6.02 Spring 2011
Lecture #16

• sharing the frequency spectrum
• modulation
• demodulation

Using Some Piece of the Spectrum

• You have: a band-limited signal $x[n]$ at baseband (i.e., centered around 0 frequency).
• You want: the same signal, but centered around some specific frequency $k_c(2\pi/N)$.
  • Modulation: convert from baseband up to $k_c(2\pi/N)$
  • Demodulation: convert from $k_c(2\pi/N)$ down to baseband

Modulation

$$x[n] \rightarrow \cos[k_c(2\pi/N)n]$$

For band-limited signal $a_k$ are nonzero only for small range of $k$

$$y[n] = \sum_{k=-k_c}^{k_c} a_k e^{jk(\frac{2\pi}{N})n} = \frac{1}{2} \sum_{k=-k_c}^{k_c} a_k e^{jk(k_c-k)\frac{2\pi}{N}} + \frac{1}{2} \sum_{k=-k_c}^{k_c} a_k e^{-jk(k_c-k)\frac{2\pi}{N}}$$

$\Omega = 2\pi \frac{f}{f_s} = 2\pi \frac{k\Omega}{N}$

Examples:
- $f_s$ = 1e6 samples/sec, $f = 10$ kHz, $N = 1000$
  - $\Omega = 0.02\pi$ and $k\Omega = 10$
- $k = 15$, $N = 100$, $f_s = 1e6$
  - $\Omega = 0.3\pi$ and $f = 150$ kHz
Example: Modulation (time)

$$x[n] = \text{band-limited}$$

$$y[n] = \cos\left[\frac{35(2\pi/N)n}{N}\right]$$

Example: Modulation (freq)

Demodulation

$$y[n] \rightarrow \times \rightarrow z[n]$$

$$z[n] = y[n] \left[ \frac{1}{2} e^{j\frac{2\pi n}{N}} + \frac{1}{2} e^{-j\frac{2\pi n}{N}} \right]$$

$$z[n] = y[n] \left[ \frac{1}{2} e^{j\frac{2\pi n}{N}} + \frac{1}{2} e^{-j\frac{2\pi n}{N}} \right]$$

$$z[n] = \frac{1}{2} \sum_{k=-k_x}^{k} a_k e^{j\left(k+2k_x\right)\frac{2\pi n}{N}} + \frac{1}{2} \sum_{k=-k_x}^{k} a_k e^{j\left(k-k_x\right)\frac{2\pi n}{N}}$$

$$z[n] = \frac{1}{4} \sum_{k=-k_x}^{k} a_k e^{j\left(k+2k_x\right)\frac{2\pi n}{N}} + \frac{1}{4} \sum_{k=-k_x}^{k} a_k e^{j\left(k-k_x\right)\frac{2\pi n}{N}}$$

$$z[n] = \frac{1}{4} \sum_{k=-k_x}^{k} a_k e^{j\left(k+2k_x\right)\frac{2\pi n}{N}} + \frac{1}{4} \sum_{k=-k_x}^{k} a_k e^{j\left(k-k_x\right)\frac{2\pi n}{N}}$$

Hmm. So $z[n]$ has what we want at baseband, but has signal we don’t want at $\pm 2f_c$.

Demodulation Frequency Diagram

What we want
Demodulation Frequency Diagram

What we want

Note combining of signals around 0 results in doubling of amplitude

Example: Demodulation (time)

Example: Demodulation (freq)

Demodulation + LPF

x[n] → y[n] → z[n] → LPF → x[n]

Cutoff @ ±k_x
Gain = 2
Demodulation with $\sin[k_c(2\pi/N)n]$

$$y[n] \rightarrow \times \rightarrow z[n]$$

$$z[n] = y[n] \left[ -\frac{j}{2} e^{\frac{j2\pi}{N}n} + \frac{j}{2} e^{-\frac{j2\pi}{N}n} \right]$$

$$= \frac{1}{2} \sum_{k=-k_c}^{k_c} a_k e^{j(k+k_c)\frac{2\pi}{N}n} + \frac{1}{2} \sum_{k=-k_c}^{k_c} a_k e^{j(k-k_c)\frac{2\pi}{N}n}$$

$$= -\frac{j}{4} \sum_{k=-k_c}^{k_c} a_k e^{j(2k)\frac{2\pi}{N}n} + \frac{j}{4} \sum_{k=-k_c}^{k_c} a_k e^{j(-2k)\frac{2\pi}{N}n}$$

Oops, no baseband signal! Lecture 16, Slide #13

Demodulation (sin) Frequency Diagram

Multiple Transmitters

Choose bandwidths and $\ell_i$’s so as to avoid overlap! Once signals combine at a given frequency, can’t be undone…