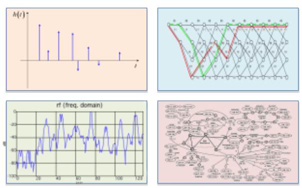
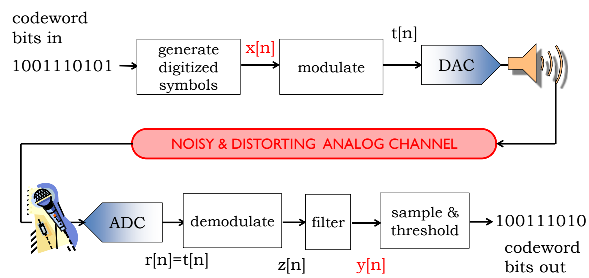


INTRODUCTION TO EECS II
DIGITAL COMMUNICATION SYSTEMS

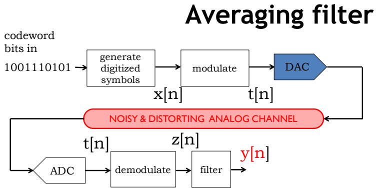
6.02 Spring 2012
Lecture #10

- Input/output descriptions of systems
- Linear time-invariant (LTI) systems
- Constructing LTI system responses to input signals

From Bits to Modulated Signal, and Back



Averaging filter



$\Omega_c = 2\pi/16$

16 samples per cycle

y[n] = (z[n]+…+z[n-L])/(L+1), L+1 length of the averaging filter

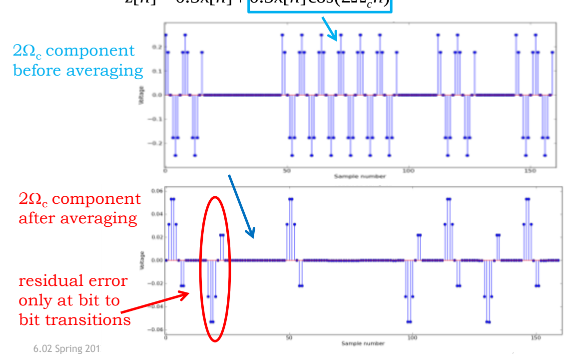
For L+1=8, $2\Omega_c$ component is at $2\pi/8$, which is 8 samples per cycle

So, the $\Sigma \cos(2\pi/8*(n-k)) = 0$ for k=0,…,L and the $2\Omega_c$ gets integrated out*

* $2\Omega_c$ component gets integrated out in regions where x[n] is constant for L+1 samples. At transitions, there is a bit of degradation, but we make decisions on the middle samples

Averaging filter in action

$$z[n] = 0.5x[n] + 0.5x[n]\cos(2\Omega_c n)$$



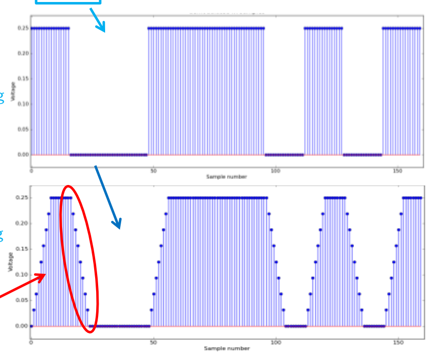
Averaging filter in action

$$z[n] = \boxed{0.5x[n]} + 0.5x[n]\cos(2\Omega_c n)$$

desired component before averaging

desired component after averaging

residual error only at bit to bit transitions

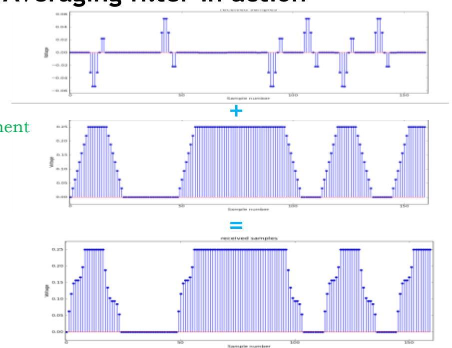


Averaging filter in action

$2\Omega_c$ component after averaging

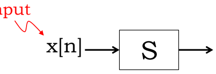
Desired component after averaging

Output of the averaging filter



Need to learn a bit more about Linear Time Invariant (LTI) systems to design a better filter!

Filter design: System Input and Output



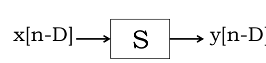
A discrete-time signal such as x[n] or y[n] is described by an infinite sequence of values, i.e., the time index n takes values in −∞ to +∞. The above picture is a snapshot at a particular time n.

In the diagram above, the sequence of *output* values y[.] is the *response* of system S to the *input* sequence x[.]

Time Invariant Systems

Let y[.] be the response of S to input x[.]

If for all possible sequences x[n] and integers D



then system S is said to be *time invariant* (TI). A time shift in the input sequence to S results in an identical time shift of the output sequence.

