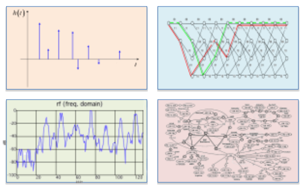
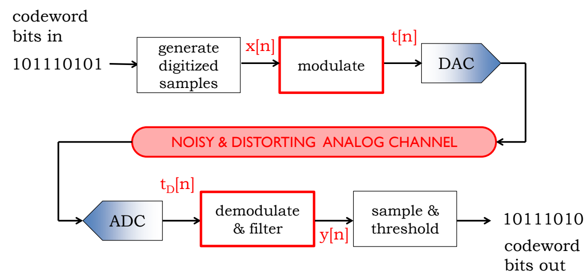


INTRODUCTION TO EECS II
DIGITAL COMMUNICATION SYSTEMS

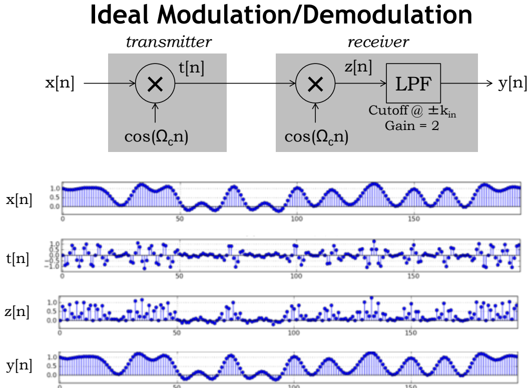
6.02 Fall 2011
Lecture #16

- More on modulation and demodulation, FDM
- Effects of phase errors and channel delays
- Quadrature demodulation and more advanced modulation formats

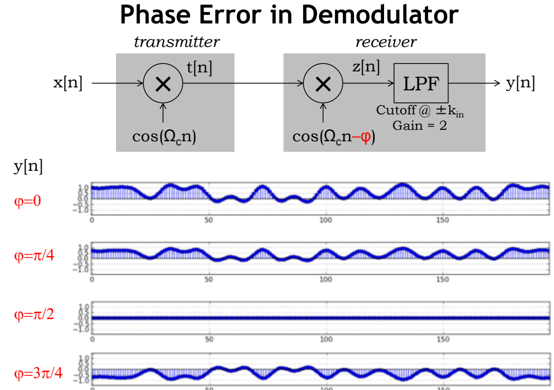
From Baseband to Modulated Signal, and Back



Ideal Modulation/Demodulation



Phase Error in Demodulator



Phase Error Math

Let's derive an equation for z[n]:

$$z[n] = t[n]\cos(\Omega_c n - \varphi) = x[n]\cos(\Omega_c n)\cos(\Omega_c n - \varphi)$$

But

$$\cos(\Omega_c n)\cos(\Omega_c n - \varphi) = 0.5(\cos(2\Omega_c n - \varphi) + \cos(\varphi))$$

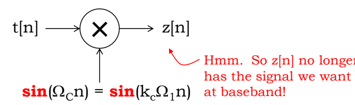
It follows that the demodulated output, after the LPF of gain 2 and cutoff frequency< $2\Omega_c$, is

$$y[n] = x[n]\cos(\varphi)$$

So a phase error of φ results in amplitude scaling by cos(φ).

Note: in the extreme case where φ=π/2, we are demodulating by a sine rather than a cosine, and we get y[n]=0 .

Demodulation with sin($k_c\Omega_1$n)



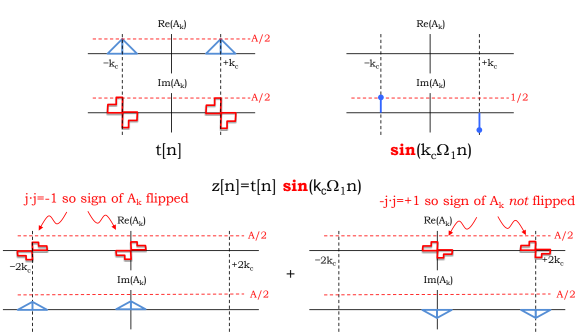
$$z[n] = t[n]\left[-\frac{j}{2}e^{jk_c\Omega_1 n} + \frac{j}{2}e^{-jk_c\Omega_1 n}\right]$$

$$= \left[\frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k+k_c)\Omega_1 n} + \frac{1}{2}\sum_{k=-k_x}^{k_x} A_k e^{j(k-k_c)\Omega_1 n}\right]\left[-\frac{j}{2}e^{jk_c\Omega_1 n} + \frac{j}{2}e^{-jk_c\Omega_1 n}\right]$$

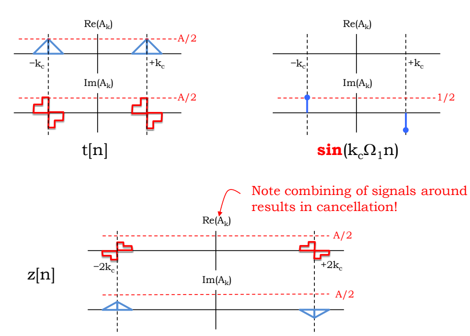
$$= -\frac{j}{4}\sum_{k=-k_x}^{k_x} A_k e^{j(k+2k_c)\Omega_1 n} + \frac{j}{4}\sum_{k=-k_x}^{k_x} A_k e^{j(k-2k_c)\Omega_1 n}$$

Oops, no baseband signal!

