 NAME: Sol

6.02 Digital Communication Systems—Spring 2010

Quiz 2 - 7:30-9:30pm (Two Hours)
Thursday, April 8th, 2010

Check your section	Section	Time	Room	Rec. Instr.
☐	1	10-11	13-5101	Lizhong Zheng
☐	2	11-12	13-5101	Lizhong Zheng
☐	3	1-2	38-166	Tania Khanna
☐	4	2-3	38-166	Tania Khanna

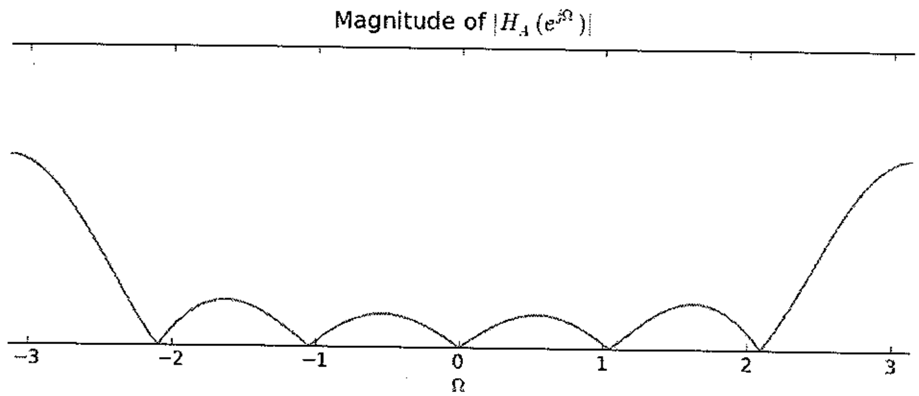
Directions: The exam consists of four problems on 18 pages. Please make sure you have all the pages. **Enter all your work and your answers directly in the spaces provided on the printed pages of this exam. Please make sure your name is on all sheets. DO IT NOW!** All sketches must be adequately labeled. Unless indicated otherwise, **answers must be derived or explained in the space provided**, not just simply written down. This examination is closed book, but students may use one 8 1/2 × 11 sheet of paper for reference. Calculators may not be used.

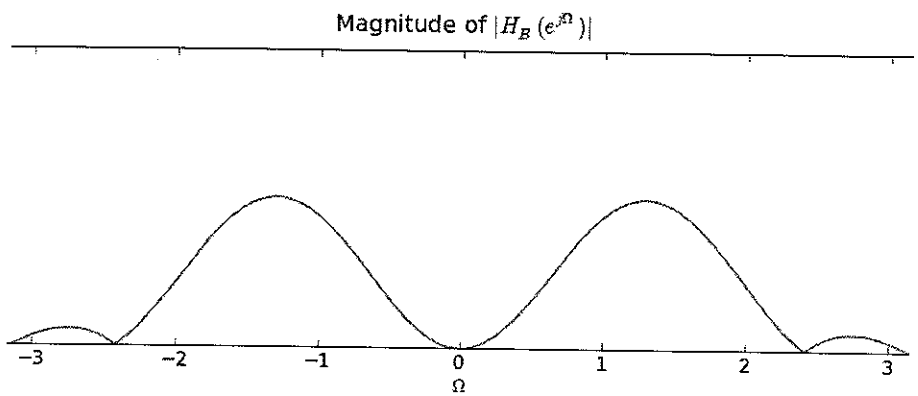
Please leave the rest of this page blank for use by the graders:

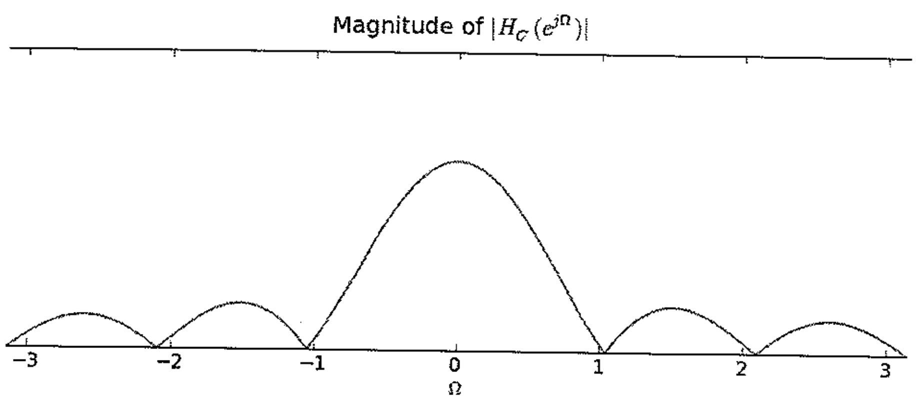
Problem	No. of points	Score	Grader
1	35		
2	10		
3	20		
4	35		
Total	100		

Problem 1. Filters [35 points]

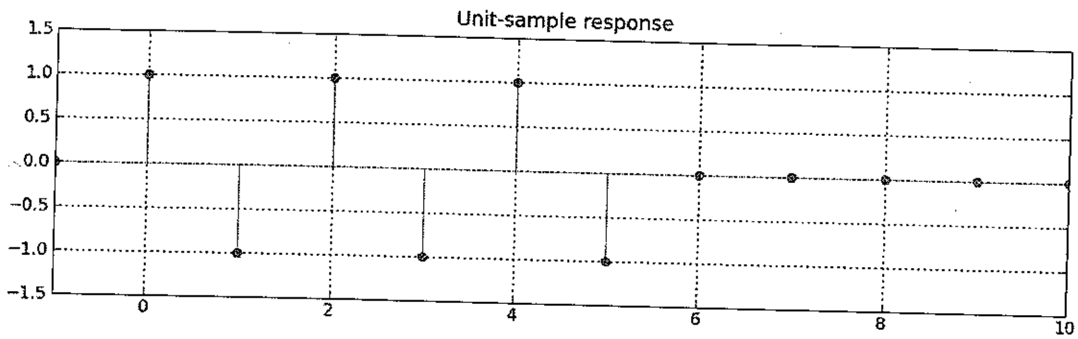
In answering the several parts of this question, consider three linear time-invariant filters, denoted A, B, and C, each characterized by the magnitude of their frequency responses, $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, and $|H_C(e^{j\Omega})|$, respectively, as given in the plots below.







