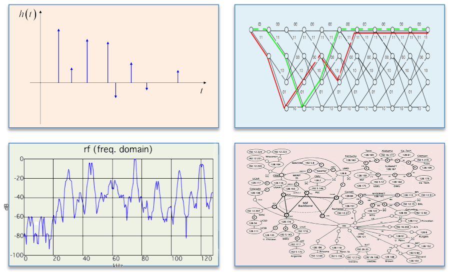


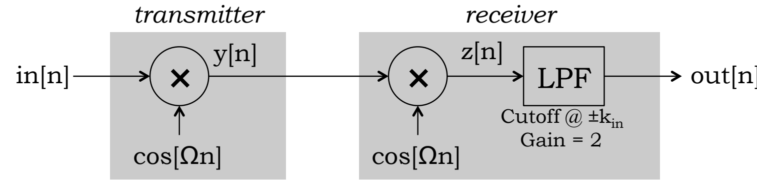
INTRODUCTION TO EECS II

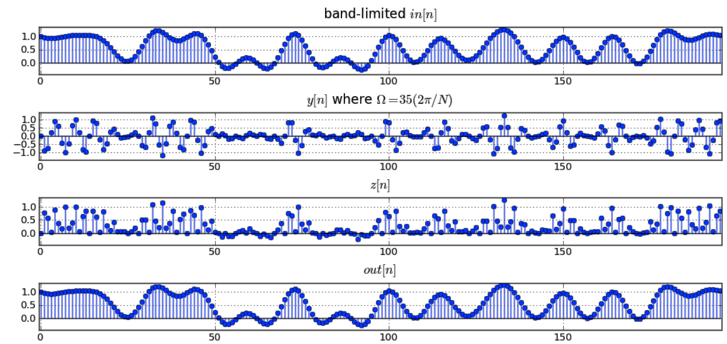
DIGITAL COMMUNICATION SYSTEMS

6.02 Spring 2011
Lecture #17

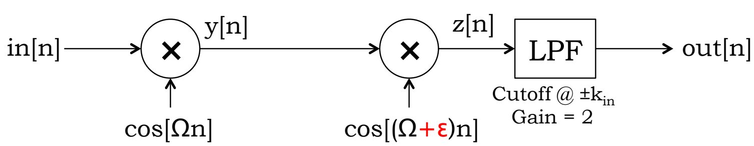
- mismatch of rcvr's freq & phase
- quadrature demod
- BPSK, QPSK, DQPSK

