1. (a) From the definition of the frequency response, \( H(\Omega) = \sum_{m=-\infty}^{\infty} h[m] e^{-j\Omega m} = 1 + 2e^{-j\Omega} + e^{-2j\Omega} \).

(b) Plug in the given \( \Omega \) values into the expression for \( H(\Omega) \), to get \( H(0) = 1, H(\pi/2) = 1 - 2j - 1 = -2j, H(\pi) = 1 - 2 + 1 = 0 \).

(c) Let \( e^{-j\Omega} = z \), so we want \( 1 + 2z + z^2 = 0 \), which gives us \( (z + 1)^2 = 0 \), or \( z = -1 \). Hence, \( e^{-j\Omega} = -1 \), or \( \Omega = \pi \).

2. (a) We’ll use the fact that if the input to an LTI with frequency response \( H(\Omega) \) is a complex exponential \( x[n] = e^{j\Omega n} \), then the output \( y[n] = H(\Omega)x[n] = H(\Omega)e^{j\Omega n} \). Plugging that into the given equation or \( y[n] \), we get \( H(z) = 1 + \alpha z + \beta z^2 + \gamma z^3 \), denoting \( e^{-j\Omega} \) by \( z \) for convenience (on both sides of the equation).

Let’s now expand the given expression, \( H(\Omega) = 1 - 0.5e^{-j2\Omega}\cos \Omega \) in terms of complex exponentials. The idea is we can then match the coefficients to obtain the values of \( \alpha, \beta, \) and \( \gamma \). Expanding the \( \cos(\Omega) \) term into complex exponential sums and multiplying, we get the expression \( H(z) = 1 - 0.25z - 0.25z^3 \). Matching coefficients, we conclude that \( \alpha = -0.25, \beta = 0, \gamma = -0.25 \).

(b) For a series (cascade) of LTI systems, the frequency response is the product of the frequency responses of the individual LTI systems. So, the frequency response of this cascade is \( G(e^{j\Omega})H(e^{j\Omega}) \). For \( w[n] \) to be equal to \( x[n] \) for all \( n \), the unit sample response of the cascade must be \( \delta[n] \). The frequency response of the cascade is related to the unit sample response by \( G(e^{j\Omega})H(e^{j\Omega}) = \sum_{m=0}^{\infty} \delta[m]e^{-j\Omega m} = 1 \). Since \( G(e^{j\Omega})H(e^{j\Omega}) = 1 \), and \( H(e^{j\Omega}) \) is never zero for \( -\pi \leq \Omega \leq \pi \), then \( G(e^{j\Omega}) = \frac{1}{h(e^{j\Omega})} \). Hence,

\[
G(e^{j\Omega}) = \frac{1}{1 - 0.5e^{-j2\Omega}\cos \Omega}.
\]

(c) First, observe that we can rewrite the given \( x[n] \) as

\[
x[n] = e^{j0n} + 0.5e^{j\pi n}.
\]

Then using the meaning of frequency response, and applying superposition, we can write

\[
y[n] = 0e^{j0n} + Ae^{j\pi n} = H(e^{j0})e^{j0n} + H(e^{j\pi n})0.5e^{j\pi n}.
\]

Matching terms, it then follows that \( H(e^{j0}) = 0 \) and \( A = 0.5H(e^{j\pi n}) \). For this difference equation, the frequency response for an arbitrary \( \Omega \) is given by

\[
H(e^{j\Omega}) = 1 + \alpha e^{-j\Omega} + \beta e^{-j2\Omega} + \gamma e^{-j3\Omega},
\]

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so for the special case of $\Omega = 0$,

$$H(e^{j0}) = 1 + \alpha + \beta + \gamma.$$ 

Given that $H(e^{j0}) = 0$, and $\alpha = \gamma = 1$, it must be true that $\beta = -3$. Then to determine $A$, consider that

$$H(e^{j\pi n}) = 1 + \alpha e^{-j\pi} + \beta e^{-j2\pi} + \gamma e^{-j3\pi} = 1 - \alpha + \beta - \gamma = 1 - 1 - 3 - 1 = -4.$$ 

Therefore, $A = 0.5H(e^{j\pi n}) = 0.5(-4) = -2$.

3. (a) From the definition of the frequency response, $H(\Omega) = \sum_{m=0}^{\infty} h[m]e^{-j\Omega m}$, we get $h[0]e^{-j\Omega 0} + h[1]e^{-j\Omega} + h[2]e^{-j2\Omega} + h[3]e^{-j3\Omega}$. Plugging in values for $h[i]$, the frequency response is $H(\Omega) = a + be^{-j\Omega} + be^{-j2\Omega} + ae^{-j3\Omega}$.

(b) Note that $x[n] = (-1)^n$ can be written as $x[n] = e^{-j\pi n}$. This input corresponds to a complex exponential with angular frequency $\Omega = \pi$. We know that for everlasting complex exponential inputs, $y[n] = H(\Omega)e^{j\Omega n}$; therefore we know that $y[n] = H(\pi)e^{j\pi n}$. Evaluating $H(\pi)$, $y[n]$ is given by $(a - b + b - a)$, or $y[n] = 0$ for all $n$.

(c) The definition of convolution follows as $y[n] = \sum_{m=0}^{\infty} h[m]x[n-m]$. We find $y[5]$ by solving $\sum_{m=0}^{\infty} h[m]x[5-m]$; this gives $-a + b - b + a = 0$. Similarly, $y[6] = \sum_{m=0}^{\infty} h[m]x[6-m]$, giving $a - b + b - a = 0$. Moreover, since these calculations are representative of what one would get for odd and even $n$ respectively, $y[n] = 0$ for all $n$.