(A) Which frequency response (A, B, or C) corresponds to the following unit sample response, and what is $\max_{\Omega} |H(e^{j\Omega})|$ for your selected filter? Please justify your selection.



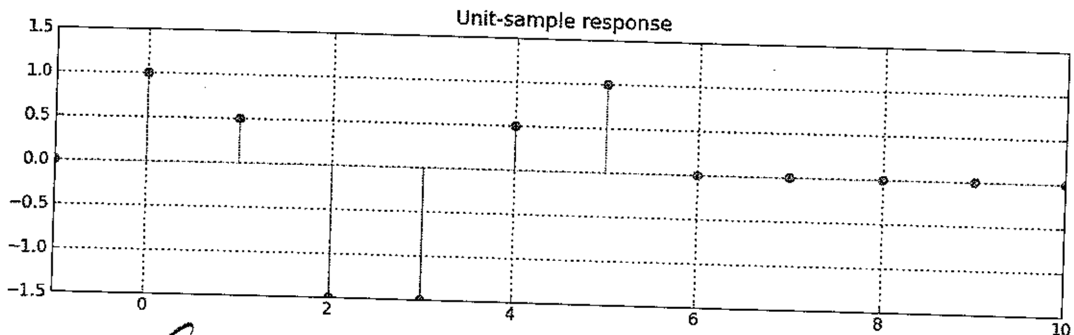
$\sum h[n] = 0 \Rightarrow |H(e^{j0})| = 0$ } Must be A
$|\sum (-1)^n h[n]| = 6 \Rightarrow |H(e^{j\pi})| = 6$

Frequency response plot (A, B, or C) (Be sure to justify your answer) = A

$$|H(e^{j\Omega})| \le \sum |h[n]| = 6 = |H(e^{j\pi})|$$

$\max_{\Omega} |H(e^{j\Omega})| =$ $|H(e^{j\pi})| = 6$

(B) Which frequency response (A, B, or C) corresponds to the following unit sample response, and is the $\max_\Omega |H(e^{j\Omega})| > 6$ for your selected filter? Please justify your answers.



Must be B

$$\begin{cases} \sum h[n] = 0 & H(e^{j0}) = 0 \\ \sum (-1)^n h[n] = \cancel{1} - \cancel{\tfrac{1}{2}} - \cancel{\tfrac{3}{2}} + \cancel{\tfrac{3}{2}} + \cancel{\tfrac{1}{2}} - \cancel{1} = 0 = H(e^{j\pi}) = 0 \end{cases}$$

Frequency response plot (A, B, or C) (Be sure to justify your answer) = B

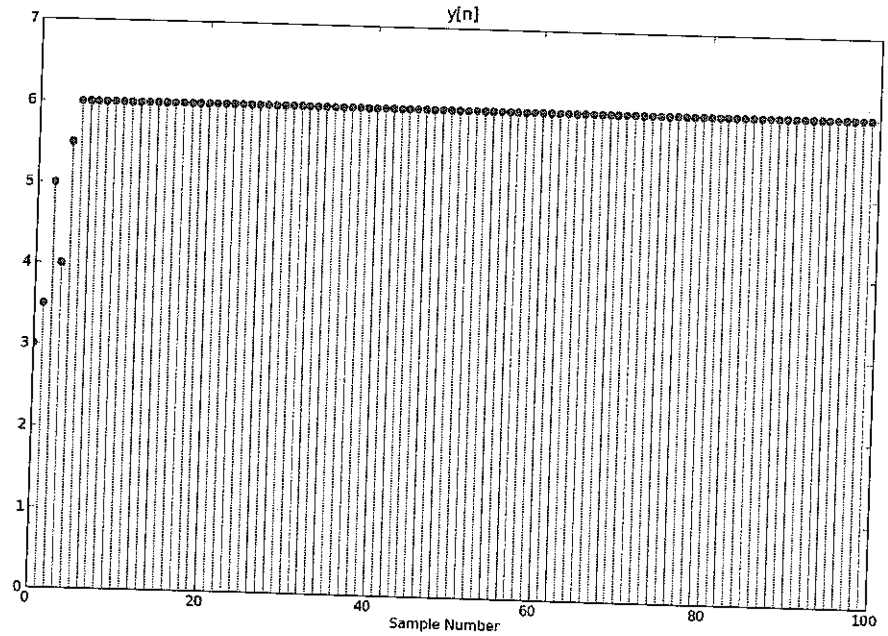
$$|H(e^{j\Omega})| = \left|\sum_{n=0}^{5} h[n] e^{-j\Omega n}\right| \le \sum_{n=0}^{5} |h[n]| \cdot 1 \le 6$$

Is $\max_\Omega |H(e^{j\Omega})| > 6$ (yes or no)? NO ≤ 6!
You must provide some justification to receive credit.

