Solutions to Quiz 2 (December 3, 2018)

This quiz uses the same abstract data type as Quiz 1.

*Mini Rush Hour* is a sliding block puzzle played on a 5×5 grid of cells numbered 1 to 25:

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← The board has an exit to the right of cell 15.

*Cars* are 1-cell × 2-cells pieces placed horizontally or vertically on the grid. Every Mini Rush Hour puzzle starts with the VIP car on cells 11 and 12. The goal is to move the VIP car to the right so it reaches cells 14 and 15, at which point it can exit the board:

Blocking the VIP car’s way are other cars, for example:

While they are labeled in this diagram, cars other than the VIP car are not differentiated except by their placement on the board, they have no distinguishing characteristics. All cars in the game move only along their axis of orientation: horizontal cars move left and right, vertical cars move up and down. Cars never overlap. They can only move through and stop on empty cells.

The player wins by making a series of moves that allow the VIP car to reach the exit. The following page shows a solution to this example puzzle, plus another example.

The problems in this quiz use *Direction* and *Car*, defined on page 6; and refer to the code for *RushHourPuzzle* on page 7 and *RushHourGame* on page 8; all of which you may detach.
Notes which are not relevant to this quiz:

- *Rush Hour®* is a trademark of ThinkFun, Inc. The full game uses varying-size cars on a larger board.
- On arbitrarily large grids, *Rush Hour* and Size-2 *Rush Hour* are PSPACE-complete (Flake & Baum, 2002; Tromp & Cilibrasi, 2005).

**Problem 1** (Recursive ADTs I) (**22 points**).
Immutable *RushHourPuzzle* represents the starting layout of a Mini Rush Hour puzzle.

Let’s implement this type as a recursive datatype with the three variants defined on page 7. Remember! Every Mini Rush Hour puzzle has the VIP car on cells 11 and 12.

(a) Write the recursive datatype definition for *RushHourPuzzle* using these variants:

**Solution.**

```
RHP = Minimal() + WithHorizontal(left:int, rest:RHP) + WithVertical(top:int, rest:RHP)
```

(b) Write a complete checkRep for *WithHorizontal* by writing several “assert ...;” statements:

```java
private void checkRep() {
    assert 1 <= left && left <= 25;
    assert left % 5 != 0;
    assert ! rest.occupied(left) && ! rest.occupied(left+1);
}
```

(c) And write an abstraction function for *WithHorizontal* that works with your checkRep:

**Solution.**

```
AF(left, rest) = the Mini Rush Hour puzzle rest with the addition of a horizontal car in cells left and left + 1
```

(d) Based on the provided code, is *WithHorizontal* safe from rep exposure? Circle YES and write the argument, or circle NO and explain why not:

**Solution.** YES

Rep fields are private and final, so they cannot be assigned; and immutable primitive or object, so they cannot be mutated.

Note that a client provides *rest*, so we cannot argue clients are never given a reference to it.

**Problem 2** (Recursive ADTs II) (**22 points**).

Complete each method below by returning a single expression. Remember, every puzzle has the VIP car.

(a) Complete `occupied(..)` for the *Minimal* variant:

```java
@override public boolean occupied(int cell) {
    return cell == 11 || cell == 12;
}
```
(b) Complete occupied(..) for the WithHorizontal variant:

```
@override public boolean occupied(int cell) {
}
```

Solution.

```
return cell == left || cell == left+1 || rest.occupied(cell);
```

(c) Using the static Stream helper methods on page 6, complete cars() for the Minimal variant:

```
@override public Stream<Car> cars() {
}
```

Solution.

```
return Stream.of(new Car(11, Direction.RIGHT));
```

(d) Again using the static Stream helpers on page 6, complete cars() for the WithHorizontal variant:

```
@override public Stream<Car> cars() {
}
```

Solution.

```
return Stream.concat(rest.cars(), Stream.of(new Car(left, Direction.RIGHT)));
```

**Problem 3 (Equality & Streams) (20 points).**

Ben is thinking about equality for RushHourPuzzle instances. Remember! Every Mini Rush Hour puzzle has the VIP car on cells 11 and 12.

