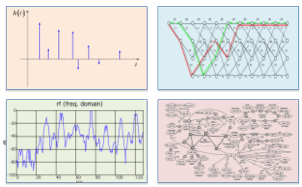


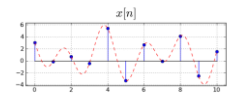
INTRODUCTION TO EECS II
DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2011
Lecture #15

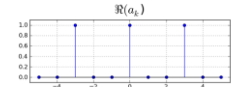
- More on signal spectra
- Modulation & demodulation

$$x[n] = 1 + 2\cos(3\frac{2\pi}{11}n) - 3\sin(5\frac{2\pi}{11}n)$$

Again, by inspection: since the cos and sin are at different frequencies, we can analyze them separately.



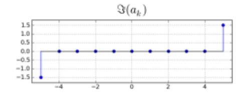
A_0 = average value = 1



$A_{\pm 3}$ = 2(1/2) = 1 [from cos term]

A_{-5} = –3(j/2) = –1.5j [from sin term]
A_5 = –3(–j/2) = 1.5j

A_k = 0 otherwise



Again, P is *odd* here (=11), so the end points of the frequency scale are at ±(π– (π/P)), not ±π.

The DTFS is also good for finite-duration signals!

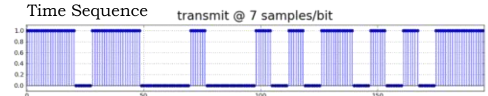
Claim: Over any contiguous interval of length P that we may be interested in --- say n=0,1,…,P–1 for concreteness --- an arbitrary DT signal x[n] can be written in the form

$$x[n] = \sum_{k=\langle P \rangle} A_k e^{j\Omega_k n}$$

What's going on here? If we know we will only be interested in the interval [0,P–1], then it doesn't matter that our representation above will create periodically repeating extensions outside the interval of interest.

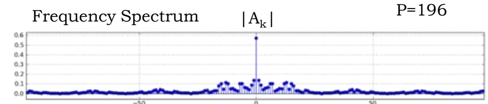
Spectrum of Digital Transmissions

Time Sequence transmit @ 7 samples/bit



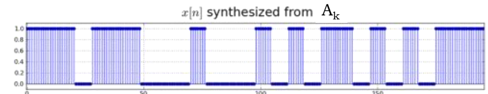
n

Frequency Spectrum $|A_k|$ P=196

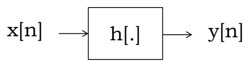


k

$x[n]$ synthesized from A_k



Application



Suppose x[n] is nonzero only over the time interval $[0, n_x]$, and h[n] is nonzero only over the time interval $[0, n_h]$.

In what time interval can the non-zero values of y[n] be guaranteed to lie? The interval $[0, n_x + n_h]$.

Since all the action we are interested in is confined to this interval, choose $P - 1 \geq n_x + n_h$, then use the DTFS to represent x[n] and y[n] over this interval.

This is actually the much more common use of the DTFS!

Spot Quiz

$$x[n] = \cos(\frac{2\pi}{5} n)$$

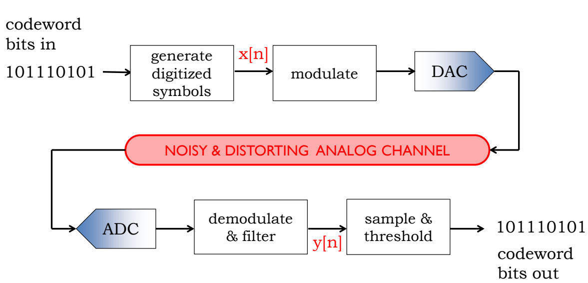
Determine non-zero spectral components:

1. A_k=? , when P=5, k= -2 … 2
2. A_k=?, when P=10, k= -5 … 4

For P=5, $\Omega_1 = 2\pi/5$, and $x[n] = 0.5e^{j\Omega_1 n} + 0.5e^{-j\Omega_1 n}$
so $A_{-1} = 0.5$ and $A_1 = 0.5$ and $A_k = 0$ for other k.

For P=10, $\Omega_1 = 2\pi/10$, and $x[n] = 0.5e^{j2\Omega_1 n} + 0.5e^{-j2\Omega_1 n} = 0.5e^{j\Omega_2 n} + 0.5e^{-j\Omega_2 n}$
so $A_{-2} = 0.5$ and $A_2 = 0.5$ and $A_k = 0$ for other k.

