Name: ____

Department of Electrical Engineering and Computer Science

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.02 Fall 2010

Quiz II

November 2, 2010

"×" your section	Section	Time	Room	Recitation Instructor
☐	1	10-11	36-112	Tania Khanna
☐	2	11-12	36-112	Tania Khanna
☐	3	12-1	36-112	George Verghese
☐	4	1-2	36-112	George Verghese
☐	5	2-3	26-168	Alexandre Megretski
☐	6	3-4	26-168	Alexandre Megretski

There are **17 questions** (some with multiple parts) and **10 pages** in this quiz booklet. Answer each question according to the instructions given. You have **120 minutes** to answer the questions.

If you find a question ambiguous, please be sure to write down any assumptions you make. **Please be neat and legible.** If we can't understand your answer, we can't give you credit! *And please show your work for partial credit.*

Use the empty sides of this booklet if you need scratch space. You may also use them for answers, although you shouldn't need to. *If you use the blank sides for answers, make sure to say so!*

Please write your name CLEARLY in the space at the top of this page. NOW, please!

One two-sided "crib sheet" allowed. No other notes, books, calculators, computers, cell phones, PDAs, information appliances, carrier pigeons carrying messages, etc.!

Do not write in the boxes below

1-2 (x/18)	3-5 (x/14)	6-9 (x/18)	10-12 (x/15)	13-14 (x/15)	15-17 (x/20)	Total (x/100)

I Linear Block Codes

1. [6 points]: For each of the three codes below, circle whether the code is a linear block code over $\mathbb{F}_2$ or not. Also fill in the rate of the code.

A. $\{111, 100, 001, 010\}$. Linear / Not linear. Code rate = ____________.

B. $\{00000, 01111, 10100, 11011\}$. Linear / Not linear. Code rate = ____________.

C. $\{00000\}$. Linear / Not linear. Code rate = ____________.

2. [12 points]: Recall that a block code takes a set of k-bit messages and produces n-bit codewords, with a minimum Hamming distance of d between any two codewords. For each (n, k, d) combination below, state whether a linear block code with those parameters exists or not. *Please provide a brief explanation for each case*: **if such a code exists, give an example; if not, you may rely on a suitable necessary condition.**

A. $(31, 26, 3)$: **Yes / No** (circle one)

B. $(32, 27, 3)$: **Yes / No** (circle one)

C. $(43, 42, 2)$: **Yes / No** (circle one)

D. $(27, 18, 3)$: **Yes / No** (circle one)

E. $(11, 5, 5)$: **Yes / No** (circle one)

II "Pairwise" Codes

Pairwise Communications has developed a linear block code over $\mathbb{F}_2$ with three data and three parity bits:

$$\begin{aligned} P_1 &= D_1 + D_2 \qquad \text{(Each } D_i \text{ is a data bit; each } P_i \text{ is a parity bit.)} \\ P_2 &= D_2 + D_3 \\ P_3 &= D_3 + D_1 \end{aligned}$$

3. [4 points]: Fill in the values of the following three attributes of this code:

1. Code rate = ______________________.

2. Number of 1s in a minimum-weight non-zero codeword = ______. (Explain your answer.)
   Note: The *weight* of a codeword is the number of 1s in it.

3. Minimum Hamming distance of code = ______________.

4. [6 points]: The receiver computes three syndrome bits from the (possibly corrupted) received data and parity bits: $E_1 = D_1 + D_2 + P_1, E_2 = D_2 + D_3 + P_2$, and $E_3 = D_3 + D_1 + P_3$. The receiver performs maximum likelihood decoding using the syndrome bits. For the combinations of syndrome bits in the table below, state what the maximum-likelihood decoder believes has occured: no errors, a single error in a specific bit (state which one), or multiple errors.

$E_3 E_2 E_1$	**Error pattern** ["No errors" / "Error in bit ..." (specify the bit) / "Multiple errors"]
0 0 0	
0 1 0	
1 0 1	
1 1 1	

5. [4 points]: Alyssa P. Hacker extends Pairwise's code by adding an *overall parity bit*. That is, she computes $P_4 = \sum_{i=1}^{3}(D_i + P_i)$, and appends P_4 to each original codeword to produce the new set of codewords. What improvement in error correction or detection capabilities, if any, does Alyssa's extended code show over Pairwise's original code? **(Explain your answer in the space below.)**

III Convolutional Coding

Consider a convolutional code whose parity equations are

$$\begin{aligned} p_0[n] &= x[n] + x[n-1] + x[n-3] \\ p_1[n] &= x[n] + x[n-1] + x[n-2] \\ p_2[n] &= x[n] + x[n-2] + x[n-3] \end{aligned}$$

6. [3 points]: What is the rate of this code? How many states are in the state machine representation of this code as discussed in 6.02?

