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1. Dijkstra's algorithm is a greedy algorithm. We add nodes in order of non-decreasing cost of the minimum-path to the nodes. We get:
A: Order 1, Cost 2.
E: Order 2, Cost 3.
B: Order 3, Cost 4.
C: Order 4, Cost 5.
D: Order 5, Cost 6.
2. In the picture below, the grey nodes (A in particular) run Bob's algorithm (shortest number of hops), while the white nodes (B in particular) run Alice's (minimum-cost).


Suppose the destination is E. A will pick B as its next hop because ABE is the shortest path. B will pick A as its next hop because BACDE is the minimum-cost path (cost of 4, compared to 11 for the ABE path). The result is a routing loop ABABAB...
3. (a) Yes. Any shortest path in $G$ is also a shortest path in $G^{\prime}$ because if the cost of a path $P$ in $G$ is $c$, then the cost of $P$ in $G^{\prime}$ is $k c$, for all $P$. Also, there's a one-to-one correspondence between paths in $G$ and $G^{\prime}$.
(b) No. A counter-example is easy to construct. For example, suppose $G$ is a triangle, $A, B, C$, where $\operatorname{cost}(A B)=1, \operatorname{cost}(B C)=1$, and $\operatorname{cost}(C A)=3$. The shortest path between $A$ and $C$ in $G$ is $A-B-C$, with cost 2 . But now suppose $k=1$ and $h=2$. The link costs become $\operatorname{cost}(A B)=4, \operatorname{cost}(B C)=4$, and $\operatorname{cost}(A C=5)$. Now, the shortest path between $A$ and $C$ is the direct link, $A C$.
4. Statements (a) and (b) are false, statement (c) is true, and statement (d) is false. Statement (a) is false because $u$ could propagate an incorrect cost to its neighbors causing the neighbor to have an incorrect route. In fact, $u$ 's neighbors could do the same. Statement (c) is correct; a simple example is when the network is a tree, where there is exactly one path between any two nodes.
Statement (d) is false; no routing loops can occur under the stated condition. We can demonstrate this property by contradiction. Consider the shortest path from any node $s$ to any other node $t$ running the flawed routing protocol. If the path does not traverse $u$, no node on that path can have a loop because distance vector routing without any packet loss or failures is loop-free. Now consider the nodes for which the computed paths go through $u$; all these nodes are correctly implemented except for $u$, which means the paths between $u$ and each of them is loop-free. Moreover, the path to $u$ is itself loop-free because $u$ picks one of its neighbors with smaller cost, and there is no possibility of a loop.
5. Reverse-engineering routing trees: See PSet.
6. This question asks for the update rule in the Bellman-Ford integration step. The cost in $S$ 's routing table for $D$ should be set to $\min _{i}\left\{c_{i}+p_{i}\right\}$.
7. FishNet: See PSet.
8. (a) B,C,E,A,F,D and B,C,E,F,A,D.
(b) No effect. The edge AC is not in any shortest path.
(c) Can affect route to $\mathbf{A}$. If $\operatorname{cost}_{A C} \leq 3$, then we can start using this edge to go to A instead of the edge BA.
(d) Can affect route to $\mathbf{C}, \mathbf{F}, \mathbf{E}$. If $\operatorname{cost}_{B C} \geq 7$, then we can use $\mathrm{BE}-\mathrm{EC}$ to go to C instead of BC . If $\operatorname{cost}_{B C} \geq 5$, then we can use BE-EF to go to F . If $\operatorname{cost}_{B C} \geq 3$, can use BE to go to $B$.
(e) Can affect route to $\mathbf{D}$. If $\operatorname{cost}_{B C} \leq 1$, then we can use BC-CE-ED to go to D instead of BD.
9. (a) The most correct answer is True, given how the problem was stated. Consider the example where $A, B$, and $C$ are connected in a triangle with equal weight edges. If $A$ and $C$ are running the new algorithm, $A$ will, at some point in time, hear an advertisement of cost zero from $B$ and $C$. Similarly, $C$ will hear a zero cost path coming from $A$ and $B$. The problem does not give any indication of how ties are broken, so it is possible that $C$ will route packets to B through $A$, and $A$ will route packets to $B$ through $C$, causing a routing loop: an invalid path.
If, however, ties are broken in a way that prefers the routing table entry that already exists, then no routing loops will ever happen, because every advertisement will come from a node that has a valid (although not necessarily min-cost) path across a link that has a finite communication time (because otherwise it would not hear the advertisement.). Under this assumption, the correct response is False.
(b) True. For nodes to have incorrect routes, packets will eventually reach D, but not at a minimum cost. Since some nodes do not account for their own link cost, incorrect advertised costs will propagate through the network, leading to incorrect routes. There are several examples; one is a simple triangle $A, B, D$ where the cost from $A$ to $D$ is 100 , the cost from $A$ to $B$ is 1 and the cost from $B$ to $D$ is 1 . $A$ will pick the direct link to $D$ as its route, but the correct route is the link to $B$, corresponding to the path $A B D$.
(c) True. A network where there is only one path between any two nodes will have a correct routes (for example, where the network is a tree). There are other examples too; for instance, imagine a connected network where the set of nodes with the incorrect DV update have no links between each other at all. In such networks, the routes everywhere will be correct.
10. See PSet.