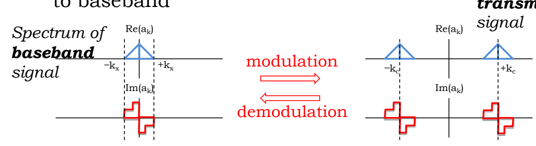
From Baseband to Modulated Signal, and Back



Using Some Piece of the Spectrum

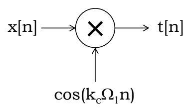
- You have: a band-limited signal x[n] at *baseband* (i.e., centered around 0 frequency).
- You want: the same signal, but centered around some specific frequency $k_c\Omega_1$.
- Modulation: convert from baseband up to $k_c\Omega_1$.
- Demodulation: convert from $k_c\Omega_1$ down to baseband

Spectrum of ***transmitted*** *signal*



Signal centered at 0 Signal centered at k_c

Modulation

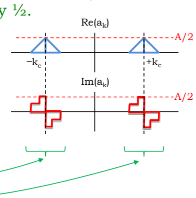


For band-limited signal A_k are nonzero only for small range of $\pm k_x$

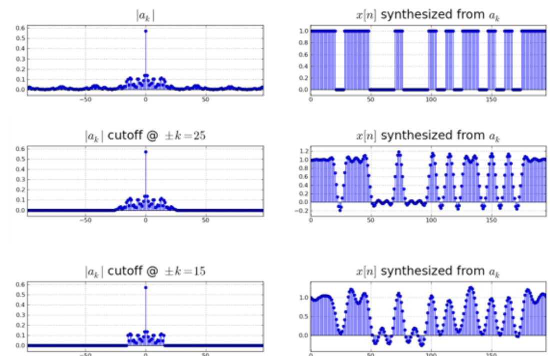
i.e., just replicate baseband signal at $\pm k_c$, and scale by ½.

$$t[n] = \left[\sum_{k=-k_x}^{k_x} A_k e^{jk\Omega_1 n}\right]\left[\frac{1}{2}e^{jk_c\Omega_1 n} + \frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

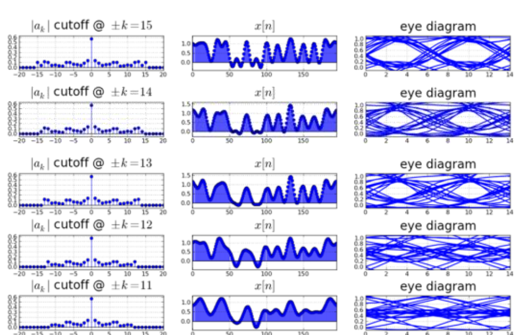
$$= \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k+k_c)\Omega_1 n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k-k_c)\Omega_1 n}$$



Effect of Band-limiting a Transmission



How Low Can We Go?

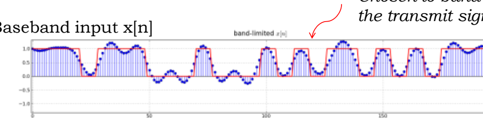


7 samples/bit → 14 samples/period → k=(N/14)=(196/14)=14

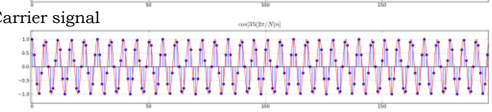
Example: Modulation (time)

Shaped pulses! Chosen to band-limit the transmit signal

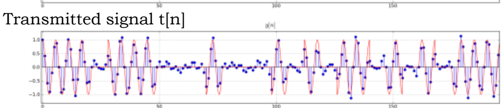
Baseband input x[n]



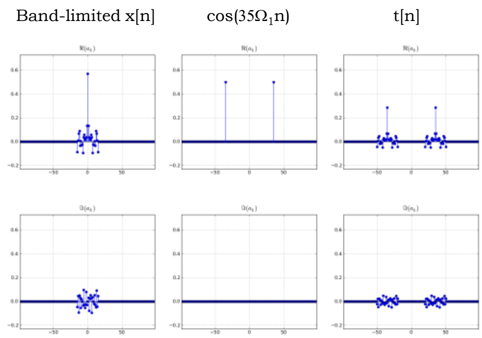
Carrier signal



Transmitted signal t[n]

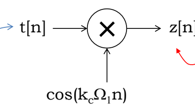


Example: Modulation (freq domain picture)



Demodulation

Assuming no distortion or noise on channel, so what was transmitted is received



Hmm. So z[n] has what we want at baseband, but has signal we don't want at $\pm 2k_c\Omega_1$

