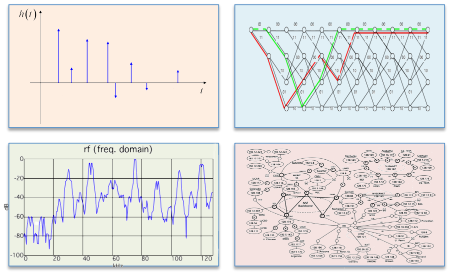


INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Spring 2011
Lecture #2

- Adaptive variable-length codes: LZW
- Perceptual coding

Example from Last Lecture

$choice_i$	p_i	$log_2(1/p_i)$	p_i * $log_2(1/p_i)$	*Huffman encoding*	*Expected length*
"A"	1/3	1.58 bits	0.528 bits	10	0.667 bits
"B"	1/2	1 bit	0.5 bits	0	0.5 bits
"C"	1/12	3.58 bits	0.299 bits	110	0.25 bits
"D"	1/12	3.58 bits	0.299 bits	111	0.25 bits
			1.626 bits		1.667 bits

Entropy is 1.626 bits/symbol, expected length of Huffman encoding is 1.667 bits/symbol.

How do we do better?

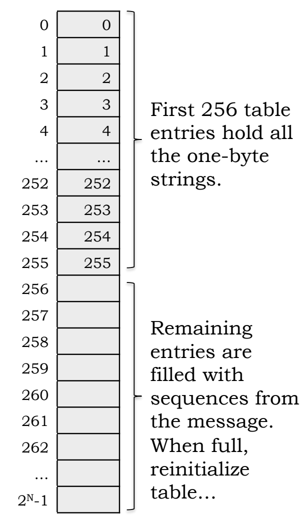
16 Pairs: 1.646 bits/sym
64 Triples: 1.637 bits/sym
256 Quads: 1.633 bits/sym

Huffman Codes – the final word?

- Given static symbol probabilities, the Huffman algorithm creates an optimal encoding when each symbol is encoded separately. (optimal ≡ no other encoding will have a shorter expected message length)
- Huffman codes have the biggest impact on average message length when some symbols are substantially more likely than other symbols.
- You can improve the results by adding encodings for symbol pairs, triples, quads, etc. But the number of possible encodings quickly becomes intractable.
- Symbol probabilities change message-to-message, or even within a single message.
- Can we do adaptive variable-length encoding?

Adaptive Variable-length Codes

- Algorithm first developed by Lempel and Ziv, later improved by Welch. Now commonly referred to as the "LZW Algorithm"
- As message is processed a "string table" is built which maps symbol sequences to an N-bit fixed-length code. Table size = 2^N
- Transmit table indices, usually shorter than the corresponding string → compression!
- Note: String table can be reconstructed by the decoder based on information in the encoded stream – the table, while central to the encoding and decoding process, is never transmitted!



LZW Encoding

```
STRING = get input symbol
WHILE there are still input symbols DO
   SYMBOL = get input symbol
   IF STRING + SYMBOL is in the string table THEN
       STRING = STRING + SYMBOL
   ELSE
       output the code for STRING
       add STRING + SYMBOL to the string table
       STRING = SYMBOL
   END
END

output the code for STRING
```

1. Accumulate message bytes in S as long as S appears in table.
2. When S+b isn't in table: send code for S, add S+b to table.
3. Reinitialize S with b, back to step 1.