(d) Decompose into cosines. Dividing both sides by $e^{-j3\Omega 2}$ we know that $G(\Omega) = \frac{1}{e^{-j3\pi 2}}H(\Omega)$. Dividing out $e^{-j3\Omega 2}$ from each term in $H(\Omega)$, we get $ae^{j3\Omega 2} + be^{j\Omega 2} + be^{-j\Omega 2} + ae^{j3\Omega 2}$. Converting this expression to cosines, we find that

$$G(\Omega) = 2[a \cos(\frac{3\Omega}{2}) + b \cos(\frac{\Omega}{2})].$$

(e) By superposition, the response is the sum of the responses to $(-1)^n$ and to $\cos(\frac{\pi}{2} n + \theta_0)$. But we have already seen in parts (b) and (c) that the response to $(-1)^n$ is 0 for all $n$. So what remains is to find the response to $\cos(\pi n + \theta_0 + \Omega/2)$. This response is simply

$$y[n] = |H(\pi/2)| \cos(\frac{\pi}{2} n + \theta_0 + \angle H(\pi/2)).$$

Note from the form of $H(\Omega)$ in part (d) that

$$H(\pi/2) = G(\pi/2)e^{-j3\pi/4} = 2[a \cos(\frac{3\pi}{4}) + b \cos(\frac{\pi}{4})] \sqrt{2}(b - a)e^{-j3\pi/4}$$

Since $b > a$, it follows that $|H(\pi/2)| = \sqrt{2}(b - a)$ and $\angle H(\pi/2) = -3\pi/4$. Hence,

$$y[n] = \sqrt{2}(b - a) \cos(\frac{\pi}{2} n + \theta_0 - \frac{3\pi}{4}).$$

As a check, consider the case $\theta_0 = 0$, so the input of interest is $\cos(\pi n)$, which alternates between $+1$ and $-1$ at even values of $n$, and is 0 at all odd values of $n$. Convolving this sequence with the given unit sample response $h[n]$ shows that $y[0] = a - b$. And this answer matches what we get on evaluating the above general expression for $y[n]$ at $n = 0$ and $\theta_0 = 0$. 

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4. (a) There are two key things to notice about this frequency response function. First off, it has no zeros. Second, looking at the denominator, when $\Omega \approx \pm \frac{\pi}{2}$ the denominator becomes very small, causing $H_A(e^{j\Omega})$ to become large. The only graph that satisfies these two criteria is $H_I$.

(b) We can solve this problem by setting $x[n] = e^{j\Omega n}$ and knowing that the frequency response is a complex exponential with the same frequency. We get

$$H(\Omega)e^{j\Omega n} + a_1H(\Omega)e^{j(\Omega(n-1))} + a_2H(\Omega)e^{j(\Omega(n-2))} = e^{j\Omega n}.$$ 

Dividing both sides by $e^{j\Omega n}$ and solving for $H(\Omega)$, we get

$$H(\Omega) = \frac{1}{1 + a_1e^{-j\Omega} + a_2e^{-j2\Omega}}.$$ 

It is easier to expand $H_A(\Omega)$ than to factor $H(\Omega)$, so we factor $H_A(\Omega)$ to find the denominator is $1 + 0.95e^{j(\Omega+\frac{\pi}{2})} + 0.95e^{j(\Omega-\frac{\pi}{2})}$. The middle two terms cancel each other out (because of the $\frac{\pi}{2}$ phase shifts) and the last term is equal to $0.9025e^{-j2\Omega}$. Matching coefficients with $H(\Omega)$, we find that $a_1 = 0$ and $a_2 = \left(\frac{19}{20}\right)^2 = 0.9025$.

5. (a) We can make the following observations:

$$\max_n (\cos \frac{\pi}{6} n) = 1, n = 0, 12, \ldots$$

$$\max_n (\cos \frac{5\pi}{6} n) = 1, n = 0, 12, \ldots$$

$$\max_n (3(-1)^n) = 1, n \text{ even}.$$ 

Hence, the maximum value is 7, and the smallest positive $n$ at which the maximum occurs is $n = 12$.

(b) The frequency response at $\Omega = \frac{\pi}{6}$ and at $\Omega = \frac{5\pi}{6}$ must be zero, which means that the only possibility is $H_{III}$.

$$y[n] = H(0) \cdot 2 \cdot (1)^n + H(\pi) \cdot 3 \cdot (-1)^n = 4 \cdot 2 \cdot (1)^n + 4 \cdot 3 \cdot (-1)^n.$$ 

The numerical value of $M$ is 4.

6. (a) The first equation tells us that the DC component of the frequency response, i.e., $H(0)$ is 5. The second and third equations tell us that $H(\pi/2)$ and $H(-\pi/2)$ are both 0.

(b) $H_{II}$ is the frequency response that best describes the above information, as it is the only curve that meets the constraints of part (a). The numerical value of $M$ must be 5.

(c) $y[n]/x[n]$ is the frequency response $H(\Omega)$. Because the input is an everlasting exponential with frequency $\Omega = \frac{\pi}{6}$, $y[n]/x[n]$ is simply $H(\frac{\pi}{6}) = h[0] + h[2]e^{-j\pi/3} = 3.75 - j(5\sqrt{3}/4)$.