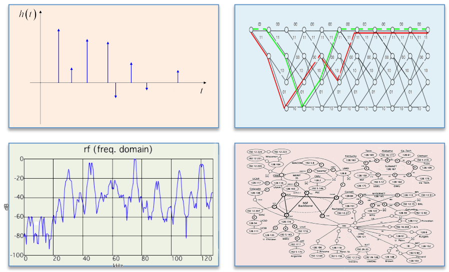


INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Spring 2011
Lecture #16

- sharing the frequency spectrum
- modulation
- demodulation

f_s, frequency, Ω and k

Various frequency specifications we'll use

- f_s, the sample frequency in samples/sec
- f, the signal frequency in Hz = cycles/sec
  - $-f_s/2 \le f \le f_s/2$
- Ω, the angular frequency in radians/sample
  - $-\pi \le \Omega \le \pi$
- k, the spectral coefficient index
  - $-N/2 \le k \le N/2$

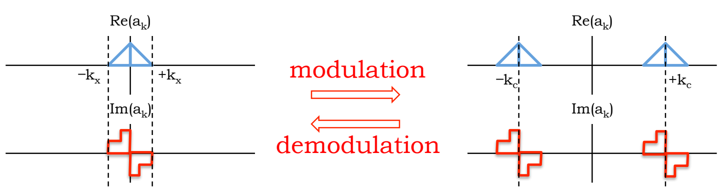
$$\boxed{\Omega = 2\pi \frac{f}{f_s} = 2\pi \frac{k_\Omega}{N}}$$

Examples: f_s=1e6 samples/sec, f = 10 kHz, N = 1000
so Ω = .02π and k_Ω = 10

k = 15, N = 100, f_s = 1e6
so Ω = .3π and f = 150 kHz

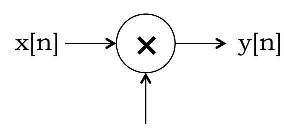
Using Some Piece of the Spectrum

- You have: a band-limited signal x[n] at *baseband* (i.e., centered around 0 frequency).
- You want: the same signal, but centered around some specific frequency $k_c(2\pi/N)$.
- Modulation: convert from baseband up to $k_c(2\pi/N)$
  Demodulation: convert from $k_c(2\pi/N)$ down to baseband



Signal centered at 0 Signal centered at k_c

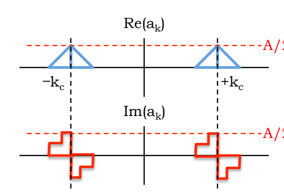
Modulation



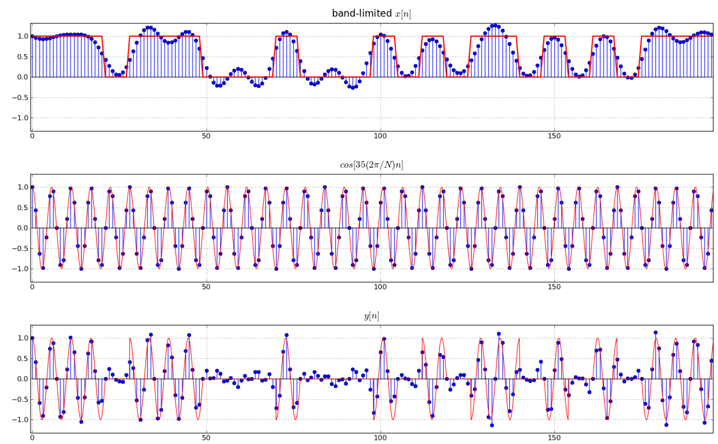
For band-limited signal a_k are nonzero only for small range of ±k

$$y[n] = \left[\sum_{k=-k_x}^{k_x} a_k e^{jk\frac{2\pi}{N}n}\right]\left[\frac{1}{2}e^{jk_c\frac{2\pi}{N}n} + \frac{1}{2}e^{-jk_c\frac{2\pi}{N}n}\right]$$

$$= \frac{1}{2}\sum_{k=-k_x}^{k_x} a_k e^{j(k+k_c)\frac{2\pi}{N}n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} a_k e^{j(k-k_c)\frac{2\pi}{N}n}$$