From http://marknelson.us/1989/10/01/lzw-data-compression/

Example: Encode "abbbabbbab…"

256	ab
257	bb
258	bba
259	abb
260	bbab
261	
262	

1. Read a; string = a
2. Read b; ab not in table
   output 97, add ab to table, string = b
3. Read b; bb not in table
   output 98, add bb to table, string = b
4. Read b; bb in table, string = bb
5. Read a; bba not in table
   output 257, add bba to table, string = a
6. Read b, ab in table, string = ab
7. Read b, abb not in table
   output 256, add abb to table, string = b
8. Read b, bb in table, string = bb
9. Read a, bba in table, string = bba
10. Read b, bbab not in table
    output 258, add bbab to table, string = b

Encoder Notes

- The encoder algorithm is greedy – it's designed to find the longest possible match in the string table before it makes a transmission.
- The string table is filled with sequences actually found in the message stream. No encodings are wasted on sequences not actually found in the file.
- Note that in this example the amount of compression increases as the encoding progresses, i.e., more input bytes are consumed between transmissions.
- Eventually the table will fill and then be reinitialized, recycling the N-bit codes for new sequences. So the encoder will eventually adapt to changes in the probabilities of the symbols or symbol sequences.

LZW Decoding

```
Read CODE
output CODE
STRING = CODE

WHILE there are still codes to receive DO
    Read CODE
    IF CODE is not in the translation table THEN
        ENTRY = STRING + STRING[0]
    ELSE
        ENTRY = get translation of CODE
    END
    output ENTRY
    add STRING+ENTRY[0] to the translation table
    STRING = ENTRY
END
```

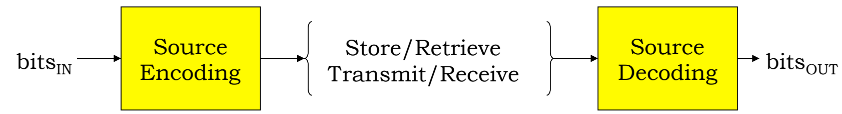
Easy: use table lookup to convert code to message string
Less easy: build table that's identical to that in encoder

Example: Decode 97, 97, 257, 256, 258

256	ab
257	bb
258	bba
259	abb
260	
261	
262	

1. Read `97`;
   output `a`; string = `a`
2. Read `98`; entry = `b`
   output `b`; add `ab` to table; string = `b`
3. Read `257`; entry = `bb`
   output bb; add `bb` to table; string = `bb`
4. Read `256`; entry = `ab`
   output `ab`; add `bba` to table; string = `ab`
5. Read `258`; entry = `bba`
   output `bba`; add `abb` to table; string = `bba`
   …

Lossless vs. Lossy Compression



- Huffman and LZW encodings are *lossless*, i.e., we can reconstruct the original bit stream exactly: $\text{bits}_{\text{OUT}} = \text{bits}_{\text{IN}}$.
  - What we want for "naturally digital" bit streams (documents, messages, datasets, …)
- Any use for *lossy* encodings: $\text{bits}_{\text{OUT}} \approx \text{bits}_{\text{IN}}$?
  - "Essential" information preserved
  - Appropriate for sampled bit streams (audio, video) intended for human consumption via imperfect sensors (ears, eyes).

Perceptual Coding

- Start by evaluating input response of bitstream consumer (eg, human ears or eyes), i.e., how consumer will perceive the input.
  - Frequency range, amplitude sensitivity, color response, …
  - Masking effects
- Identify information that can be removed from bit stream without perceived effect, e.g.,
  - Sounds outside frequency range, or masked sounds
  - Visual detail below resolution limit (color, spatial detail)
  - Info beyond maximum allowed output bit rate
- Encode remaining information efficiently
  - Use DCT-based transformations (real instead of complex)
  - Quantize DCT coefficients
  - Entropy code (eg, Huffman encoding) results

Perceptual Coding Example: Images

- Characteristics of our visual system
  ⇒ opportunities to remove information from the bit stream
  - More sensitive to changes in luminance than color
    ⇒ spend more bits on luminance than color (encode separately)
  - More sensitive to large changes in intensity (edges) than small changes
    ⇒ quantize intensity values
  - Less sensitive to changes in intensity at higher spatial frequencies
    ⇒ use larger quanta at higher spatial frequencies
- So to perceptually encode image, we would need:
  - Intensity at different spatial frequencies
  - Luminance (grey scale intensity) separate from color intensity

JPEG Image Compression

JPEG = Joint Photographic Experts Group

Lenna Söderberg, Miss November 1972

RGB to YCbCr Conversion → Group into 8x8 blocks of pixels → Convert to energy at different spatial freqs. → Quantizer → Entropy Encoder → 011010...