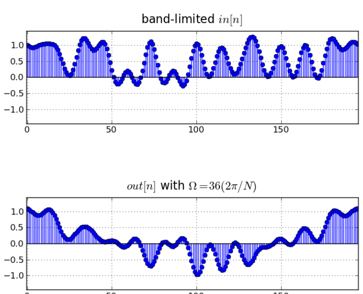
Ideal Modulation/Demodulation

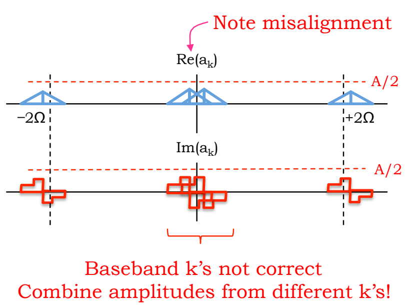




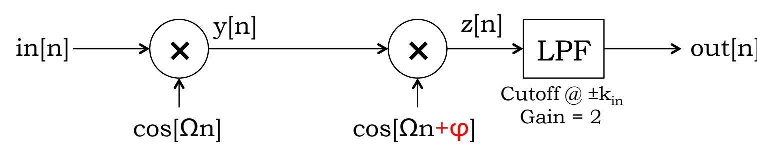
Frequency Error in Demodulator

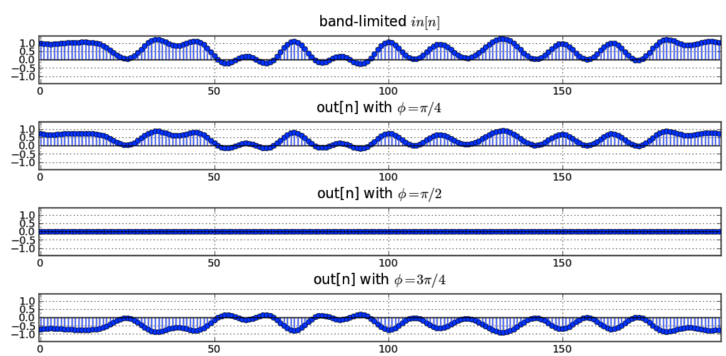






Phase Error in Demodulator





Phase Error Math

Let's derive an equation for z[n]:

$$z[n] = y[n]\cdot\cos[\Omega n+\varphi] = \text{in}[n]\cdot\cos[\Omega n]\cdot\cos[\Omega n+\varphi]$$

$$= \text{in}[n]\cdot\frac{1}{2}\left[e^{j\Omega n}+e^{-j\Omega n}\right]\cdot\frac{1}{2}\left[e^{j\varphi}e^{j\Omega n}+e^{-j\varphi}e^{-j\Omega n}\right]$$

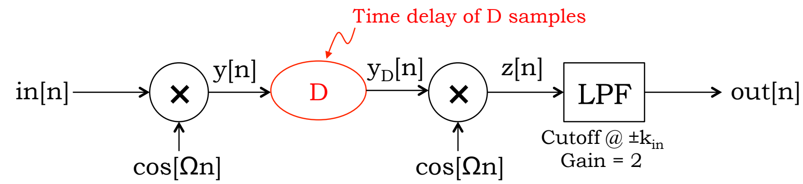
$$= \text{in}[n]\cdot\frac{1}{4}\left[e^{j\varphi}e^{j2\Omega n}+e^{j\varphi}+e^{-j\varphi}+e^{-j\varphi}e^{-j2\Omega n}\right]$$

Passing this through the LPF with a gain of 2 eliminates the high-frequency terms and doubles the amplitude:

$$\text{out}[n] = \text{in}[n]\cdot\frac{2}{4}\left[e^{j\varphi}+e^{-j\varphi}\right] = \text{in}[n]\cdot\cos(\varphi)$$

So a phase error of φ results in amplitude scaling of cos(φ).

Channel Delay



$$z[n] = y_D[n]\cdot\cos[\Omega n] = y[n-D]\cdot\cos[\Omega n] = \text{in}[n-D]\cdot\cos[\Omega(n-D)]\cdot\cos[\Omega n]$$

$$= \text{in}[n-D]\cdot\frac{1}{2}\left[e^{-j\Omega D}e^{j\Omega n}+e^{j\Omega D}e^{-j\Omega n}\right]\cdot\frac{1}{2}\left[e^{j\Omega n}+e^{-j\Omega n}\right]$$

$$= \text{in}[n]\cdot\frac{1}{4}\left[e^{-j\Omega D}e^{j2\Omega n}+e^{-j\Omega D}+e^{j\Omega D}+e^{j\Omega D}e^{-j2\Omega n}\right]$$

Passing this through the LPF:

Looks like a phase error

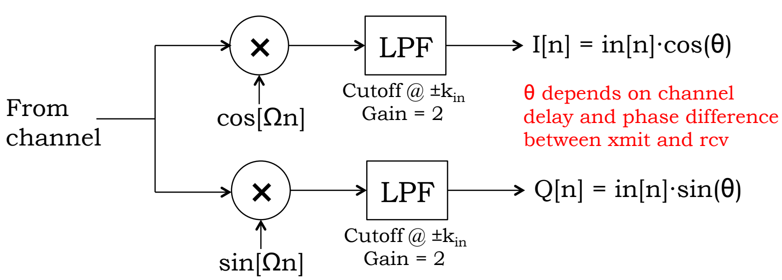
$$\text{out}[n] = \text{in}[n]\cdot\frac{2}{4}\left[e^{-j\Omega D}+e^{j\Omega D}\right] = \text{in}[n]\cdot\cos(\Omega D)$$

