Name: ______________________________

DEPARTMENT OF EECS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.02 Spring 2012

Quiz II

April 20, 2012

"×" your section	Section	Time	Recitation Instructor	TA
☐	1	10-11	Vincent Chan	Jared Monnin
☐	2	11-12	Vincent Chan	Anirudh Sivaraman
☐	3	12-1	Sidhant Misra	Sungwon Chung
☐	4	1-2	Sidhant Misra	Omid Aryan / Nathan Lachenmyer
☐	5	2-3	Katrina LaCurts	Sunghyun Park
☐	6	3-4	Katrina LaCurts	Muyiwa Ogunnika

Please read and follow these instructions:

0. Please write your name in the space above and × your section.
1. There are **9 questions** (some with multiple parts) and **14 pages** in this quiz booklet.
2. Answer each question according to the instructions given, within **120 minutes**.
3. **Please answer legibly. Explain your answers, especially when we ask you to.** If you find a question ambiguous, write down your assumptions. Show your work for partial credit.
4. Use the empty sides of this booklet if you need scratch space. *If you use the blank sides for answers, make sure to say so!*

Two two-sided "crib sheets" and a calculator allowed. No other aids.

PLEASE NOTE: PLEASE DON'T DISCUSS THIS QUIZ WITH ANYONE IN THE CLASS, UNLESS YOU'RE SURE THEY HAVE TAKEN IT WITH YOU TODAY.

Do not write in the boxes below

1-2(x/20)	3-4 (x/30)	5-6 (x/30)	7-9 (x/20)	Total(x/100)

I Analyzing Linear Time Invariant Systems in Time Domain

1. [2 + 2 = 4 points]: A *line-of-sight* channel can be represented as an LTI system with a unit sample response:

$$h[n] = a\delta[n - M] \ ,$$

where M is the channel delay, and $M > 0$.

A. (2 points) Is this channel causal (Yes/No)? Explain.

B. (2 points) Write an expression for the unit step response $s[n]$ of this system.

2. [3 + (4 + 3) + 6 = 16 points]: A *channel with echo* can be represented as an LTI system with a unit sample response:

$$h[n] = a\delta[n - M] + b\delta[n - N] \ , \tag{1}$$

where M is the channel delay, N is the echo delay, and $N > M$.

A. (3 points) Derive the unit step response $s[n]$ of this channel with echo.

B. Two such channels, with unit sample responses

$$h_1[n] = \delta[n] + 0.1\delta[n-2] \quad \text{and} \quad h_2[n] = \delta[n-1] + 0.2\delta[n-2],$$

are cascaded in **series**.

(a) (4 points) Derive the unit **sample** response $h_{12}[n]$ of the cascaded system.

(b) (3 points) Derive the unit **step** response $s_{12}[n]$ of the cascaded system.

C. (6 points) The transmitter maps each bit to N_b samples using bipolar signaling (bit **0** maps to N_b samples of value -1, and bit **1** maps to N_b samples of value $+1$). The mapped samples are sent over the *channel with echo*, with unit sample response given by **Equation (1)** on the previous page, with $a > b > 0$, $N_b = 4$, $N = N_b$, and $M = 0$.

Sketch the output of the channel for the input bit sequence **01**. The initial condition before the **01** bit sequence is that the input to the channel was a long stream of zeroes. Clearly mark the signal levels on the y-axis, as well as sample indices on the x-axis.

II Analyzing Linear Time Invariant Systems in Frequency Domain

3. [3 + 4 + (2 + 2 + 3) = 14 points]: A *channel with echo* can be represented as an LTI system with a unit sample response:

$$h[n] = a\delta[n-M] + b\delta[n-M-kN_b] \ ,$$

where a is a positive real constant, M is the channel delay, k is an integer greater than 0, N_b is the number of samples per bit, and $M + kN_b$ is the echo delay.

A. (3 points) Derive the expression for the frequency response of this channel, $H(\Omega)$, as a function of a, b, M, k, and N_b.

B. (4 points) If $b = -a$, $k = 1$, and $N_b = 4$, find the values of $\Omega \in [-\pi, +\pi]$ for which $H(\Omega) = 0$.