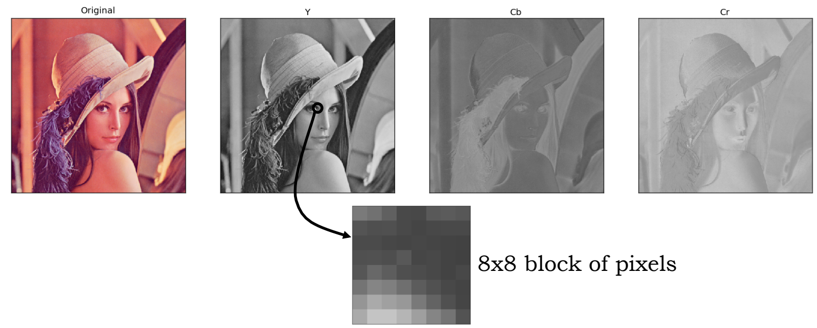
Performed for each 8x8 block of pixels

YCbCr Color Representation

JPEG-YCbCr (601) from "digital 8-bit RGB"

Y = 16 + 0.299*R + 0.587*G + 0.114*B
Cb = 128 - 0.168736*R - 0.331264*G + 0.5* B
Cr = 128 + 0.5*R - 0.418688*G - 0.081312*B

All values are in the range 16 to 235

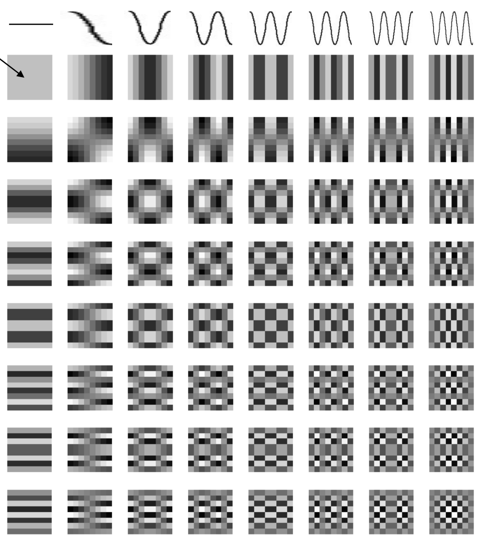


8x8 block of pixels

http://en.wikipedia.org/wiki/YCrCb

2D DCT Basis Functions

DC Component

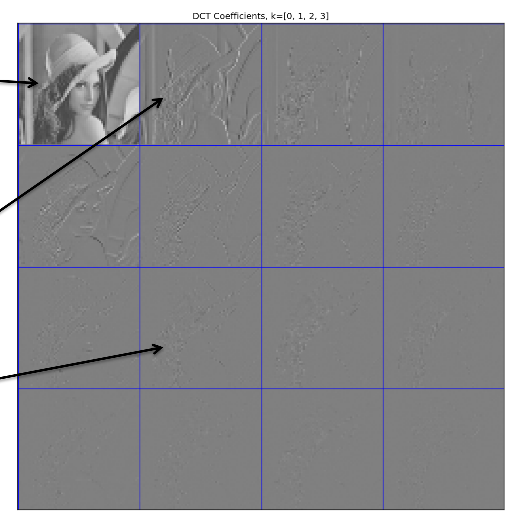


Lenna DCT Coeffs from each 8x8 block

DC coeffs carry a lot of the picture info!

Low freqs contain major edge info (note more H and V...)