(C) Suppose the input to each of the above three filters is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

$$x[n] = \cos \frac{\pi}{3.0} n + \cos \pi n + 1.0.$$

Which filter (A, B,or C) produced the output, $y[n]$ below, and what is $\max_\Omega |H(e^{j\Omega})|$ for your selected system?



Frequency response plot (A, B, or C) (Be sure to justify your answer) = C

Output is eventually $y[n] = 6$ $(n > 5)$

$H(e^{j0}) = 6$ $H(e^{j\pi/3})$ must $= 0$ $H(e^{j\pi})$ must $= 0$

$\max_\Omega |H(e^{j\Omega})| =$ 6 $\Leftarrow$ From plot of $|H_c(e^{j\Omega})|$ max is at $\Omega = 0$

(D) Suppose the input to each of the above three filters is $x[n] = 0$ for $n < 0$ and for $n \geq 0$ is

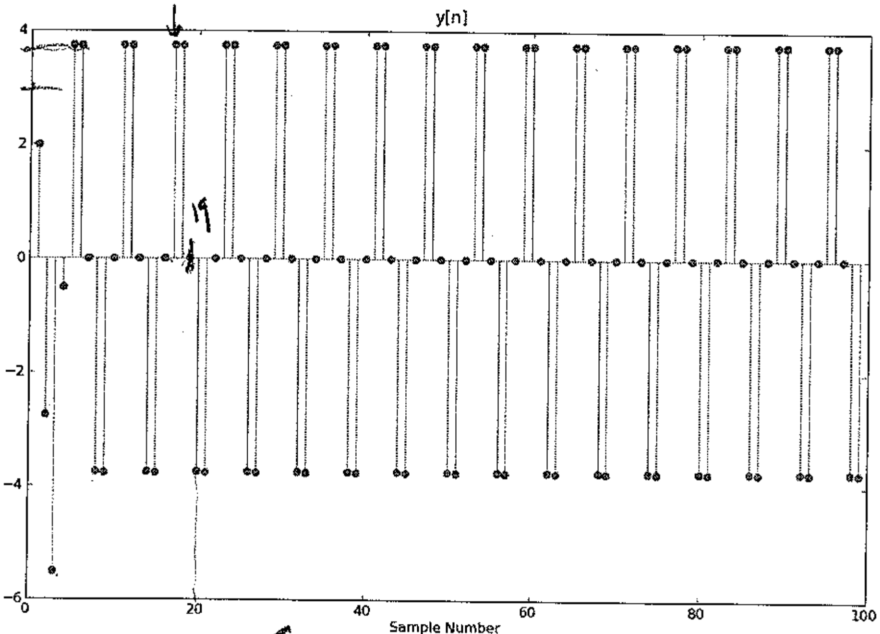
$$x[n] = \cos \frac{\pi}{3.0} n + \cos \pi n + 1.0.$$

(same input as part C).

Which filter, (A, B, or C), produced the output, $y[n]$ below, and is $\max_\Omega |H(e^{j\Omega})| < 4.0$ for your selected filter?

17

3.8



$\underline{n > 5}$

$y[n] = |H(e^{j\pi/3})| \cos(\frac{\pi}{3}n + \frac{\pi}{6})$

$\left\{ \begin{array}{ll} y[17] = \cos -\pi/6 & y[18] = \cos \pi/6 \\ y[19] = \cos \frac{3\pi}{6} & y[20] = \cos \frac{5\pi}{6} \end{array} \right.$

Frequency response plot (A, B, or C) (Be sure to justify your answer) = B

$|H(e^{j\pi/3})| \underbrace{\cos \pi/6}_{\sqrt{3}/2 \simeq 0.87} \simeq 4 - \varepsilon$

$\left. \begin{array}{l} H(e^{j0}) = 0 \\ H(e^{j\pi}) = 0 \end{array} \right\}$ Must be B

Is $\max_\Omega |H(e^{j\Omega})| < 4$ (yes or no)? No

You must provide some justification to receive credit.

$|H(e^{j\pi/3})| \simeq \frac{3.8}{0.87} > 4$

(E) Six new filters were generated using the unit sample responses of filters A, B and C, denoted $h_A[n]$, $h_B[n]$, and $h_C[n]$ respectively. The unit sample responses of the new filters were generated in the following way:

$$h_1[n] = h_A[n] + h_B[n]$$

$$h_2[n] = h_A[n] + h_C[n]$$

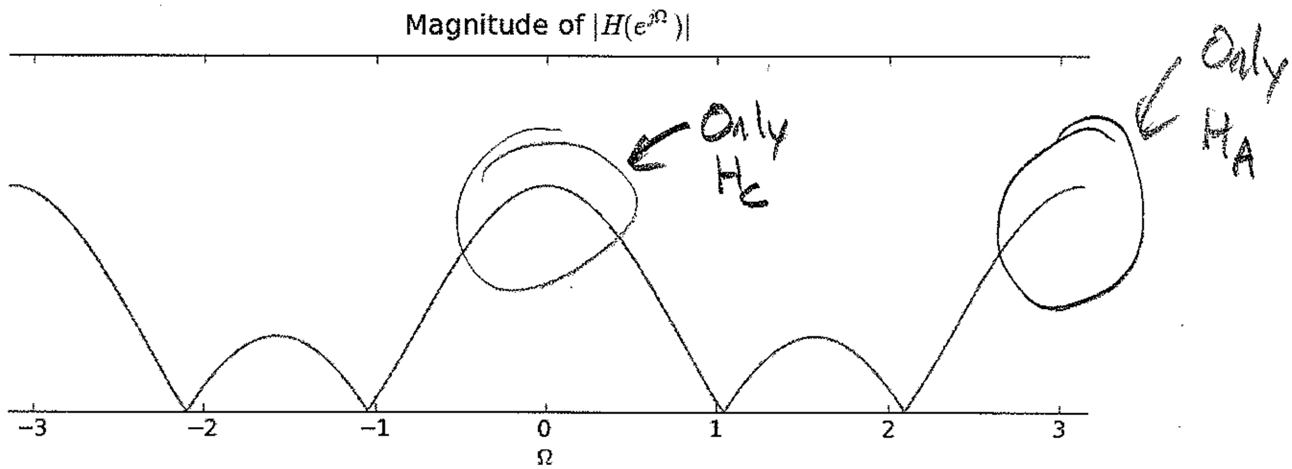
$$h_3[n] = h_B[n] + h_C[n]$$

$$h_4[n] = \sum_{m=0}^{m=n} h_A[m]h_B[n-m].$$

$$h_5[n] = \sum_{m=0}^{m=n} h_A[m]h_C[n-m].$$

$$h_6[n] = \sum_{m=0}^{m=n} h_B[m]h_C[n-m].$$

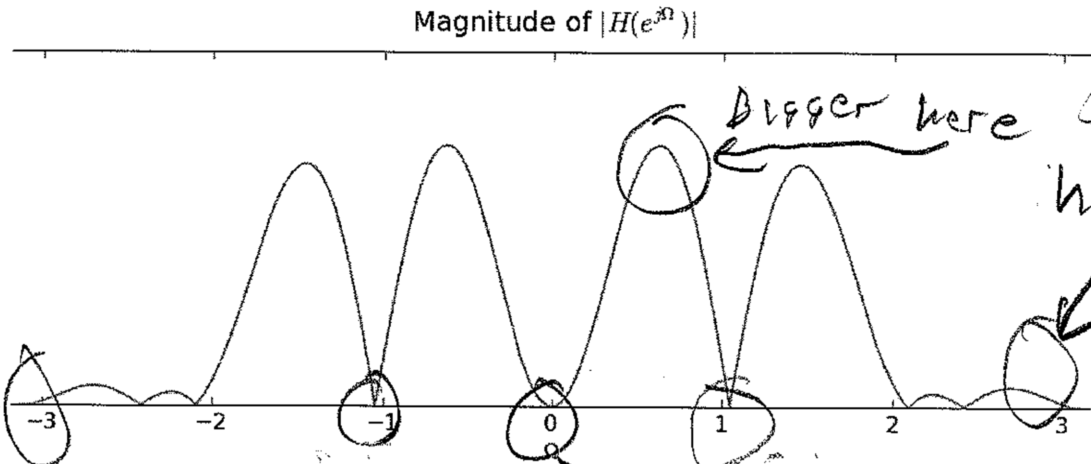
Which of the six new filters has a frequency response as plotted below?



Must be $h_A + h_C$ ($h_A * h_C$ would have zeros at 0 & π)

Which filter (1,2,3,4,5 or 6) and you must justify your answer: 2

(F) Which of the six new filters described in part E has a frequency response as plotted below?



Bigger here than here H_C

H_B & H_C H_A or H_C H_A & H_B H_A or H_C

Must be $|H_B||H_C|$ or $h_B * h_C$

Which filter (1,2,3,4,5 or 6) and you must justify your answer: ___6___

Problem 2. Block Codes [10 points]

(A) [3 points] The Registrar has asked for an encoding of class year ("Freshman", "Sophomore", "Junior", "Senior") that will allow single error correction. In the table below please fill in an appropriate 5-bit binary encoding for each of the four years.

Year	Multi-bit binary encoding for year
Freshman	00000
Sophomore	01101
Junior	10110
Senior	11011

P_1 P_2 P_3

min HD=3

(B) [2 points] Dos Equis Encodings, Inc. specializes in codes that use 20-bit transmit blocks. They are trying to design a (20, *k*, 3) block code for single error correction, for some integer *k*. What's the largest integer value of *k* they can use? Briefly explain your reasoning.

$n+1 \le 2^{n-k}$

$21 \le 2^{20-k}$

largest value for *k*: 15

(C) [5 points] The following matrix shows a rectangular single error correcting code consisting of 9 data bits, 3 row parity bits and 3 column parity bits. For each of the examples that follow, please indicate the correction the receiver must perform: give the position of the bit that needs correcting (e.g., D7, R1), or "no" if there are no errors, or "M" if there is a multi-bit uncorrectable error.

D1	D2	D3	R1
D4	D5	D6	R2
D7	D8	D9	R3
C1	C2	C3	—

1	1	1	1
1	1	0	0
0	1	1	0
0	1	1	-

Error: C3

0	0	1	0
0	1	1	1
0	1	1	0
0	0	1	-

Error: M

1	1	0	0
1	0	0	1
0	0	1	0
0	0	1	-

Error: D8

1	0	1	0
1	0	0	1
1	1	0	0
1	1	1	-

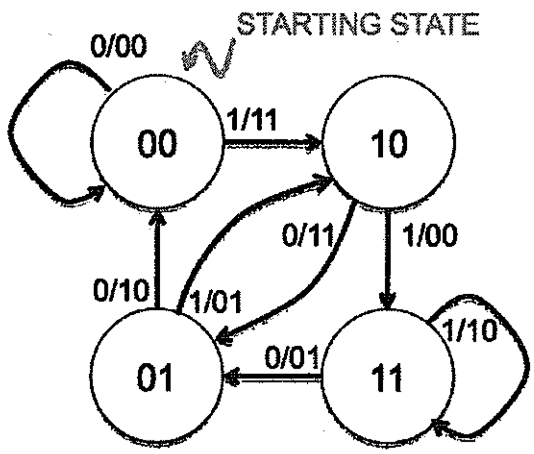
Error: no

1	1	1	0
1	1	1	1
1	0	0	0
0	0	1	-

Error: M

Problem 3. Convolutional Codes [20 points]