C. For $b = -a$, $M = 4$, and $kN_b = 12$, derive the expression for the output of the channel (time sequence $y[n]$) for the input $x[n]$ when

(a) (2 points) $x[n] = 1$, for all n.

(b) (2 points) $x[n] = 2\cos(\frac{\pi}{4}n)$, for all n.

(c) (3 points) For $x[n] = 3\sin(\frac{\pi}{8}n + \frac{\pi}{4})$, for all n, circle the correct expression for $y[n]$ among the following choices and explain your answer in the space below:

i. $y[n] = 3\sqrt{2}\sin(\frac{\pi}{8}n + \frac{3\pi}{2})$

ii. $y[n] = 3a\sin(\frac{\pi}{8}n + \frac{\pi}{4})$

iii. $y[n] = 3a\sqrt{2}\sin(\frac{\pi}{8}n - \frac{\pi}{2})$

iv. $y[n] = -3a\sqrt{2}\sin(\frac{\pi}{8}n)$

4. [10 + 4 + 2 = 16 points]: A wireline channel has unit sample response $h_1[n] = e^{-an}$ for $n \geq 0$, and 0 otherwise, where $a > 0$ is a real number.

Ben Bitdiddle is an MIT student who recently got a job at WireSpeed, Inc. Having taken 6.02, he is trying to convince his manager that he can significantly improve the signalling speed (and hence transfer the bits faster) over this wireline channel, by placing a filter with unit sample response

$$h_2[n] = A\delta[n] + B\delta[n - D],$$

at the receiver, so that

$$(h_1 * h_2)[n] = \delta[n].$$

A. (10 points) What values of A, B and D satisfy Ben's goal? Show your work in the space below.

B. (4 points) Suppose $a = 0.1$. Then, does $H_2(\Omega)$ behave like a (1) low-pass filter, (2) high-pass filter, (3) all-pass filter? Explain your answer.

C. (2 points) Under what noise conditions will Ben's idea work reasonably well? Please give a brief, qualitative explantion for your answer in the space below. There's no need to calculate anything here.

III Signal Spectra and Modulation

5. [3 + 2 = 5 points]: Consider an audio channel with a sampling rate of 8000 samples/second.

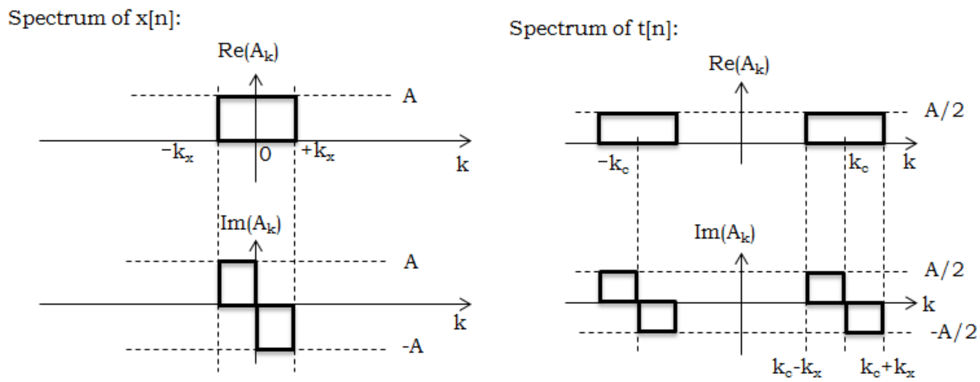
A. (3 points) What is the angular frequency of the piano note $\mathcal{A}$ (in radians/sample), given that its continuous time frequency is 880 Hz?

B. (2 points) What is the smallest number of samples, P, needed to represent the note $\mathcal{A}$ as a spectral component at $\Omega_k = \frac{2\pi}{P}k$, for integer k? And what is the value of k?