Linear Systems

Let $y_1[.]$ be the response of S to input $x_1[.]$, and $y_2[.]$ be the response to $x_2[.]$

If the response to linear combinations of these two inputs equals the same linear combination of the individual responses, then system S is said to be *linear*.

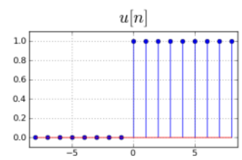
$$a_1x_1[n]+a_2x_2[n] \longrightarrow \boxed{\text{S}} \longrightarrow a_1y_1[n]+a_2y_2[n]$$

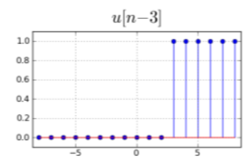
If the input is the weighted sum of several signals, the response is the corresponding *superposition* (i.e., weighted sum with same weights) of the response to those signals.

Unit Step

A simple but useful discrete-time signal is the *unit step* signal or function, u[n], defined as

$$u[n]=\begin{cases} 0, & n<0 \\ 1, & n\geq 0 \end{cases}$$

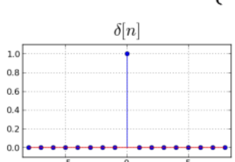


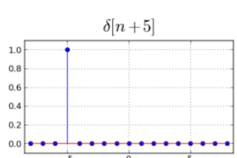


Unit Sample

Another simple but useful discrete-time signal is the *unit sample* signal or function, δ[n], defined as

$$\delta[n]=u[n]-u[n-1]=\begin{cases} 0, & n\neq 0 \\ 1, & n=0 \end{cases}$$





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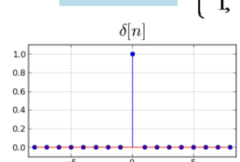
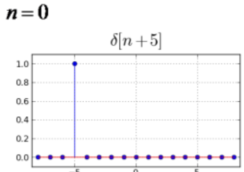
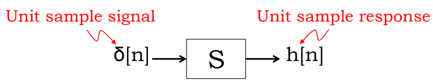




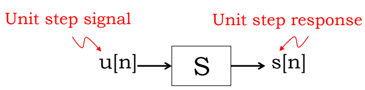
Note that standard algebraic operations on signals (e.g. subtraction, addition, scaling by a constant) are defined in the obvious way, instant by instant.

Unit Sample Response & Unit Step Response

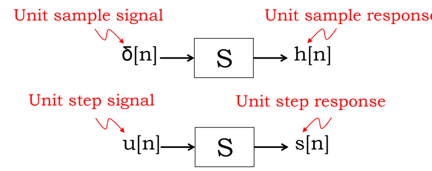


The *unit sample response* of a system S is the response of the system to the unit sample input. We will typically denote the unit sample response as h[n].

Similarly, the *unit step response* s[n]:



Relating h[n] and s[n] of an LTI System

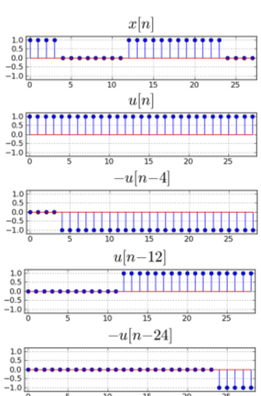


$\delta[n] = u[n] - u[n-1] \quad \Rightarrow \quad h[n] = s[n] - s[n-1]$

from which it follows that $s[n] = \sum_{k=0}^{n} h[k], \;\; n \geq 0$

(assuming $s[k] = 0, k < 0$, i.e., a causal* LTI system)

*causal system has h[n]=0 and s[n]=0 for n<0.



Unit Step Decomposition

"Rectangular-wave" digital signaling waveforms, of the sort we have been considering, are easily decomposed into time-shifted, scaled unit steps --- each transition corresponds to another shifted, scaled unit step.

e.g., if x[n] is the transmission of 1001110 using 4 samples/bit:

$$\begin{aligned} x[n] \\ &= u[n] \\ &\quad - u[n-4] \\ &\quad + u[n-12] \\ &\quad - u[n-24] \end{aligned}$$

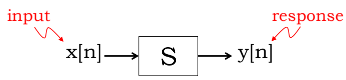
… so the corresponding response is

$$\begin{aligned} x[n] &= u[n] - u[n-4] + u[n-12] - u[n-24] \\ \Rightarrow \quad y[n] &= s[n] - s[n-4] + s[n-12] - s[n-24] \end{aligned}$$

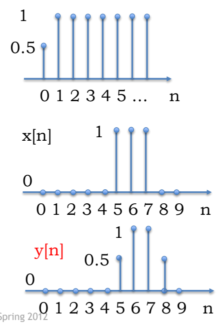
Note how we have invoked linearity and time invariance!

Let's apply to our averaging filter example …

Spot Quiz



Unit step response: s[n]



Find y[n]:

1. Write x[n] as a function of unit steps
x[n] = u[n-5] – u[n-8]

2. Write y[n] as a function of unit step responses
y[n] = s[n-5] – s[n-8]

3. Draw y[n]