Fixing Phase Problems in the Receiver

So phase errors and channel delay both result in a scaling of the output amplitude, where the magnitude of the scaling can't necessarily be determined at system design time:

- channel delay varies on mobile devices
- phase difference between transmitter and receiver is arbitrary

One solution: *quadrature demodulation*



Quadrature Demodulation

If we let

$$w[n] = I[n] + jQ[n]$$

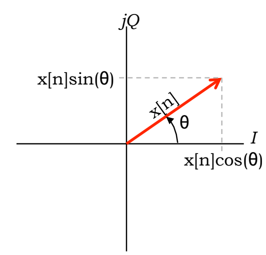
then

$$|w[n]| = \sqrt{I[n]^2 + Q[n]^2}$$

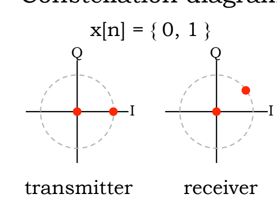
$$= \sqrt{x[n]^2\cdot\cos^2\theta + x[n]^2\cdot\sin^2\theta}$$

$$= \sqrt{x[n]^2\cdot(\cos^2\theta+\sin^2\theta)}$$

$$= x[n]$$



Constellation diagrams:



Phase Modulation

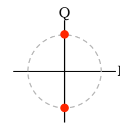
A sinusoid is characterized by its frequency, amplitude and phase – one can modulate anyone of these using x[n], which represents the message to be transmitted.

- Amplitude modulation (AM) – what we've done so far
- Frequency modulation (FM)
- Phase modulation – our next topic

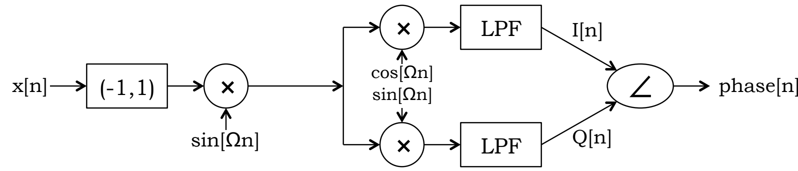
Using AM the signal can have zero amplitude, indistinguishable from no signal at all, which can confound the circuitry that "fine tunes" the amplitude and frequency at the receiver.

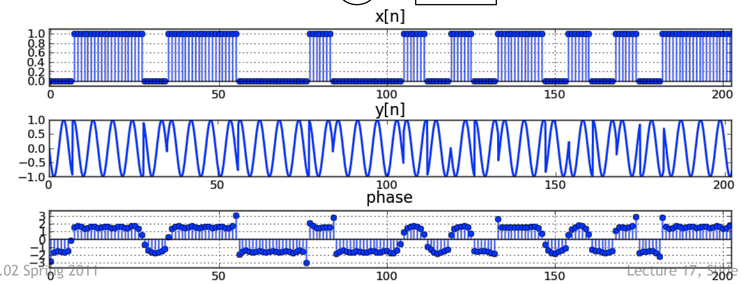
Using phase modulation, aka phase-shift keying (PSK), the transmitted signal has constant amplitude; information is encoded in the phase of the carrier sinusoid.

BPSK

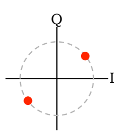


In binary phase-shift keying (BPSK), the message bit selects one of two phases for the carrier, e.g., $\pi/2$ and $-\pi/2$.



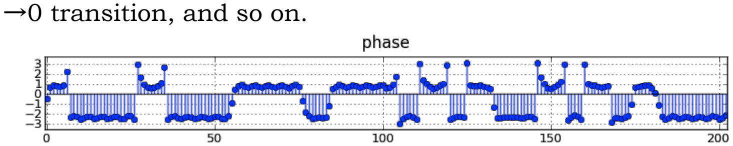


Dealing With Phase Ambiguity



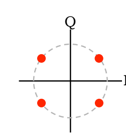
BPSK is also subject phase changes introduced by channel delays or phase difference between xmit and rcv: the received constellation will be rotated with respect to the transmitter's constellation. Which phase corresponds to which bit?