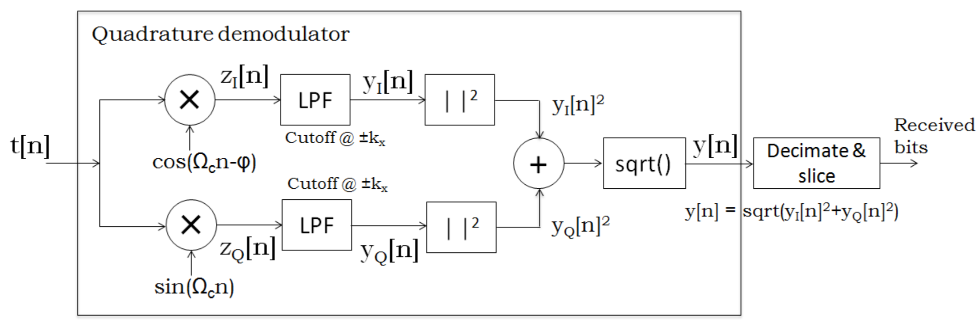
$P =$ __________________.

$k =$ __________________.

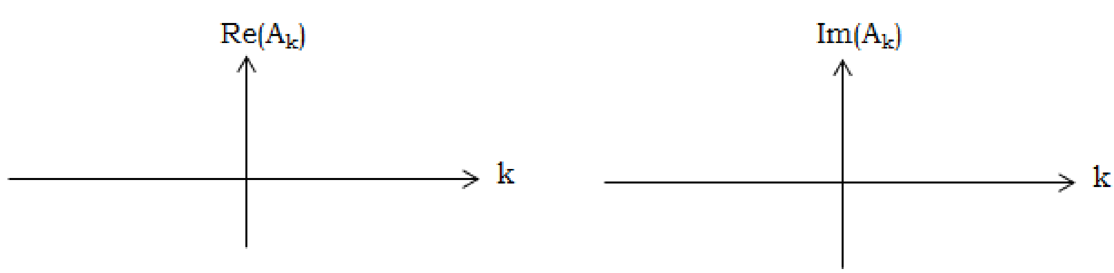
6. [5 + 8 + 3 + 5 + 4 = 25 points]: A baseband signal $x[n]$ has spectra $X(k)$ with nonzero components between $-k_x$ and k_x, as sketched in the figure below (left). At the transmitter, $x[n]$ is modulated over a carrier $\cos(\Omega_c n)$ to produce signal $t[n] = x[n]\cos(\Omega_c n)$, which is transmitted, where $\Omega_c = k_c\Omega_1$, and Ω_1 is the fundamental frequency. For this problem, assume that the received signal is equal to $t[n]$, with a spectrum shown in the figure below (right).



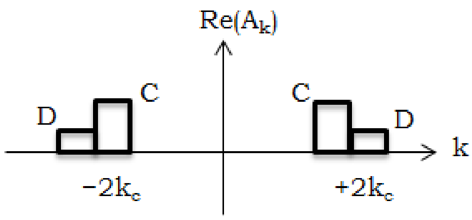
The baseband signal at the transmitter, $x[n]$, is generated using **on/off** signaling. John Bitdiddle (Ben's twin working at AirSpeed, Inc., a wireless communication startup) decides to use a **quadrature demodulator** in his wireless receiver. However, due to a bug in the implementation, his receiver ends up looking like in the figure below, with phase error φ between the $\cos()$ and $\sin()$ branches. Assume that $\varphi \neq 0$, and $\varphi \in \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$.

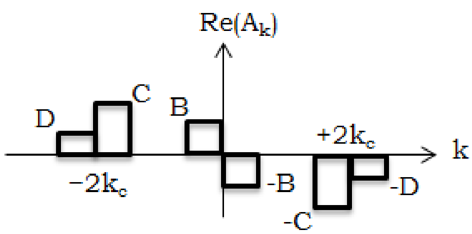


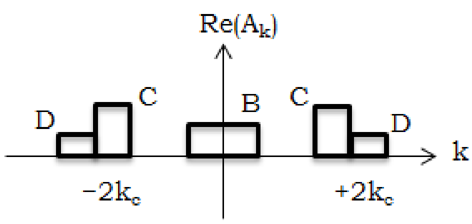
A. (5 points) Draw the real and imaginary parts of the spectrum of $\cos(\Omega_c n - \varphi)$. Clearly label the axes (the k-index and the spectral value) for all nonzero spectral component values.