Code rate =

Number of states in the state machine representation =

7. [7 points]: Suppose the decoder reaches the state "110" during the forward pass of the Viterbi algorithm with this convolutional code.

A. How many predecessor states (i.e., immediately preceding states) does state "110" have?

B. What are the bit-sequence representations of the predecessor states of state "110"?

C. What are the expected parity bits for the transitions from each of these predecessor states to state "110"? Specify each predecessor state and the expected parity bits associated with the corresponding transition below.

Predecessor state → "110"	Expected parity bits

Suppose we want to send a message M using a convolutional code over some channel. Let P be the sequence of parity bits produced by the encoder using the convolutional code. The receiver gets a sequence of voltage samples. It digitizes each sample to produce a sequence of received parity bits, R. It decodes this sequence using the Viterbi decoder with the Hamming distance between the expected and received parity bits as the branch metric. Let D denote the decoder's final result (i.e., the message output by the decoder). We find that the minimum path metric among all the states in the final stage of the trellis is 2010.

8. [5 points]: Please answer the following questions.

A. The Hamming distance between _____________ and _____________ is 2010.

B. Suppose the final state from which the Viterbi decoder begins its backward pass has the bit representation $b_1 b_2 \dots b_\ell$, where ℓ is the number of bits required to represent all the states.
Then, D *must* **start / end** with the bit sequence __________________________.
(*Circle "start" or "end" and fill in the blank so that this sentence is always true.*)

9. [3 points]: To increase the rate of the given code, Lem E. Tweakit punctures the p_0 parity stream using the vector (1 0 1 1 0), which means that every second and fifth bit produced on the stream are *not sent*. In addition, she punctures the p_1 parity stream using the vector (1 1 0 1 1). She sends the p_2 parity stream unchanged. What is the rate of the punctured code?

(Explain your answer in the space below.)

IV Time Division Duplex (TDD) MAC Protocol

Bluetooth is a wireless technology found on many mobile devices, including laptops, mobile phones, GPS navigation devices, headsets, and so on. It uses a MAC protocol called *Time Division Duplex (TDD)*. In TDD, the shared medium network has 1 master and N slaves. You may assume that the network has already been configured with one device as the master and the others as slaves. Each slave has a unique identifier (ID) that serves as its address, an integer between 1 and N. Assume that no devices ever turn off during the operation of the protocol. Unless otherwise mentioned, assume that no packets are lost.

The MAC protocol works as follows. Time is slotted and each packet is one time slot long.

- In every *odd* time slot $(1, 3, 5, \ldots, 2t-1, \ldots)$, the master sends a packet addressed to *some* slave for which it has packets backlogged, in round-robin order (i.e., cycling through the slaves in numeric order).
- In every *even* time slot $(2, 4, 6, \ldots, 2t, \ldots)$, the slave that received a packet from the master in the immediately preceding time slot gets to send a packet to the master, if it has a packet to send. If it has no packet to send, then that time slot is left unused, and the slot is wasted.

10. [3 points]: Alyssa P. Hacker finds a problem with the TDD protocol described above, and implements the following rule in addition:

> From time to time, in an odd time slot, the master sends a "dummy" packet addressed to a slave even if it has no other data packets to send to the slave (and even if it has packets for other slaves).

Why does Alyssa's rule improve the TDD protocol?

(Explain your answer in the space below.)

Henceforth, the term "TDD" will refer to the protocol described above, augmented with Alyssa's rule. Moreover, whenever a "dummy" packet is sent, that time slot will be considered a <u>*wasted*</u> *slot.*

11. [3 points]: Alyssa's goal is to emulate a round-robin TDMA scheme amongst the N slaves. Propose a way to achieve this goal by specifying the ID of the slave that the master should send a data or dummy packet to, in time slot $2t-1$ (note that $1 \le t < \infty$).