The fix? Think of the phase encoding as *differential*, not absolute: a change in phase corresponds to a change in bit value. Assume that, by convention, messages start with a single 0 bit, i.e., prepend a 0 to each to message. Then the first phase change represents a 0→1 transition, the second phase change a 1→0 transition, and so on.



Differential PSK

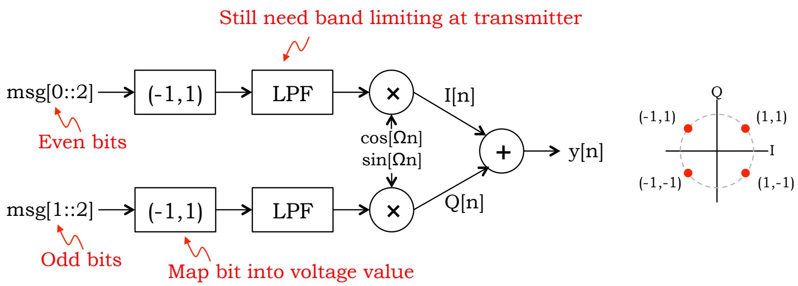
- Approach 1: encode bits
  - DBPSK – process one message bit at a time
    - "0": phase change is 0
    - "1": phase change is π
  - DQPSK – process two message bits at a time
    - "00": phase change is 0
    - "01": phase change is $\pi/2$
    - "10": phase change is π
    - "11": phase change is $3\pi/2$
- Approach 2: encode transitions
  - DBPSK
    - No transition: phase change is 0
    - Transition: phase change is π
  - DQPSK: assume cyclic order is 00, 01, 11, 10
    - No advance in order: phase change is 0
    - Advance one position in order: phase change is $\pi/2$
    - And so on…



What about 3 bits at time, i.e., 8 constellation points?

QPSK Modulation

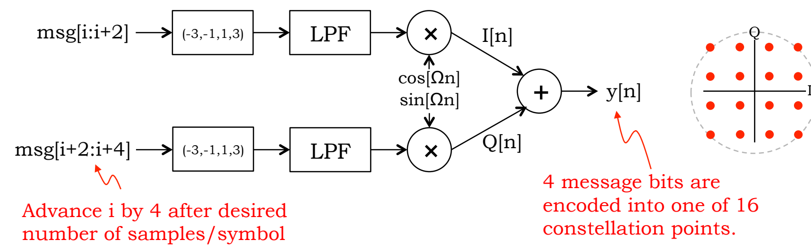
We can use the quadrature scheme at the transmitter too:



When mapping bits to voltage values, we should choose the values so that the maximum amplitude of y[n] is 1. For QPSK (also referred to as QAM-4) that would mean $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) = (.707,.707)$

Quadrature Amplitude Modulation (QAM)

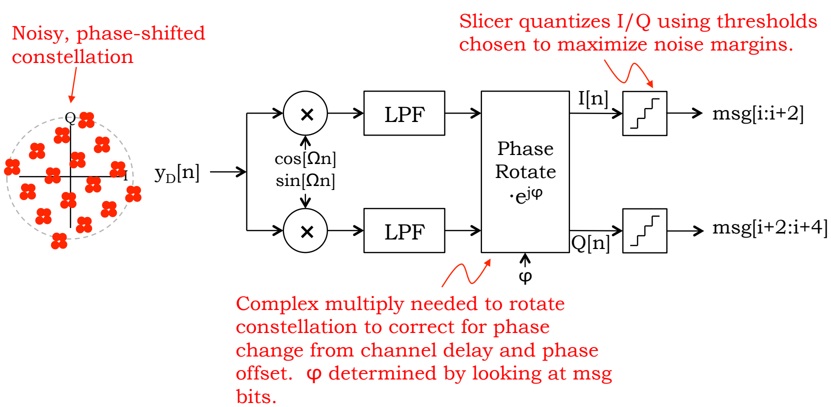
Using more message bits at a time, we can generate larger constellations using quadrature amplitude modulation. Here's a diagram of a QAM-16 system with 16 constellation points:



Larger constellations mean points are closer together, so there's more sensitivity to noise. Some systems adapt the size of the constellation to the noise level (QAM-4, QAM-16, QAM-64, …), i.e., use 2, 4, 6, … bits/symbol.