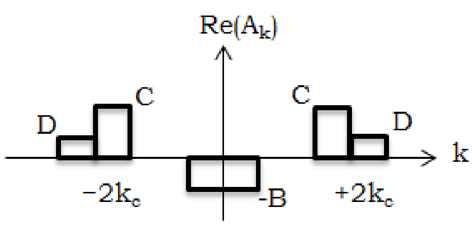


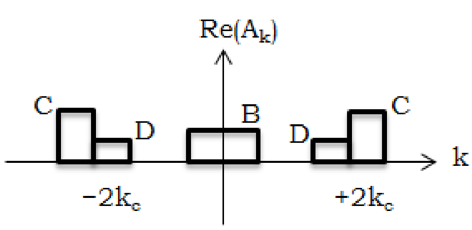
B. (8 points) Circle the plots that represent the real and imaginary parts of the spectrum of $z_I[n]$, from these choices (the left column has choices for the real part and the right column has choices for the imaginary part). The expressions for the amplitude values shown in the pictures below are $B = \frac{A}{2}\cos(\varphi)$, $C = \frac{A}{4}(\cos(\varphi) + \sin(\varphi))$, and $D = \frac{A}{4}(\cos(\varphi) - \sin(\varphi))$.

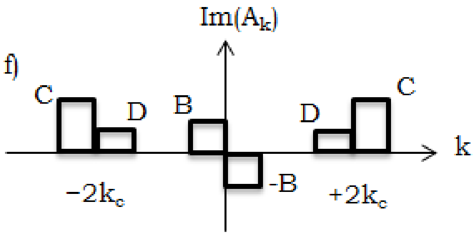
a)


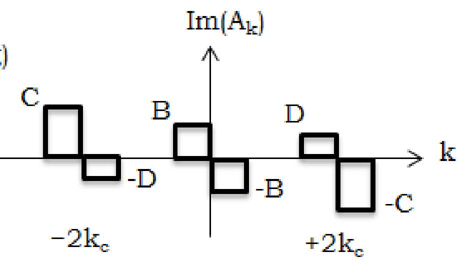
b)


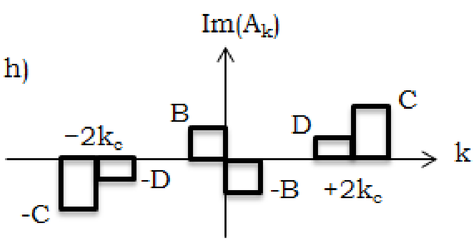
c)


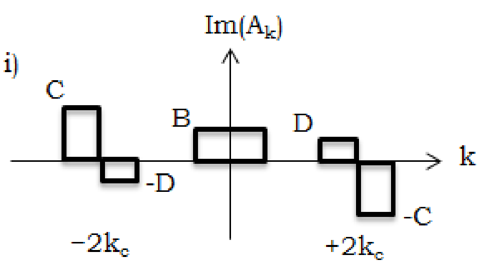
d)


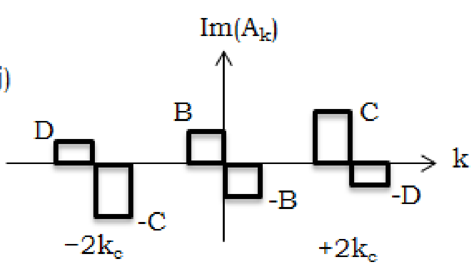
e)


f)


g)


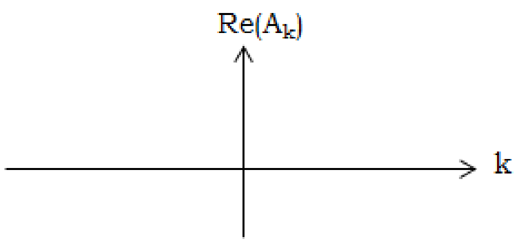
h)


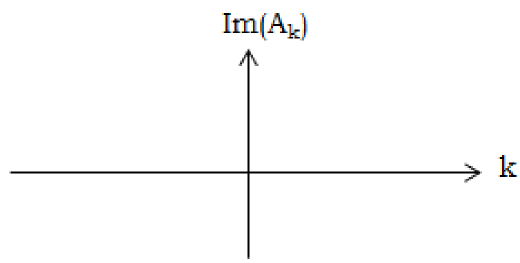
i)


j)


k) None of the above

C. (3 points) Draw the real and imaginary parts of the spectrum of $y_I[n]$. Use the same spectral shapes as used in the figures above for the spectrum of $t[n]$, but clearly mark the new center frequencies and amplitude scaling.