(Explain your answer in the space below.)

Henceforth, assume that the TDD scheme implements round-robin TDMA amongst the slaves.

12. [4+5=9 points]: Suppose the master always has data packets to send *only* to an arbitrary (but fixed) subset of the N slaves. In addition, a (possibly different) subset of the slaves always has packets to send to the master. Each subset is of size r, a fixed value. Answer the questions below (you may find it helpful to think about different subsets of slaves).

A. What is the *maximum possible utilization* of such a configuration?

(Explain your answer in the space below.)

B. What is the *minimum possible utilization* (for a given value of r) of such a configuration? Assume that $r > N/2$. Note that if the master does not have a data packet to send to a slave in a round, it sends a "dummy" packet to that slave instead. (A dummy packet does not count toward the utilization of the medium.)

(Explain your answer in the space below.)

V Contention Protocols

13. [5 points]: Recall the MAC protocol with contention windows. Here, each node maintains a contention window, W, and sends a packet t idle time slots after the current slot, where t is an integer picked uniformly in $[1, W]$. Assume that each packet is 1 slot long, and that $W_1 \geq W_2$.

Suppose there are two backlogged nodes in the network with contention windows W_1 and W_2, respectively. What is the probability that the two nodes will collide the next time they each transmit?

(Explain your answer in the space below.)

14. [10 points]: Eager B. Eaver gets a new computer with *two* radios. There are N other devices on the shared medium network to which he connects, but each of the other devices has only one radio. The MAC protocol is slotted Aloha with a packet size equal to 1 time slot. Each device uses a fixed transmission probability, and only one packet can be sent successfully in any time slot. All devices are backlogged.

Eager persuades you that because he has paid for two radios, his computer has a moral right to get twice the throughput of any other device in the network. You begrudgingly agree.

Eager develops two protocols:

Protocol A: Each radio on Eager's computer runs its MAC protocol independently. That is, each radio sends a packet with fixed probability p. Each other device on the network sends a packet with probability p as well.

Protocol B: Eager's computer runs a single MAC protocol across its two radios, sending packets with probability $2p$, and alternating transmissions between the two radios. Each other device on the network sends a packet with probability p.

A. With which protocol, A or B, will Eager achieve higher throughput?

(Explain your answer in the space below.)

B. Which of the two protocols would you allow Eager to use on the network so that his expected throughput is double any other device's? **(Explain your answer in the space below.)**

VI Filters

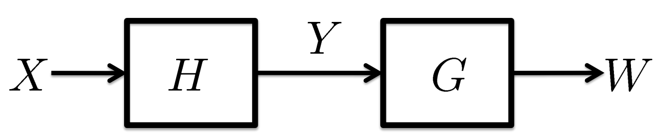
Suppose a causal linear time invariant (LTI) system, denoted H, is described by the difference equation relating input X to output Y,

$$y[n] = x[n] + \alpha x[n-1] + \beta x[n-2] + \gamma x[n-3]. \tag{1}$$

15. [7 points]: Please determine the values of α, β and γ so that the frequency response of system H is $H(e^{j\Omega}) = 1 - 0.5e^{-j2\Omega}\cos\Omega$.

$\alpha =$ ____________________; $\beta =$ ____________________; $\gamma =$ ____________________.

16. [4 points]: Suppose that Y, the output of H, is used as the input to a second causal LTI system G, with output W, as shown below.



If $H(e^{j\Omega}) = 1 - 0.5e^{-j2\Omega}\cos\Omega$, what should the frequency response, $G(e^{j\Omega})$, be so that $w[n] = x[n]$ for all n?

$G(e^{j\Omega}) =$ ________________________________.

The difference equation from the previous page, Eq. (1), is reproduced below for convenience:

$$y[n] = x[n] + \alpha x[n-1] + \beta x[n-2] + \gamma x[n-3].$$

17. [9 points]: Suppose $\alpha = 1$ and $\gamma = 1$ in the above equation for an H with a **different** frequency response than the one you obtained in Problem 15. For this different H, you are told that $y[n] = A\,(-1)^n$ when $x[n] = 1.0 + 0.5\,(-1)^n$ for all n. Please use this information to determine the value of β in Eq. (1) and the value of A in the formula for $y[n]$.

$\beta =$ ____________________________; $A =$ ____________________________.

FIN