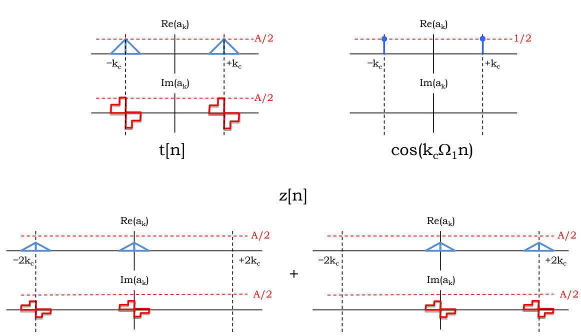
$$z[n]=t[n]\left[\frac{1}{2}e^{jk_c\Omega_1 n}+\frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

$$=\left[\frac{1}{2}\sum_{k=-k_x}^{k_x}A_k e^{j(k+k_c)\Omega_1 n}+\frac{1}{2}\sum_{k=-k_x}^{k_x}A_k e^{j(k-k_c)\Omega_1 n}\right]\left[\frac{1}{2}e^{jk_c\Omega_1 n}+\frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

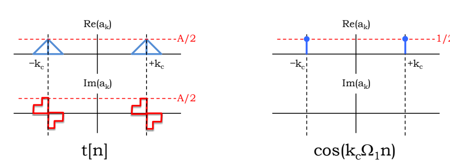
$$=\frac{1}{4}\sum_{k=-k_x}^{k_x}A_k e^{j(k+2k_c)\Omega_1 n}+\frac{1}{2}\sum_{k=-k_x}^{k_x}A_k e^{jk\Omega_1 n}+\frac{1}{4}\sum_{k=-k_x}^{k_x}A_k e^{j(k-2k_c)\Omega_1 n}$$

What we want

Demodulation Frequency Diagram

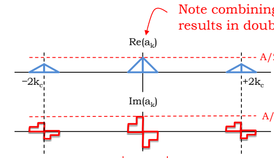


Demodulation Frequency Diagram



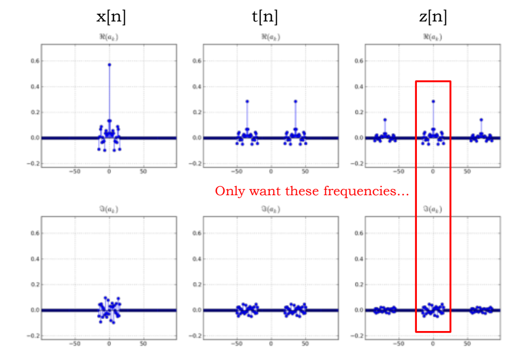
Note combining of signals around 0 results in doubling of amplitude

z[n]

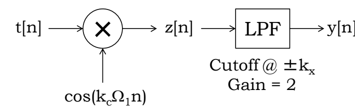


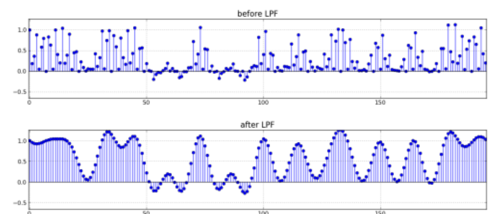
What we want

Example: Demodulation (freq)

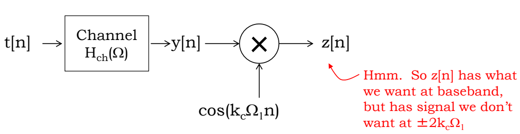


Demodulation + LPF





Demodulation



$$z[n] = y[n]\left[\frac{1}{2}e^{jk_c\Omega_1 n} + \frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \left[\frac{1}{2}\sum_{k=-k_x}^{k_x} H_{ch}((k+k_c)\Omega_1)A_k e^{j(k+k_c)\Omega_1 n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} H_{ch}((k-k_c)\Omega_1)A_k e^{j(k-k_c)\Omega_1 n}\right]\left[\frac{1}{2}e^{jk_c\Omega_1 n} + \frac{1}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \frac{1}{4}\sum_{k=-k_x}^{k_x} H_{ch}((k+k_c)\Omega_1)A_k e^{j(k+2k_c)\Omega_1 n} + \frac{1}{4}\sum_{k=-k_x}^{k_x} H_{ch}((k-k_c)\Omega_1)A_k e^{j(k-2k_c)\Omega_1 n} +$$

$$+ \underbrace{\frac{1}{4}\sum_{k=-k_x}^{k_x} (H_{ch}((k+k_c)\Omega_1) + H_{ch}((k-k_c)\Omega_1))A_k e^{jk\Omega_1 n}}_{\text{What we want}}$$

What we want