D. (5 points) Derive the expression for $y[n] = \sqrt{y_I[n]^2 + y_Q[n]^2}$ as a function of the baseband sequence $x[n]$ and phase error φ.

E. (4 points) For $\varphi = 0$, find the **minimum** value of k_c that ensures proper transmission and demodulation (by proper, we mean that the spectral content of $x[n]$ is not affected by the modulation and transmission process and $x[n]$ can be reconstructed at the receiver). Explain your answer.

IV Noise

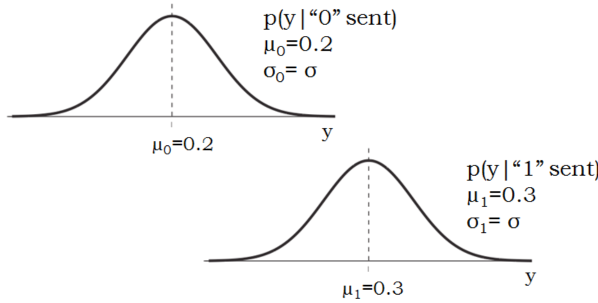
7. [2 + 2 + 2 = 6 points]: Bit samples are transmitted with amplitude $A_{TX} = \pm 1$ (i.e., bipolar signaling). The channel **attenuation** is 20 dB, so the power of any transmitted signal is reduced by this factor when it arrives at the receiver.

A. (2 points) What receiver noise standard deviation value (σ) corresponds to a signal-to-noise ratio (SNR) of 20 dB at the receiver? (Note that the SNR at the receiver is defined as the ratio of the received signal power to σ^2.)

B. (2 points) Express the bit error rate at the receiver in terms of the erfc() function when the SNR at the receiver is 20 dB.

C. (2 points) Under the conditions of the previous parts of this question, suppose an amplifier with gain of 10 dB is added to the receiver *after* the signal has been corrupted with noise. Explain how this amplification affects the bit error rate.

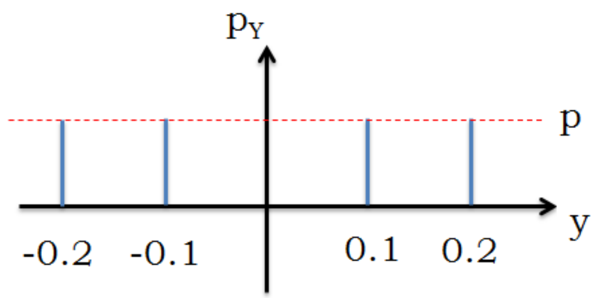
8. [2 + 3 = 5 points]: In a communication system, the received decimated signal samples are shifted by an external interferer so that the signal distributions (including noise) look like:



A. (2 points) Find the optimal receiver threshold if the prior probabilities of sending a "0" and "1" are equal, and the noise standard deviations on sending a "0" and "1" are also equal ($\sigma_0 = \sigma_1$).

B. (3 points) Derive the expression for the bit error rate in terms of the erfc() function.

9. [2 + 2 + 5 = 9 points]: Due to ISI, the received signal distribution (probability mass function) without noise looks like in the diagram below.



A. (2 points) Determine the value p marked on the graph above.

$p =$ ____________________.

B. (2 points) Determine the optimal decision threshold $V_{threshold}$, assuming that the prior probabilities of sending a "0" and a "1", and the noise standard deviations on sending a "0" and a "1" are also equal ($\sigma_0 = \sigma_1$).

C. (5 points) Derive the expression for the bit error rate in terms of the erfc() function if $\sigma = 0.025$.

FIN