Consider the following state transition diagram for a $k = 3$, $rate = \frac{1}{2}$ convolutional encoder. The message stream is the bit sequence $X = x[0]x[1] \ldots x[n]$; the states are labeled with $x[n-1]x[n-2]$ and the transition arcs are labeled at their near end with $x[n]/p_0p_1$. Assume that the output of the encoder is created by concatenating the parity bits generated from processing each message bit in turn, i.e., $p_0[0]p_1[0]p_0[1]p_1[1] \ldots p_0[n]p_1[n]$.

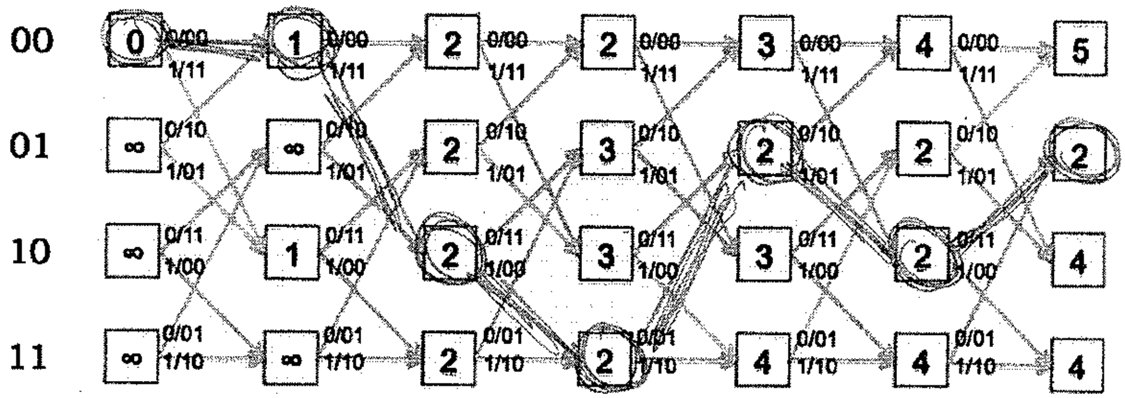


(A) [3 points] If $X = 01011\ldots$, what is the sequence of bits produced by the convolutional encoder?

sequence produced by encoder: 00 11 11 01 00

The receiver determines the most-likely transmitted message by using the Viterbi algorithm to process the (possibly corrupted) received parity bits. The path metric trellis generated from a particular set of received parity bits is shown below. The boxes in the trellis contain the minimum path metric as computed by the Viterbi algorithm.

Time step	1	2	3	4	5	6
Received	01	01	00	01	01	11



(B) [3 points] Referring to the trellis above, what is the receiver's estimate of the most-likely transmitter state after processing the bits received at time step 6?

most-likely final state: 01

(C) [5 points] Referring to the trellis above, show the most-likely path through the trellis by placing a circle around the appropriate state box at each time step and darkening the appropriate arcs. What is the receiver's estimate of the most-likely transmitted message?

most-likely transmitted message: 011010

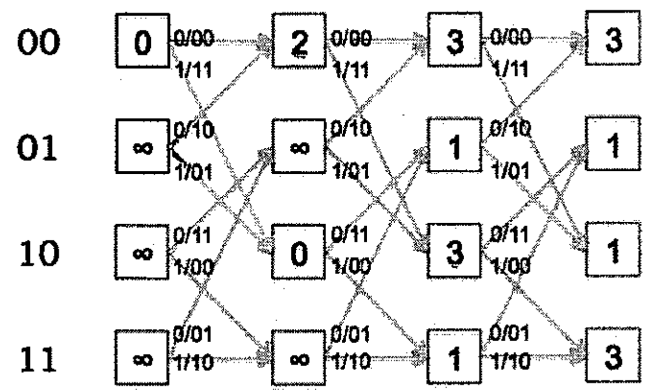
(D) [3 points] Referring to the trellis above, and given the receiver's estimate of the most-likely transmitted message, at what time step(s) were errors detected by the receiver? Briefly explain your reasoning.

time steps at which errors detected: 1, 2

where PM increments along most likely path.

Now consider the path metric trellis generated from a *different* set of received parity bits.

Time step	1	2	3
Received	??	??	??



(E) [6 points] Referring to the trellis above, determine which pair(s) of parity bits *could* have been been received at time steps 1, 2 and 3. Briefly explain your reasoning.