(a) Write two different RushHourPuzzle reps that represent the same Mini Rush Hour puzzle. Use as few RushHourPuzzle-type objects as possible. As examples:

- Minimal() and Minimal() are the same rep, representing the same puzzle
- WithHorizontal(1, Minimal()) and WithHorizontal(2, Minimal()) are different reps, representing different puzzles

Solution. E.g., WH(1, WH(6, Minimal())) and WH(6, WH(1, Minimal()))

(b) Alyssa doesn’t like having multiple representations for the same puzzle, so she plans to make more reps illegal. Write an additional statement to include in the WithHorizontal rep invariant to achieve this goal.

Solution. E.g., no car in rest has topLeft greater than left, except for the VIP car.

(c) Ben is going to implement equals by comparing abstract values, not concrete reps.

He’s super jazzed about functional programming with streams, so he wants to write the sameValue(..) helper for the Minimal variant using map, filter, and reduce. Fill in a correct implementation below, returning true if and only if that is also the minimal Mini Rush Hour puzzle based on its cars() stream.

Do not use an identity function or a function that returns a constant – those aren’t jazzy enough.
private boolean sameValue(RushHourPuzzle that) {
    return that.cars().map(__________________________)
        .filter (__________________________)
        .reduce (__________________________,
                 _________________________) == __________________________;
}

Use reduce: Stream<T>×T×(T×T→T)→T by providing an identity value and an accumulator function.
Hint: what cars are in the minimal puzzle, and what cars are in a puzzle that is not the minimal puzzle?

Solution. E.g.,
that.cars().map(car -> car.topLeft())
    .filter(cell -> cell != 11) // find any non-VIP cars
    .reduce(0, (a,b) -> a+b)   // sum
    == 0;                     // should be none

Problem 4 (Thread Safety) (18 points).
Mutable RushHourGame (on page 8) allows the client to play a Mini Rush Hour game, starting from an initial RushHourPuzzle and making a series of moves with the move(..) method.

(a) Based on the provided code, circle one or more of CONFINED, IMMUTABLE, and/or THREADSAFE TYPE if they apply; or circle NONE otherwise:

The vipCar reference: CONFINED IMMUTABLE THREADSAFE TYPE NONE
Possible values of vipCar: CONFINED IMMUTABLE THREADSAFE TYPE NONE
The carsMap reference: CONFINED IMMUTABLE THREADSAFE TYPE NONE
The value of carsMap: CONFINED IMMUTABLE THREADSAFE TYPE NONE

Solution. Which of the thread safety strategies apply?

The vipCar field is not confined or final, so NONE.
Car values are IMMUTABLE. The implementation makes it a THREADSAFE TYPE but the spec doesn’t say, so either answer is OK.
The carsMap field is final, so IMMUTABLE.
It is initialized with synchronizedMap to make it a THREADSAFE TYPE.

(b) Suppose two threads A and B call move on the same RushHourGame concurrently, starting from the game state below.

Choose two different calls, neither of which moves the VIP car, and explain how the steps of move can interleave so that both calls return a value, but the rep is invalid and broken after they complete. Say how the rep is broken.
Use line numbers from page 8, and circle all line numbers (not cell numbers!). For example, “thread A runs line $N$, then thread B runs line $M$, which causes problem $P$,” etc.

Since we don’t know its details, treat the computation of newTopLeft as a single indivisible step.

Choose the two calls:

A: B:

Explain the steps:

How is the rep broken?

Solution. E.g., move(13, DOWN) and move(21, RIGHT).

If both threads compute newTopLeft, then they will both attempt to put new cars that occupy cell 23 (plus one other cell). One thread will win, so that the rep now contains overlapping cars, one of them mapped from only one of its cells.

Many other bad interleavings are possible.

Problem 5 (Queues & Locks) (18 points).
Charlie wants to implement best-score tracking for Mini Rush Hour using the producer-consumer pattern, with many clients submitting scores to a single tracker.

