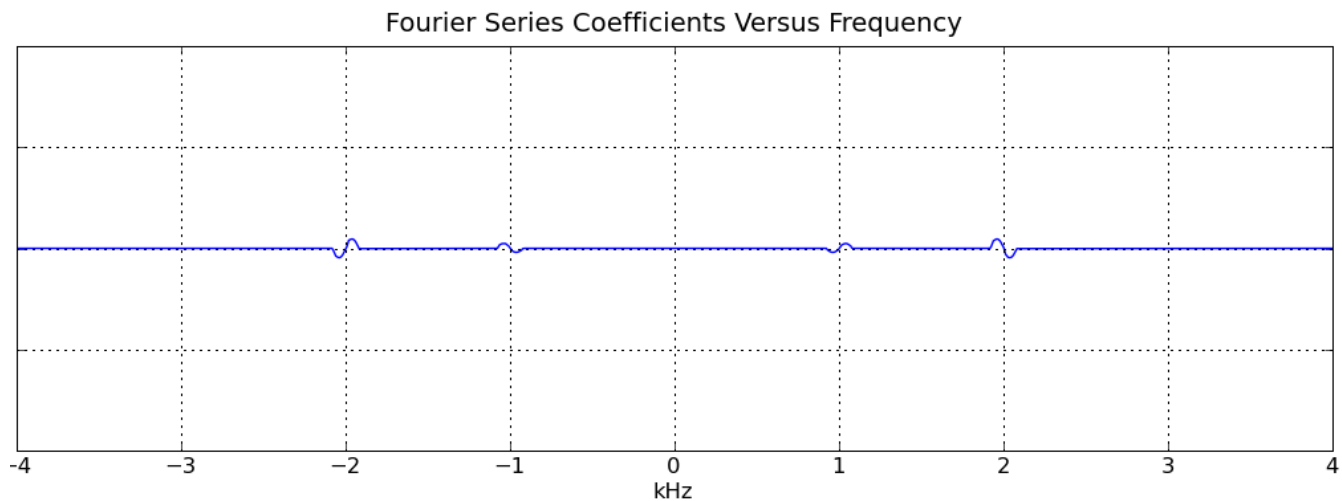


(A) Below are plots of the real and imaginary parts of the Fourier coefficients for point A in the multiple delay system. Determine the numerical values for the six peaks in the plots.

Fourier Series Coefficients Versus Frequency



(B) Use the following plot of the Fourier series coefficients for the sum of the delayed signals (point **B** in the multiple delay diagram), to determine the smallest integer value for M , the number of samples in the second delay.



Review problem 6

In this modulation problem you will be examining periodic signals and their associated discrete-time Fourier series (DTFS) coefficients. Recall that a periodic signal $x[n]$ with period N has DTFS coefficients given by

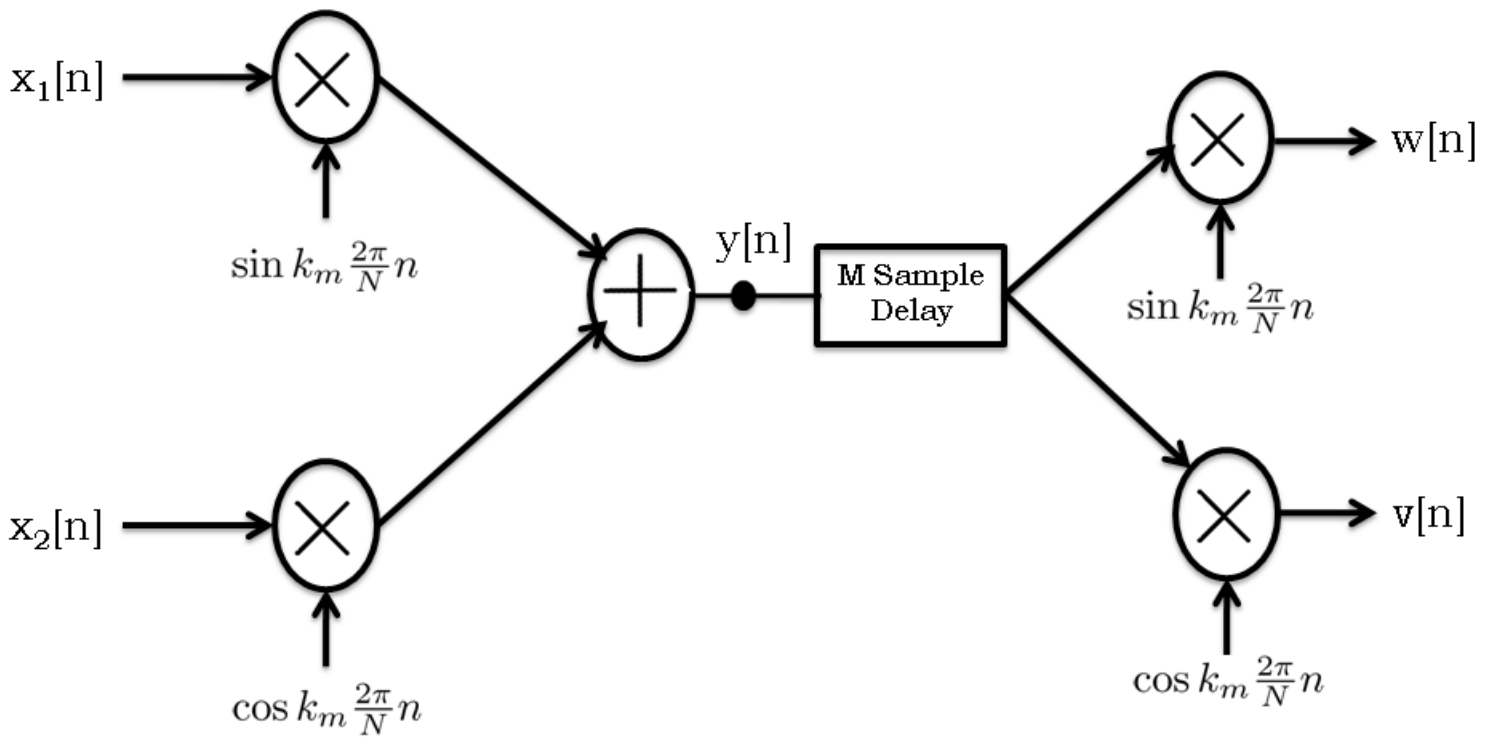
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}$$