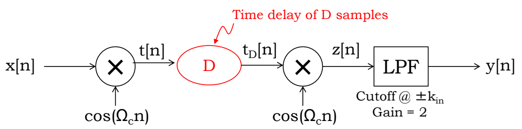
Demodulation (sin) Frequency Diagram



Demodulation (sin) Frequency Diagram



Channel Delay



Very similar math to the previous "phase error" case:

$$z[n] = t_D[n]\cos(\Omega_c n) = t[n-D]\cos(\Omega_c n)$$
$$= x[n-D]\cos[\Omega_c(n-D)]\cos(\Omega_c n)$$
$$= 0.5x[n-D]\cos(2\Omega_c n - \Omega_c D) + 0.5x[n-D]\cos(\Omega_c D)$$

Passing this through the LPF:

$$y[n] = x[n-D]\cos(\Omega_c D)$$

Looks like a phase error of Ω_cD

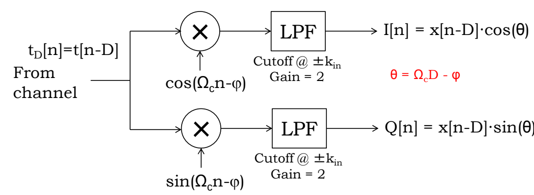
So a channel delay of *D* results in amplitude scaling by $\cos(\Omega_c D)$

Fixing Phase Problems in the Receiver

So phase errors and channel delay both result in a scaling of the output amplitude, where the magnitude of the scaling can't necessarily be determined at system design time:

- Channel delay varies on mobile devices
- Phase difference between transmitter and receiver is arbitrary

One solution: *quadrature demodulation*



Quadrature Demodulation

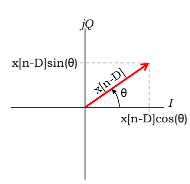
If we let

$$y[n] = I[n] + jQ[n]$$

then

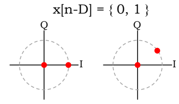
$$|y[n]| = \sqrt{I[n]^2 + Q[n]^2}$$
$$= |x[n-D]|\sqrt{\cos^2\theta + \sin^2\theta}$$
$$= |x[n-D]|$$

OK for recovering x[n] if it never goes negative, as in on-off keying

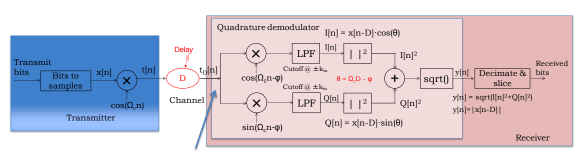


Constellation diagrams (bit decimated x[n-D]):

x[n-D] = { 0, 1 }

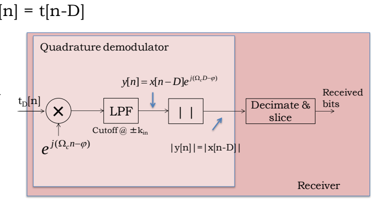


Full system view

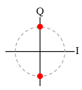


t_D[n] = t[n-D]

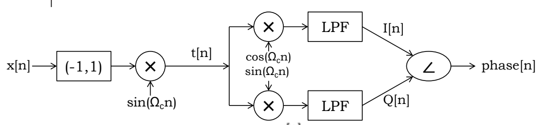
Alternative representation

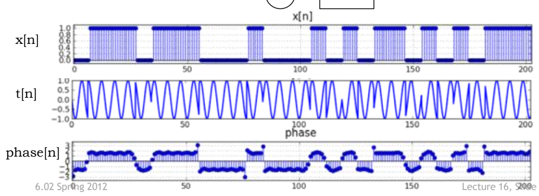


BPSK

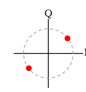


In binary phase-shift keying (BPSK), the message bit selects one of two phases for the carrier, e.g., π/2 for 0 and −π/2 for 1.



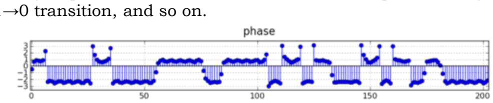


Dealing With Phase Ambiguity



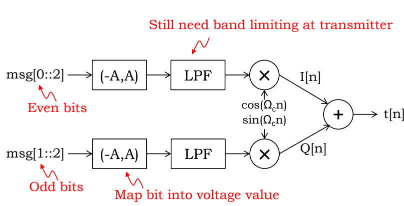
BPSK is also subject phase changes introduced by channel delays or phase difference between xmit and rcv: the received constellation will be rotated with respect to the transmitter's constellation. Which phase corresponds to which bit?

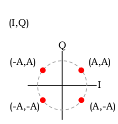
The fix? Think of the phase encoding as *differential*, not absolute: a change in phase corresponds to a change in bit value. Assume that, by convention, messages start with a single 0 bit, i.e., prepend a 0 to each to message. Then the first phase change represents a 0→1 transition, the second phase change a 1→0 transition, and so on.



QPSK Modulation

We can use the quadrature scheme at the transmitter too:

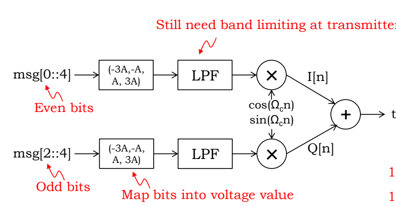


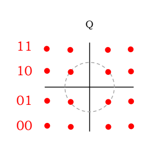


When mapping bits to voltage values, we should choose the values so that the maximum amplitude of t[n] is 1. For QPSK (also referred to as QAM-4) that would mean $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) = (.707,.707)$

QAM Modulation

Use more levels in each arm (e.g. 4 levels per arm – 16QAM):





Symbol/bits mapping table
00 -> -3A
01 -> -A
11 -> A
10 -> 3A
Gray Code (noise movement into another constellation point only causes single bit errors)

From Baseband to Modulated Signal, and Back