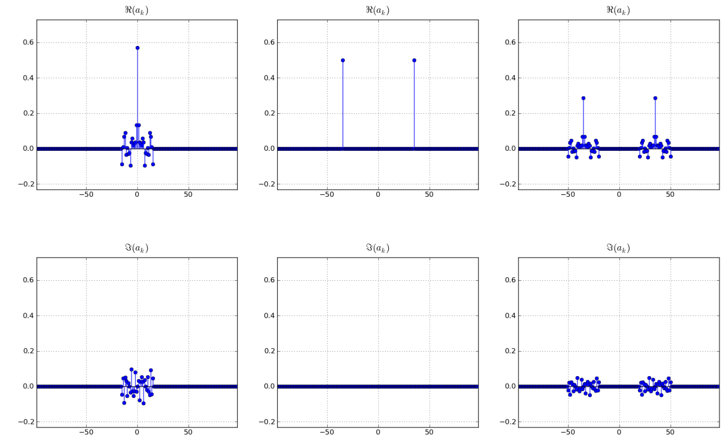


Example: Modulation (time)

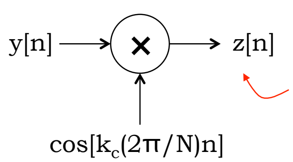


Example: Modulation (freq)

Band-limited x[n] cos[35(2π/N)n] y[n]



Demodulation



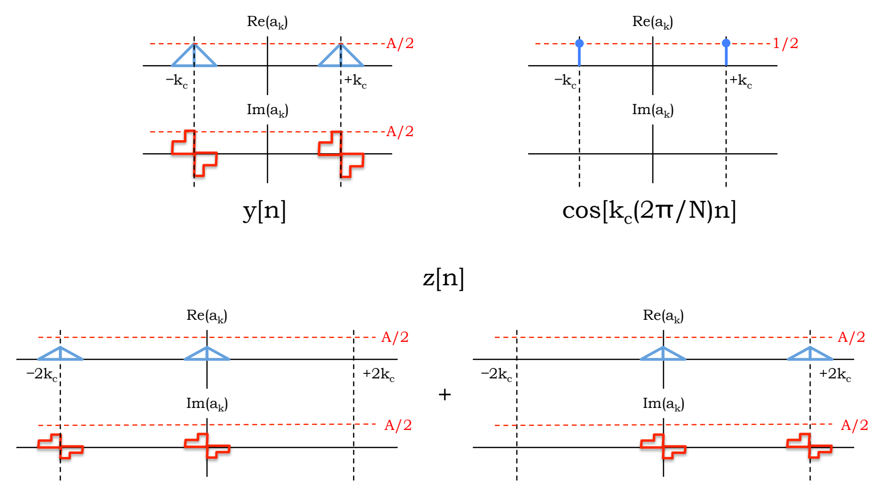
Hmm. So z[n] has what we want at baseband, but has signal we don't want at $\pm 2f_c$

$$z[n] = y[n]\left[\frac{1}{2}e^{jk_c\frac{2\pi}{N}n} + \frac{1}{2}e^{-jk_c\frac{2\pi}{N}n}\right]$$

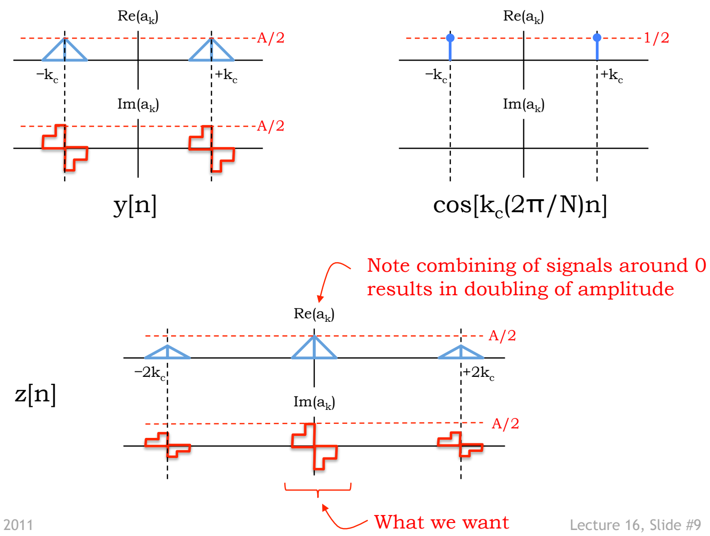
$$= \left[\frac{1}{2}\sum_{k=-k_x}^{k_x} a_k e^{j(k+k_c)\frac{2\pi}{N}n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} a_k e^{j(k-k_c)\frac{2\pi}{N}n}\right]\left[\frac{1}{2}e^{jk_c\frac{2\pi}{N}n} + \frac{1}{2}e^{-jk_c\frac{2\pi}{N}n}\right]$$