Possible pairs of parity bits time 1: 11

00→10 transition had BM=0 ⇒ bits = 11

Possible pairs of parity bits time 2: 01, 10

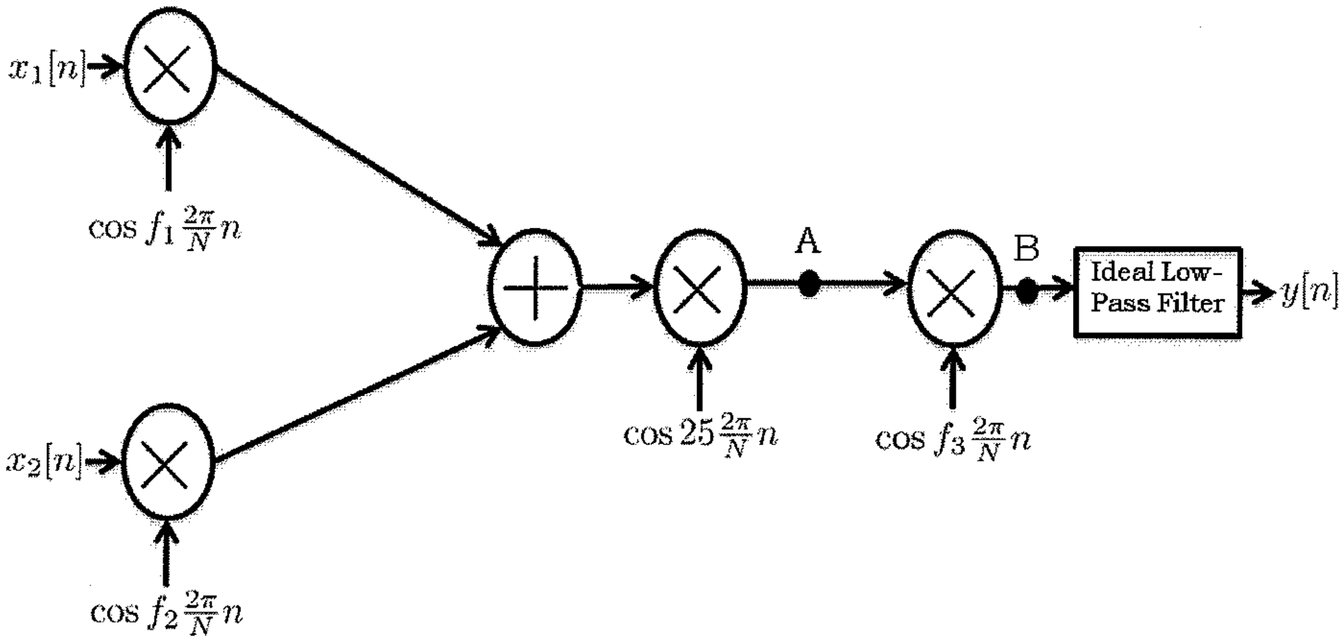
can't be 00, (00→00) nor 11 (00→10)
10→11 10→01

Possible pairs of parity bits time 3: 01

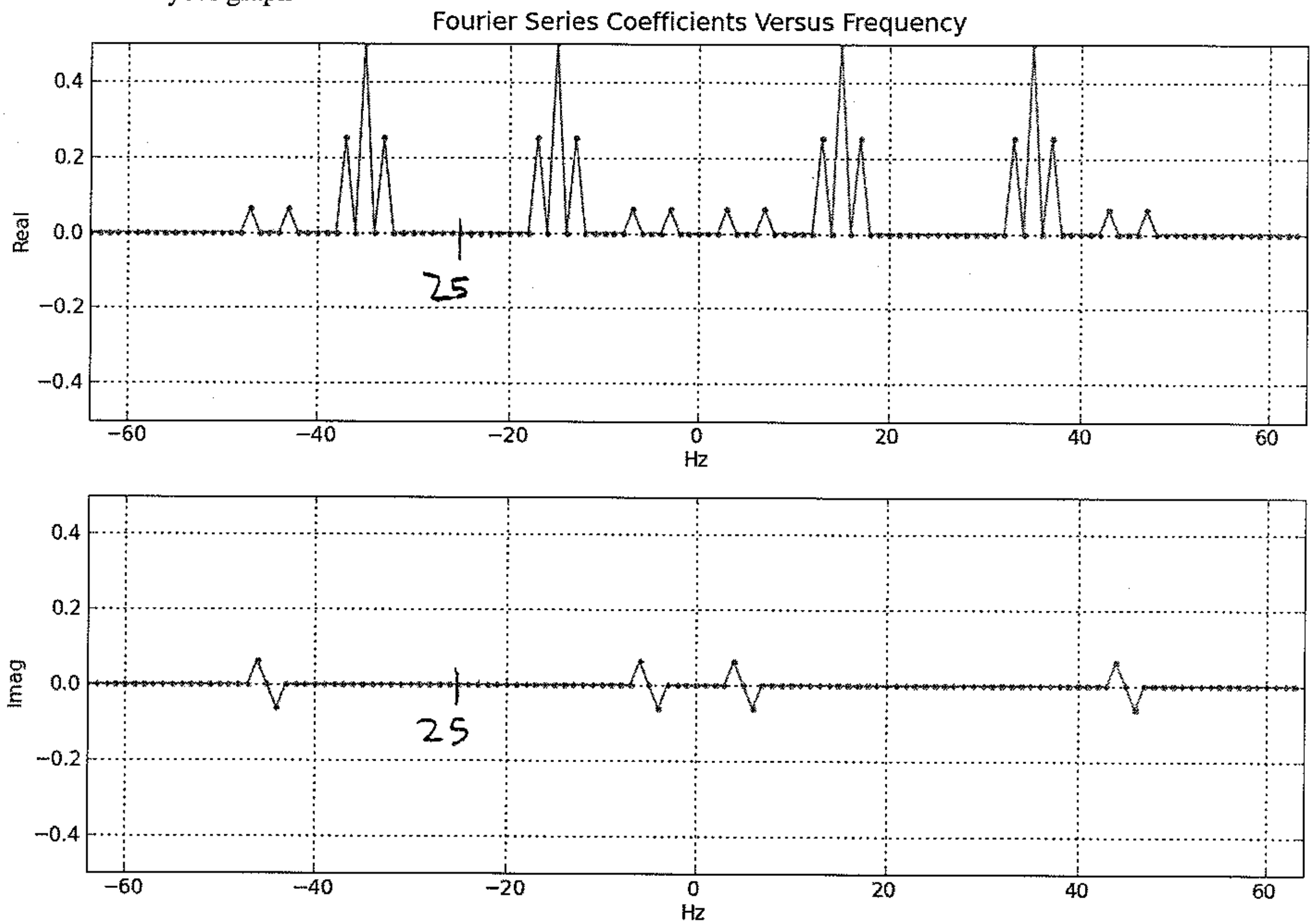
11→01 transition had BM=0 ⇒ bits = 01

Problem 4. Modulation [35 Points]

