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6.02 **Solutions to Chapter 15** Updated: April 22, 2012

Please send information about errors or omissions to hari; questions best asked on piazza.

1. (a) False. TDMA should have no collisions ever; the round-robin strategy must account for packet sizes bigger than one slot.
   (b) True. For example, a network with only one backlogged node, or a skewed workload where only a small number of nodes are backlogged.
   (c) True (mostly). Each colloding node multiplicatively halves its transmission probability, unless they are each at p_min already. If so, the retries will not experience a lower probability.
   (d) False. A skewed workload may have a higher utilizaion.
   (e) False. The probabilities vary, doubling (say) on success and halving on a collision; they don't converge to any specific value.
   (f) False. CSMA in general has fewer collisions but does not eliminate them.
2. (a) To prevent unfairness caused by some nodes being starved.
   (b) To prevent the capture effect, in which one node dominates the medium for several packets in a row.
   (c) The utilization will be (much) lower than the theoretical maximum (e.g., much lower than $1/e$ for slotted Aloha). The reason is that the collision rate will be very high.
3. (a) Let Ben's transmission probability be p_b. Then, Alyssa's transmission probability, $p_a = 3p_b/2$. The utilization, $U = p_a(1-p_b) + p_b(1-p_a)$ because exactly one node should send for a successful outcome. Plugging in $p_a = 3p_b/2$, and taking the derivative of u wrt p_b, setting it to 0, and solving for p_b, we get $p_b = 5/12$ and $p_a = 5/8$.
   (b) The maximum utilization is 25/48.
4. Let the transmission probability of node B be q. Then, the utilization,

$$U = p(1-q) + q(1-p).$$

   The throughput of B is $q(1-p)$, which should be equal to $2p(1-q)$. This means that B's transmission probability, $q = \frac{2p}{1+p}$.
5. (a) True; e.g., if one node is backlogged and the others aren't, Aloha's utilization will be 1, while TDMA will be $1/N$.
   (b) False; $U = 3 \cdot (1/3) \cdot (1-1/3)^2 = 4/9 \neq 1/e$.
   (c) False; TDMA can achieve 100% uilization, but slotted Aloha will converge to an expected value of $1/e$.
   (d) False; contention windows guarantee a transmission *attempt* within a bounded time, but there's no guarantee of success.

6. (a) $U = p(1-2p)^2 + 2(2p)(1-p)(1-2p) = 0.384$.
   (b) Note that $p < 0.5$ because p_B and p_C must be smaller than 1.

$$U = p(1-2p)^2 + 2(2p)(1-p(1-2p) = 5p - 16p^2 + 12p^3.$$

   The maximum value of U is 0.456, occurring when $p = 0.202$ (the other extremum of 0.687 is not valid because $p < 0.5$.

7. See PSet.

8. See PSet.

9. (a) All four nodes positioned so they can hear each other perfectly.
   (b) A and B can hear each other well; A and C can hear each other well; B and D can hear each other well. But D can't hear A and C can't hear B. A now sends to C and B to D. This is also called an "exposed terminal" situation.

10. See PSet.

11. See PSet.

12. See PSet.

13. (a) Protocol B. Eager's throughput with Protocol A is $2p(1-p)^{N+1}$. With Protocol B, his throughput is $2p(1-p)^N$, which is bigger than the expression for the throughput of Protocol A because $0 < 1-p < 1$. The reason is that with Protocol A, his two radios "interact" with each other and sometimes can both collide, which does not occur with Protocol B.
    (b) Protocol A, not B! The reason is that each of Eager's radios operates under the same rules as all the other ones, and he has two radios. More specifically, the throughput of any other device on the network in Protocol A is equal to $p(1-p)^{N+1}$, which is one-half of Eager's throughput in Protocol A. In Protocol B, however, any other devices get a throughput equal to $p(1-p)^{N-1}(1-2p)$, which is smaller than one-half of Eager's throughput.

14. (a) The utilization is $p(1-2p) + 2p(1-p) = 3p - 4p^2$.
    (b) The utilization is maximized for $p = \frac{3}{8}$. The maximum utilization is $\frac{9}{16}$.
    (c) We want

$$2p(1-p) = 3p(1-2p),$$

    which implies that $p = \frac{1}{4}$. The maximum utilization is $2p(1-p) + p(1-2p) = \frac{1}{2}$.

15. (a) A, of course, for its contention window is half as large.
    (b) See PSet (or #12, for a generalization of this question).
    (c) This is the probability that A picks a number less than the one picked by B, *given* that they don't pick the same number. The number of ways of **not** picking the same number is $2W^2 - W = W(2W-1)$.

    The number of ways for A to pick a number less than the one picked by B is:

0 if B picks 1; 1 if B picks 2; 2 if B picks 3; ... ,W if B picks $W+1$; and stays at W if B picks $W+k$ for $k = 2, ..., W$. So the total number of ways is

$$(1 + 2 + \cdots + W) + W(W - 1) = W(3W - 1)/2 \ .$$

The desired probability is the ratio of the two numbers we have computed, i.e.,

$$[W(3W - 1)/2]/[W(2W - 1)] = (3W - 1)/[2(2W - 1)] \ .$$

(d) The calculation here is essentially the same as the previous part, *except* that the denominator is the set of *all* possible outcomes, *without* conditioning on there being no collision at the next packet transmission. So the denominator now is just $2W^2$ rather than $W(2W - 1)$, and the desired probability is

$$[W(3W - 1)/2]/(2W^2) = (3W - 1)/(4W) \ .$$

(e) Because we don't account for repeated collisions and retries. That calculation would be more involved.

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