$$= \frac{1}{4}\sum_{k=-k_x}^{k_x} a_k e^{j(k+2k_c)\frac{2\pi}{N}n} + \underbrace{\frac{1}{2}\sum_{k=-k_x}^{k_x} a_k e^{jk\frac{2\pi}{N}n}}_{\text{What we want}} + \frac{1}{4}\sum_{k=-k_x}^{k_x} a_k e^{j(k-2k_c)\frac{2\pi}{N}n}$$

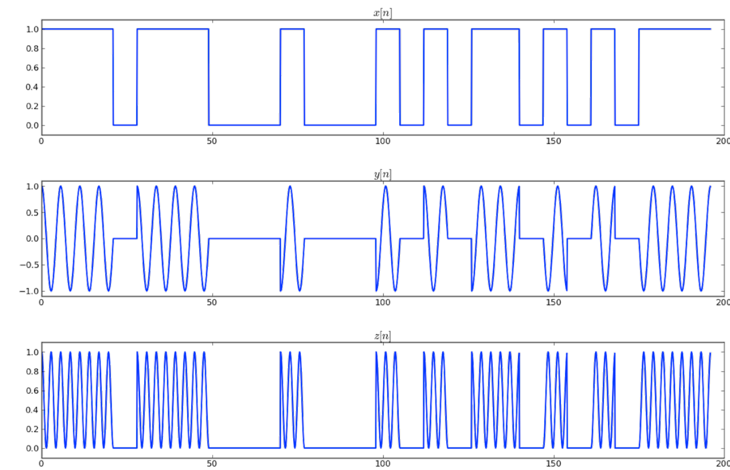
Demodulation Frequency Diagram



Demodulation Frequency Diagram

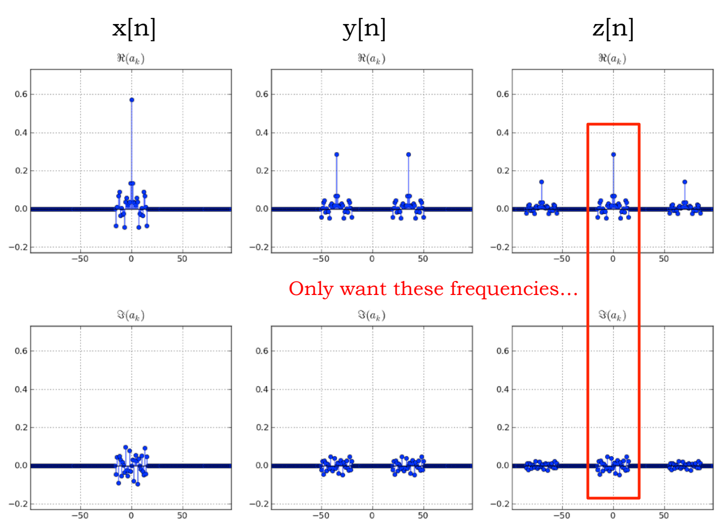


Example: Demodulation (time)

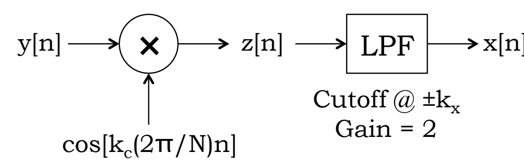


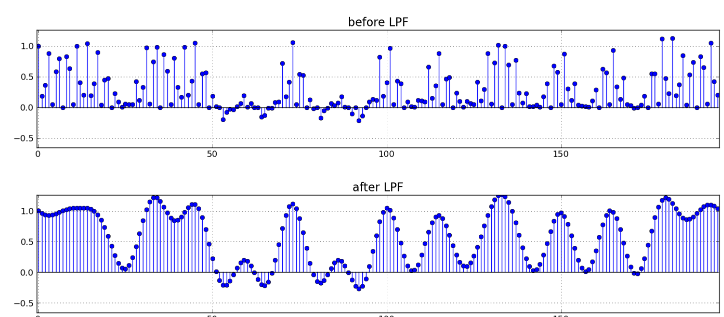
Showing idealized signals

Example: Demodulation (freq)

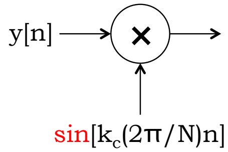


Demodulation + LPF





Demodulation with sin[k_c(2π/N)n]



Hmm. So z[n] no longer has the signal we want at baseband!