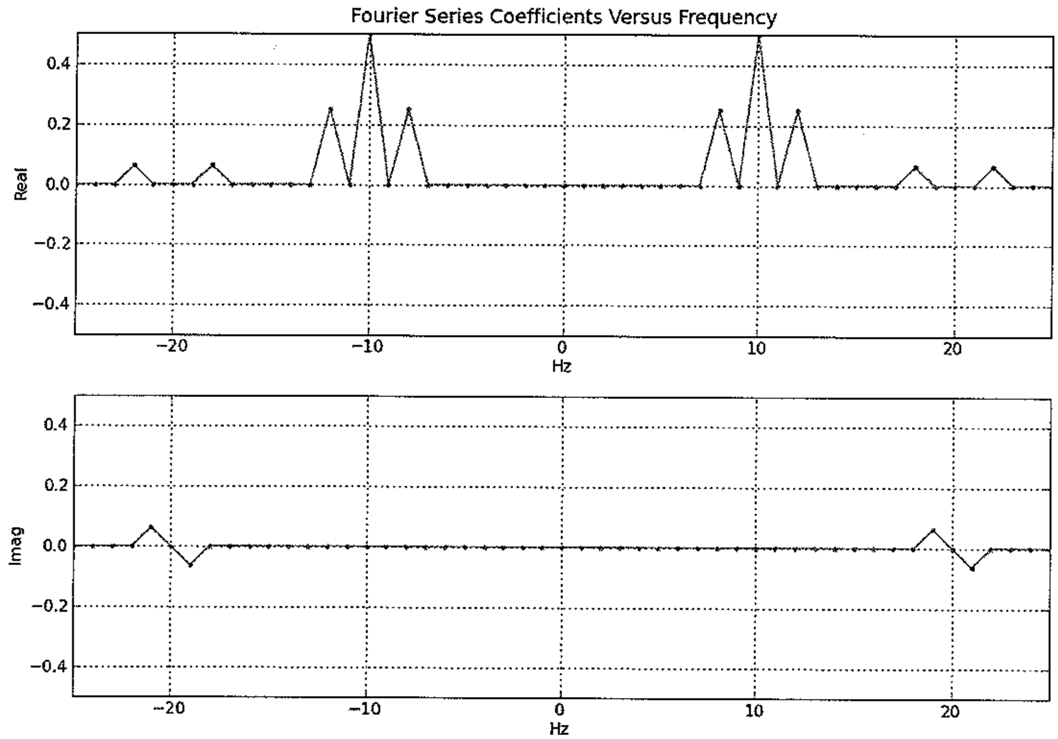
In answering the four parts of this question, please refer to the modulation-demodulation system below. For this problem, please assume $N = 128$ and that the sampling frequency, f_s, is 128 samples per second.



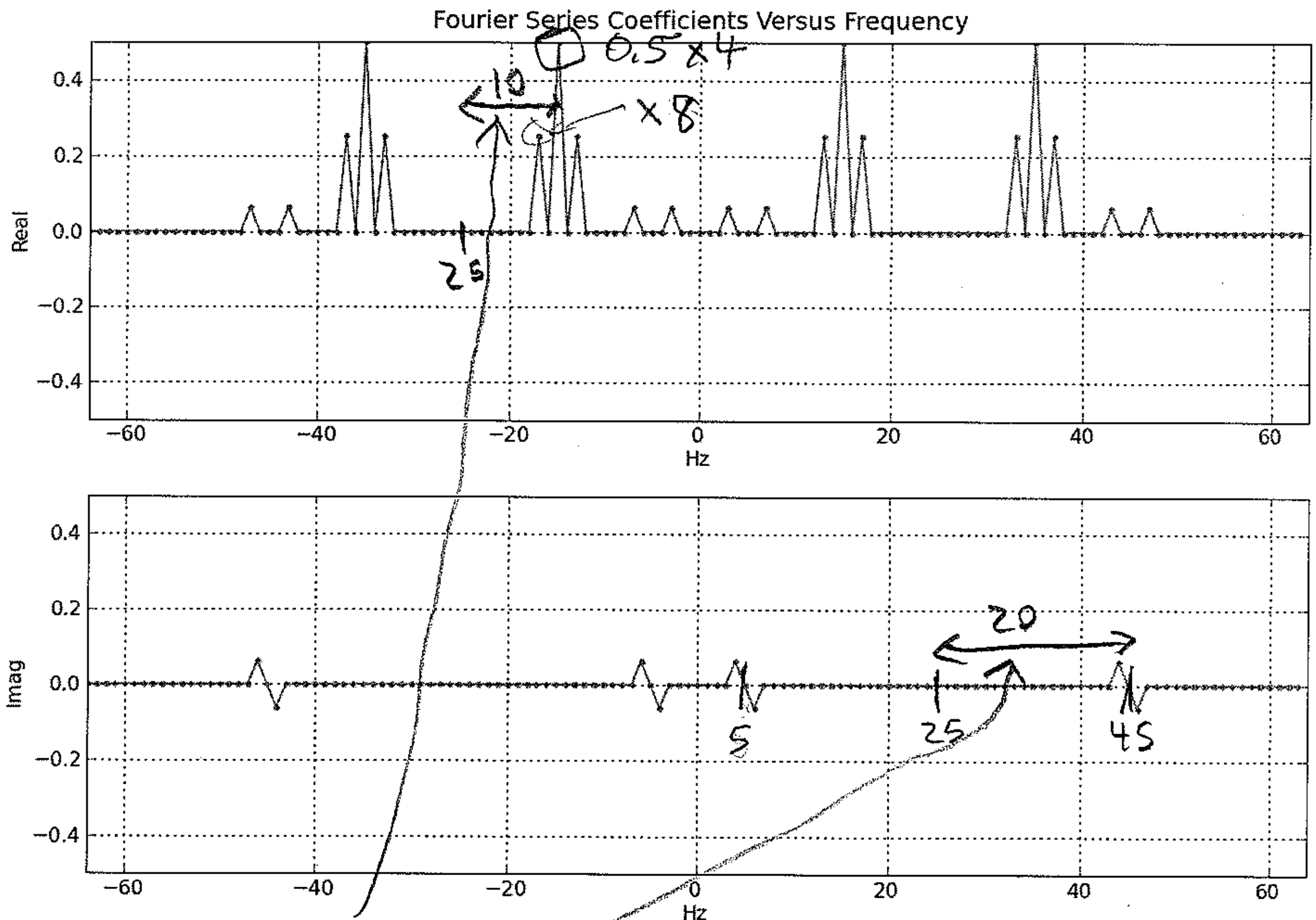
(A) Below are plots of the real and imaginary parts of the Fourier coefficients versus frequency for point **A** of the modulation-demodulation system. On the axes below, please plot the real and imaginary parts of the Fourier coefficients versus frequency for the signal at point **B**, assuming $f_3 = 25$. For this problem, you need only plot the Fourier Series coefficients for frequencies in the range -25 to 25. Please be sure to label critical frequencies and values in your graph