and that the signal $x[n]$ can be reconstructed from the DTFS coefficients using

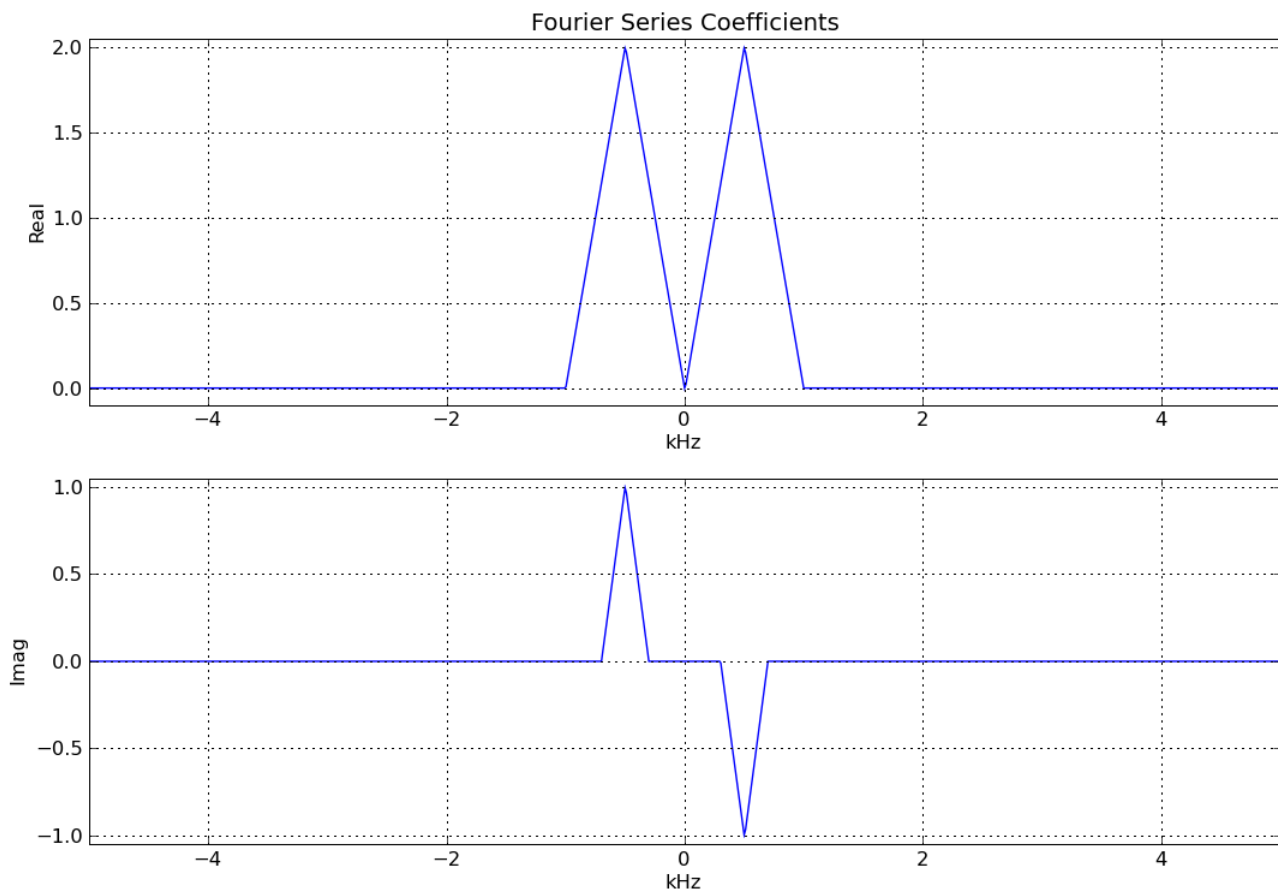
$$x[n] = \sum_{k=-K}^{K-1} X[k] e^{j \frac{2\pi}{N} kn}$$