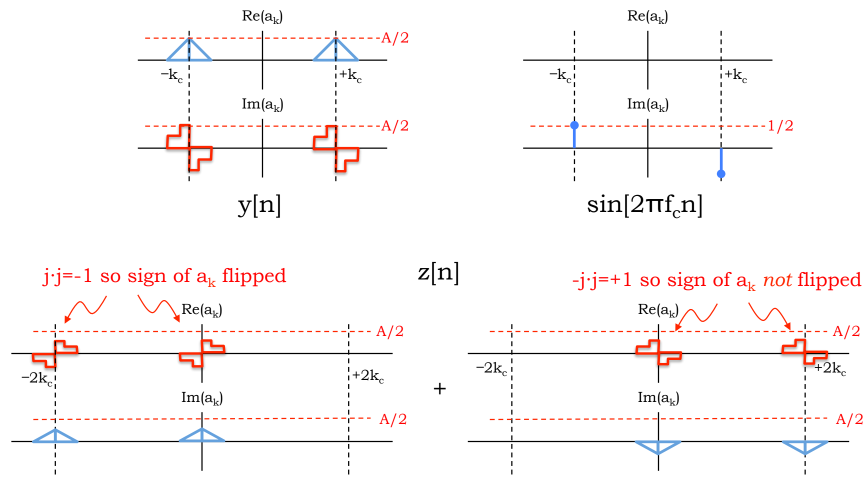
$$z[n]=y[n]\left[-\frac{j}{2}e^{jk_c\frac{2\pi}{N}n}+\frac{j}{2}e^{-jk_c\frac{2\pi}{N}n}\right]$$

$$=\left[\frac{1}{2}\sum_{k=-k_x}^{k_x}a_k e^{j(k+k_c)\frac{2\pi}{N}n}+\frac{1}{2}\sum_{k=-k_x}^{k_x}a_k e^{j(k-k_c)\frac{2\pi}{N}n}\right]\left[-\frac{j}{2}e^{jk_c\frac{2\pi}{N}n}+\frac{j}{2}e^{-jk_c\frac{2\pi}{N}n}\right]$$

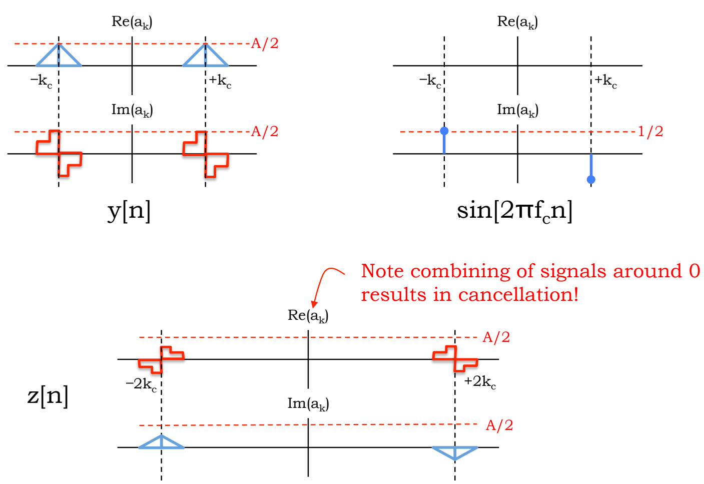
$$=-\frac{j}{4}\sum_{k=-k_x}^{k_x}a_k e^{j(k+2k_c)\frac{2\pi}{N}n}+\frac{j}{4}\sum_{k=-k_x}^{k_x}a_k e^{j(k-2k_c)\frac{2\pi}{N}n}$$

Oops, no baseband signal!

Demodulation (sin) Frequency Diagram



Demodulation (sin) Frequency Diagram



Multiple Transmitters