Signal at Point **B** Fourier Series Coefficients:



(B) Repeated below are plots of the real and imaginary parts of the Fourier coefficients versus frequency for point **A** of the modulation-demodulation system (that is, the same plot as in part A). If $x_1[n] = \alpha + \beta\cos(2\frac{2\pi}{N}n)$ and $x_2[n] = \frac{1}{2}\cos(2\frac{2\pi}{N}n) + \frac{1}{2}\sin(\frac{2\pi}{N}n)$, please determine the two modulation frequencies, f_1 and f_2, and the two amplitudes, α and β.



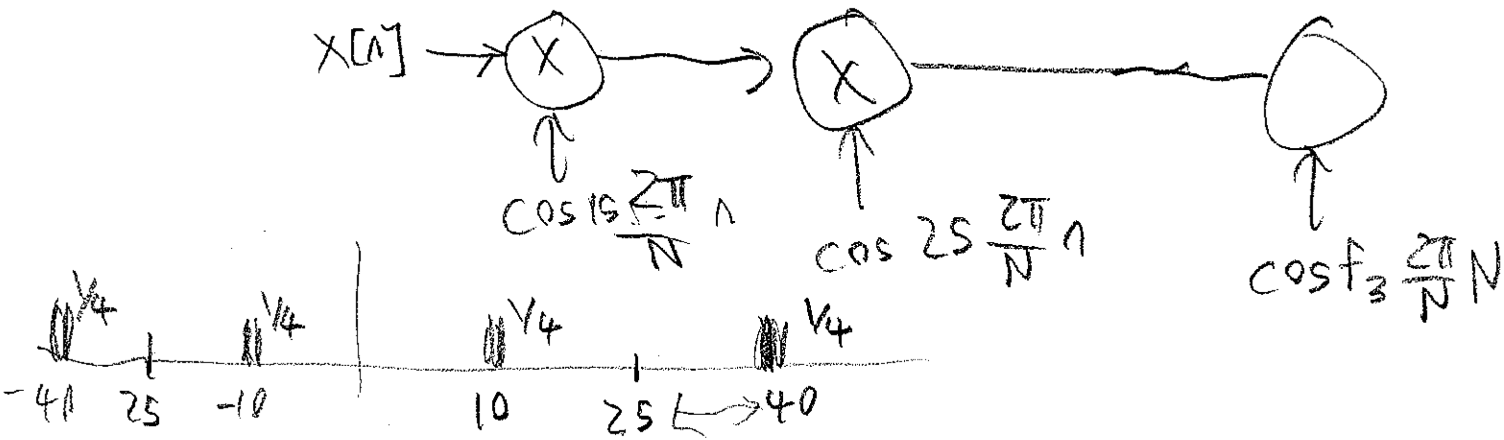
$f_1 =$ 10

$f_2 =$ 20

$\alpha =$ 2.0 (4 × 0.5)

$\beta =$ 2.0 (8 × 0.25)

(C) Now suppose $x_2[n] = 0$, $x_1[n] = \frac{1}{2}\cos(2\frac{2\pi}{N}n) + \frac{1}{2}\sin(\frac{2\pi}{N}n)$, and $f_1 = 15$ (not one of the answers to part B!). For what values of $f_3 < 64$ (there is more than one) will $y[n] = x_1[n]$, assuming the low-pass filter has been designed correctly. In addition, what should the magnitude be for the low frequency response of the low-pass filter? Please show your reasoning, with pictures if needed.



Values for f_3 = 10 and 40

Magnitude of the low-pass filter's low frequency response = 4

End of Quiz 2!