where N is the period of the signal, $-K \leq k < K$ with $K = \frac{N}{2}$.

All parts of this question pertain to the following modulation-demodulation system, where all signals are periodic with period $N = 10000$ and therefore $K = 5000$. Please also assume that the sample rate associated with this system is 10000 samples per second, so that k is both a coefficient index and a frequency. In the diagram, the modulation frequency, k_m , is 500.

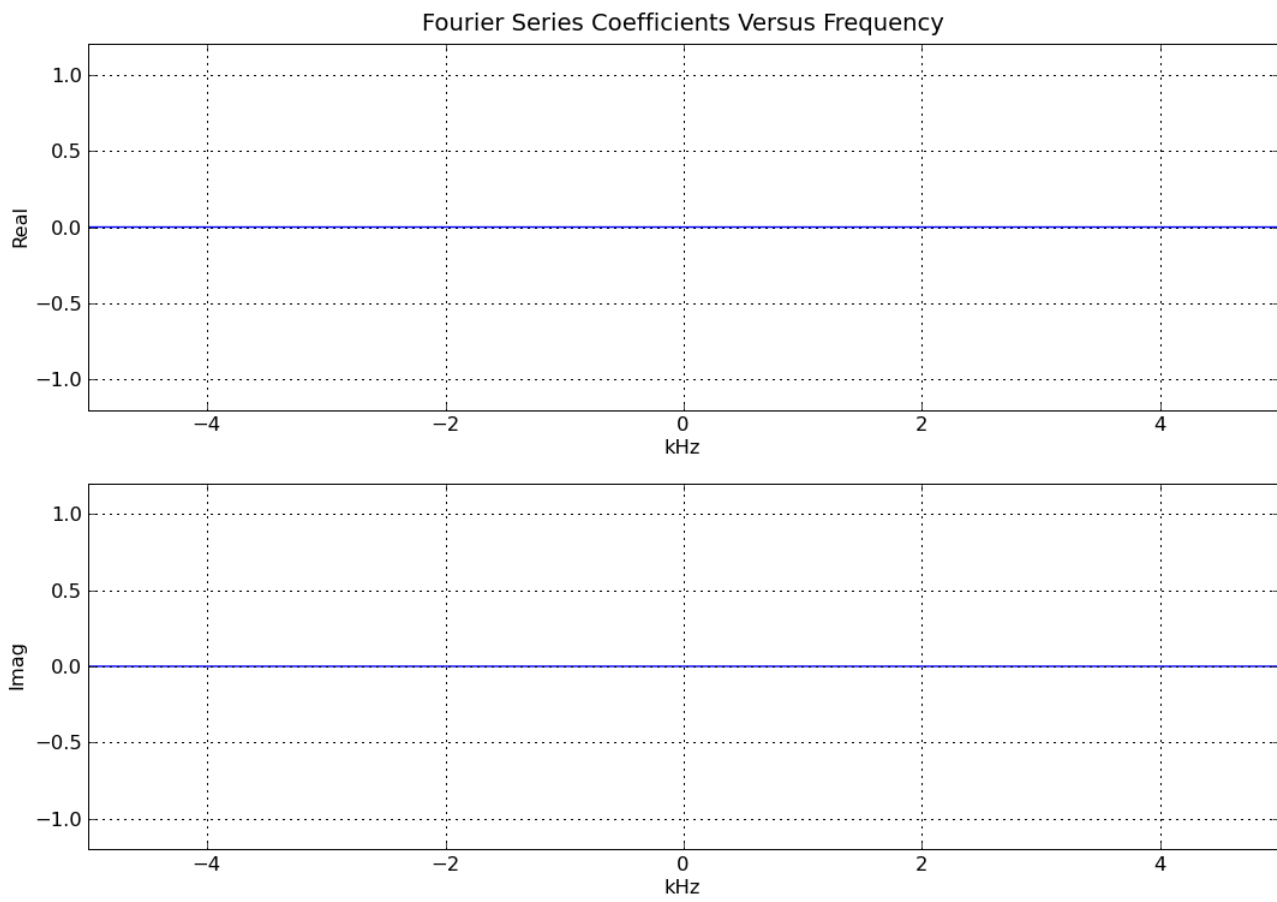


(A) Suppose the DFTS coefficients for the signal $y[n]$ in the modulation/demodulation diagram are as plotted below.

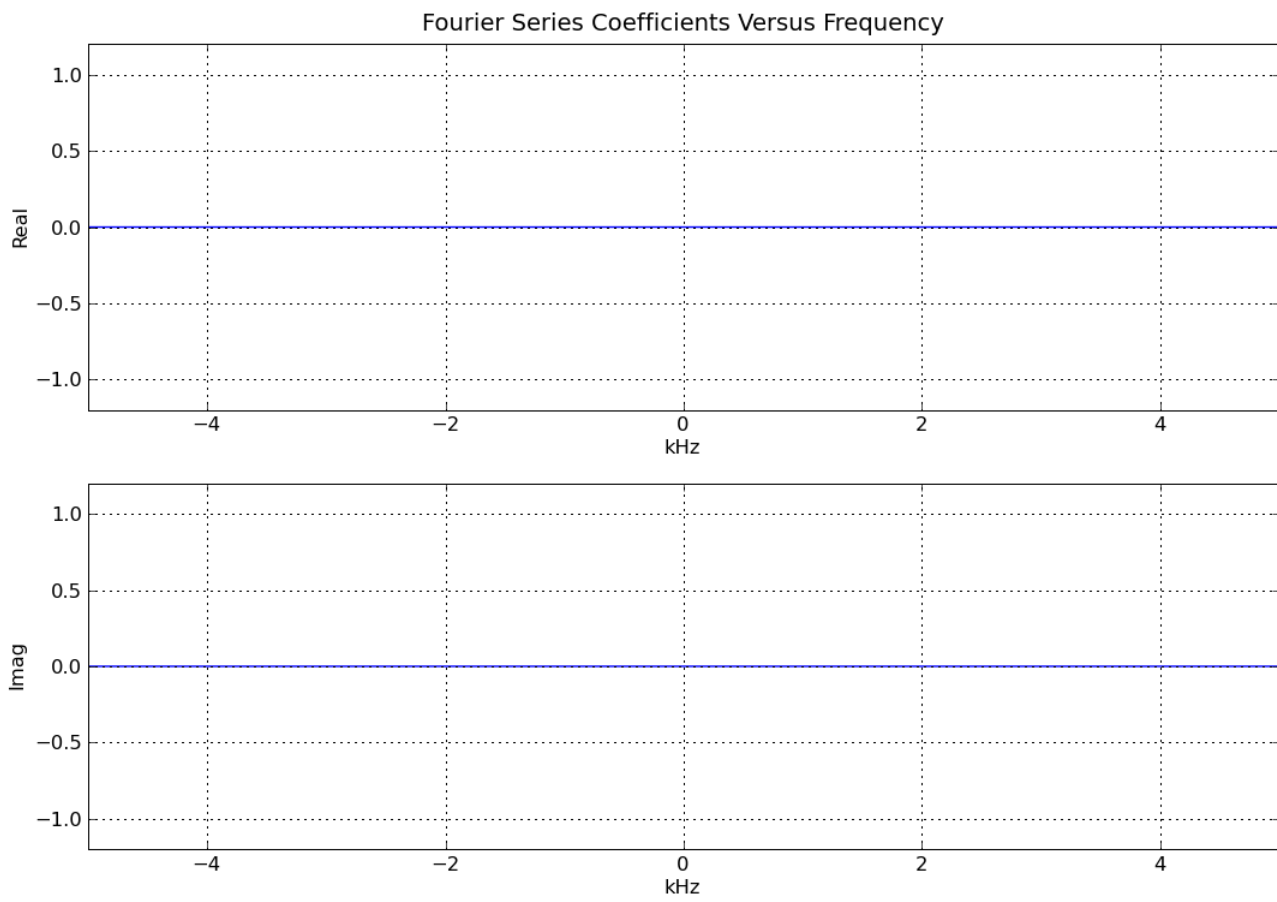


Assuming that $M = 0$ for the M -sample delay (no delay), on the two sets of axes on the next pages, please plot the DFTS coefficients for the signals \mathbf{w} and \mathbf{v} in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for w