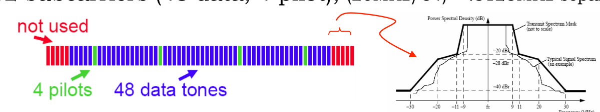
QAM Receiver

Here's a simplified diagram of a QAM-16 receiver:



802.11a

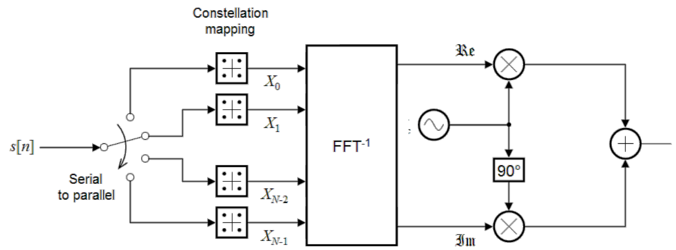
- 12 channels in 5GHz band
- 20MHz bandwidth (16.6MHz occupied)
  - Orthogonal Frequency Division Multiplexing (OFDM)
  - 52 subcarriers (48 data, 4 pilot), (20MHz/64) = .3125MHz separation



  - Modulation and channel coding scheme can be chosen to reflect actual channel capacity (choice based on SNR):
    - BPSK (6Mbps @ rate 1/2, 9Mbps @ rate 3/4, 1 bit)
    - QPSK (12Mbps @ rate 1/2, 18Mbps @ rate 3/4, 2 bits)
    - 16-QAM (24Mbps @ rate 1/2, 36Mbps @ rate 3/4, 4 bits)
    - 64-QAM (48Mbps @ rate 2/3, 54Mbps @ rate 3/4, 6 bits)
  - Symbol duration 4us (includes guard interval of 0.8us)
    - ~250k combined symbols per second, 48 data channels
    - ~Data rate = (48)(250k)(# bits/symbol)(code rate)
      = (12M)(# bits/symbol)(code rate)

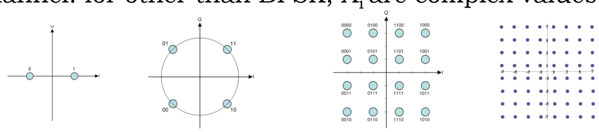
http://stellar.mit.edu/S/course/6/sp06/6.973/ -- Lecture 4

OFDM Transmitter



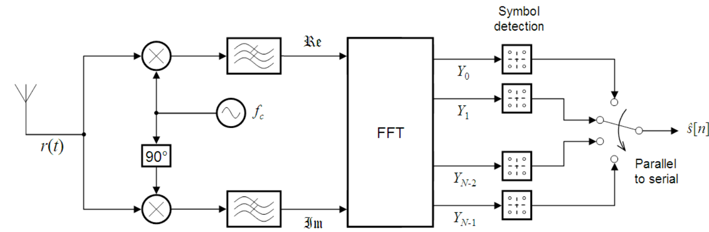
http://en.wikipedia.org/wiki/OFDM

- Signal is interleaved across multiple subchannels
  - For bidirectional links, can use some subchannels for each direction
- Constellation mapping can be chosen separately for each subchannel: for other than BPSK, X_i are complex values



BPSK QPSK 16-QAM 64-QAM

OFDM Receiver



- Need good frequency synchronization at receiver to keep subchannels orthogonal, need good gain control to keep amplitudes correct for slicing.
- Low-pass filter selects demodulated baseband signal
- Symbol detection for each subchannel is matched to modulation scheme selected by transmitter