High freqs contain fine detail (eg, texture of feather)



Quantization (the "lossy" part)

Divide each of the 64 DCT coefficients by the appropriate quantizer value (Q_{lum} for Y, Q_{chr} for Cb and Cr) and round to nearest integer ⇒ many 0 values, many of the rest are small integers.

$$Q_{\text{lum}} = \begin{pmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{pmatrix} \qquad Q_{\text{chr}} = \begin{pmatrix} 17 & 18 & 24 & 47 & 99 & 99 & 99 & 99 \\ 18 & 21 & 26 & 66 & 99 & 99 & 99 & 99 \\ 24 & 26 & 56 & 99 & 99 & 99 & 99 & 99 \\ 47 & 66 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \end{pmatrix}$$

Note fewer quantization levels in Q_{chr} and at higher spatial frequencies. Change "quality" by choosing different quantization matrices.

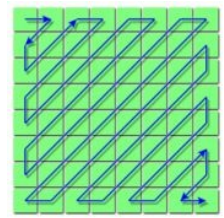
Quantization Example

```
[[-231 -148   38  -24  -15    0    4    0]
 [ 153  -11  -35   -2  -28   14   -2    0]
 [   3   73  -16  -29    2    8   -4   -3]
 [  -4   28   17  -25   -1    6   -8   -4]
 [   0    4    5    6    4    4   -2   -5]
 [   3   -4    2   10    6    0   -6   -3]
 [  -2    0   -1    6    3   -1   -5   -5]
 [  -3    1    2   -2    0    1    0    0]]
```

DCT Coefficients

```
[[-14 -13   4  -2  -1   0   0   0]
 [ 13  -1  -2   0  -1   0   0   0]
 [  0   6  -1  -1   0   0   0   0]
 [  0   2   1  -1   0   0   0   0]
 [  0   0   0   0   0   0   0   0]
 [  0   0   0   0   0   0   0   0]
 [  0   0   0   0   0   0   0   0]
 [  0   0   0   0   0   0   0   0]]
```

Quantized/Rounded Coefficients



Visit coeffs in order of increasing spatial frequency ⇒ tends to create long runs of 0s towards end of list:

```
-14
-13 13
  0 -1  4
 -2 -2  6  0
  0  2 -1  0 -1
  0 -1 -1  1  0  0
  0  0  0 -1  0  0  0…
```


Entropy Encoding Example

Quantized coeffs:

```
-14 -13 13 0 -1 4 -2 -2 6 0 0 2 -1 0 -1 0 -1 -1 1 0 0 0 0 0 -1 0 0 0…
```

DC: (N),coeff, all the rest: (run,N),coeff

```
(4)-14 (0,4)-13 (0,4)13 (1,1)-1 (0,3)4 (0,2)-2 (0,2)-2
(0,3)6 (2,2)2 (0,1)-1 (1,1)-1 (0,1)-1 (0,1)1 (5,1)-1 EOB
```

Encode using Huffman codes for N and (run,N):

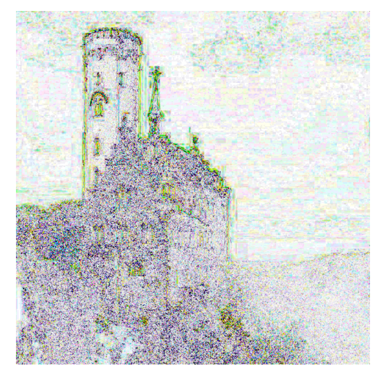
```
1010001 10110010 10111101 11000 100100 0101 0101
100110 1111101110 000 11000 000 001 11110100 1010
```

Result: 8x8 block of 8-bit pixels (512 bits) encoded as 84 bits

6x compression!

To read more see "The JPEG Still Picture Compression Standard" by Gregory K. Wallace
http://white.stanford.edu/~brian/psy221/reader/Wallace.JPEG.pdf

JPEG Results



The source image (left) was converted to JPEG (q=50) and then compared, pixel-by-pixel. The error is shown in the right-hand image (darker = larger error).

http://en.wikipedia.org/wiki/JPEG