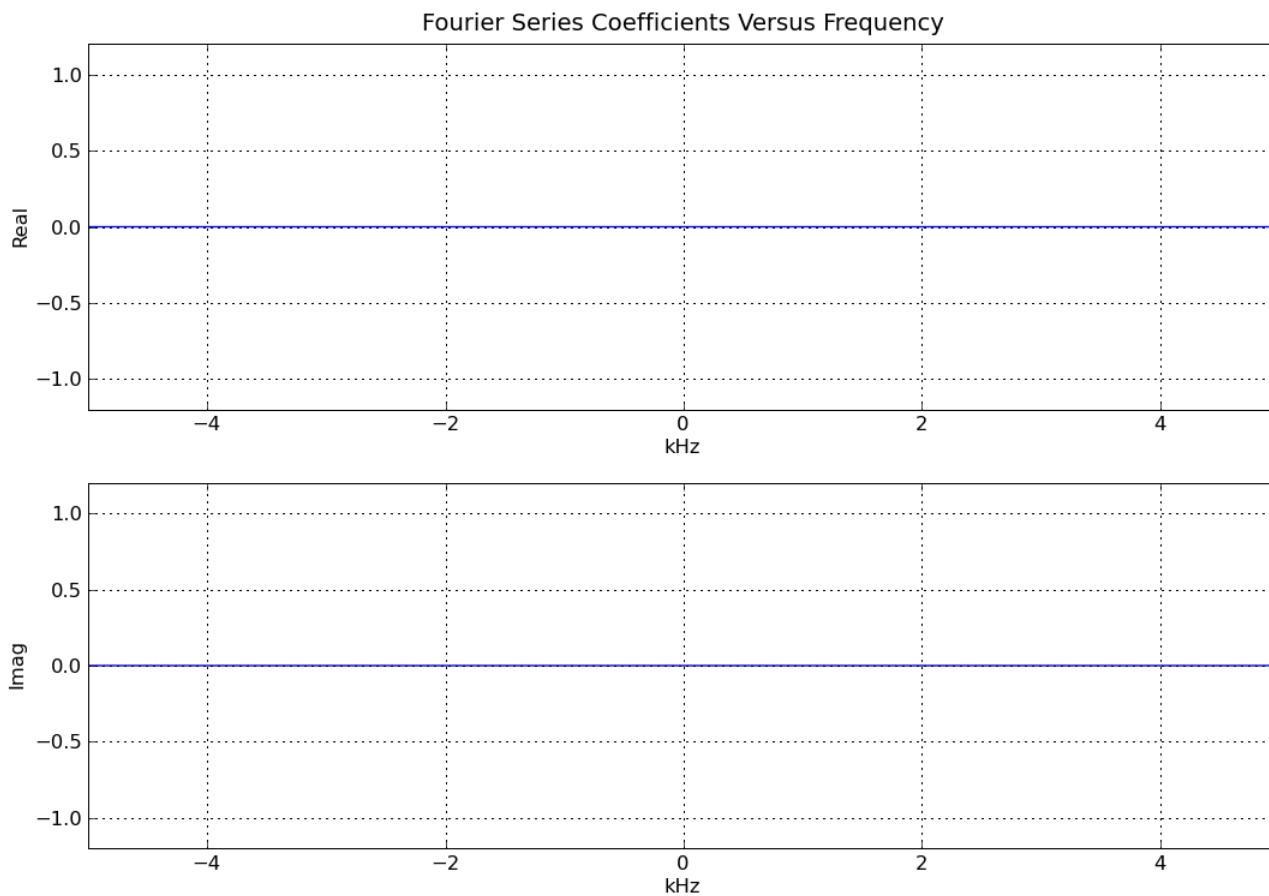


Plot of DFTS coefficients for v



- (B) Assuming the DFTS coefficients for the signal $y[n]$ are the same as in part A, on the axes below, please plot the DFT coefficients for the signal x_1 in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Plot of DFTS coefficients for x_1



- (C) If the M -sample delay in the modulation/demodulation diagram has the right number of samples of delay, then it will be possible to nearly perfectly recover $x_1[n]$ by low-pass filtering $v[n]$. Please determine the smallest positive number of samples of delay that are needed and the cut-off frequency for the low-pass filter. Please be sure to justify your answer, using pictures if appropriate.

Smallest M (number of samples of delay) $> 0 =$ _____
 Cutoff Frequency of Low Pass Filter = _____