(a) Clients will have access to two threadsafe blocking unbounded-size first-in-first-out queues:

BlockingQueue<RushHourPuzzle> puzzleSolved
BlockingQueue<Integer> movesToSolve

To record a score (the number of moves $m$ to solve a puzzle $p$), a client runs:

while (true) {
    RushHourPuzzle $p$ = puzzleSolved.take();
    int $m$ = movesToSolve.take();
    // if $m$ is the best score on $p$, record it
}

Explain a problem with this implementation and give an example:

Solution. Client put calls may interleave with one another so that puzzles and moves are out of order.

E.g. client 1 enqueues RHP $p_1$, client 2 enqueues $p_2$ and $m_2$, then client 1 enqueues $m_1$. The tracker will take $p_1$ with $m_2$.

Charlie changes the code to add synchronization on a new object shared by all clients and the tracker:

Object movesLock
To record a score, a client runs:

```java
puzzleSolved.put(p);
```

```java
synchronized (movesLock) {
    movesToSolve.put(m);
}
```

And the best-score tracker is running:

```java
while (true) {
    RushHourPuzzle p = puzzleSolved.take();
    int m;
    synchronized (movesLock) {
        m = movesToSolve.take();
    }
    // if m is the best score on p, record it
}
```

(b) Does this change fix the problem from part (a)? Circle: 

YES  NO

**Solution.** NO. Clients can acquire and release `movesLock` as part of the out-of-order sequence.

(c) What new, different kind of problem does this change introduce? Explain and give an example:

**Solution.** Deadlock.

E.g. a client enqueues `p`, then the tracker takes `p`, enters the `synchronized` block, and blocks on `take` because the queue is empty – while holding `movesLock`. Clients cannot acquire `movesLock` to call `movesToSolve.put`, so the system is deadlocked.

You may detach these pages. Write your username at the top, and hand in all pages when you leave.

```java
static <T> Stream<T> empty()
Returns an empty stream.

static <T> Stream<T> of(T t)
Returns a stream containing a single element.

static <T> Stream<T> of(T... values)
Returns a stream whose elements are the specified values.

static <T> Stream<T> iterate(T seed, Function<T,T> f)
Returns an infinite stream produced by iterative application of a function `f` to an initial element `seed`, producing a `Stream` consisting of `seed, f(seed), f(f(seed)), etc.`

static <T> Stream<T> generate(Supplier<T> s)
Returns an infinite stream where each element is generated by the provided `Supplier`.

static <T> Stream<T> concat(Stream<T> a, Stream<T> b)
Creates a concatenated stream whose elements are all the elements of the first stream followed by all the elements of the second stream.
```

```java
public enum Direction { LEFT, RIGHT, UP, DOWN }
```

```java
/** Immutable car on a Mini Rush Hour board. */
public class Car {
    private final int topLeft;
    private final Direction direction;
```
/**
 * Make a Mini Rush Hour car whose top (for a vertical car) or left (for horizontal)
 * half is in cell topLeft.
 * @param topLeft must be a valid top (for vertical) or left (for horizontal) cell
 * @param direction must be DOWN for a vertical car or RIGHT for a horizontal car
 */
public Car(int topLeft, Direction direction) {
  this.topLeft = topLeft;
  this.direction = direction;
}

/** @return the top/left cell of this car */
public int topLeft() { return topLeft; }

/** @return the bottom/right cell of this car */
public int bottomRight() { return topLeft + (direction == Direction.RIGHT ? 1 : 5); }

/** @return DOWN if this is a vertical car or RIGHT if this is a horizontal car */
public Direction direction() { return direction; }

... other observers ...
public WithHorizontal(int left, RushHourPuzzle rest) {
    this.left = left;
    this.rest = rest;
    checkRep();
}

private void checkRep() { ... }

@Override public boolean occupied(int cell) { ... }

@Override public Stream<Car> cars() { ... }

... other observers ...

class WithVertical implements RushHourPuzzle {

    private final int top;
    private final RushHourPuzzle rest;

    public WithVertical(int top, RushHourPuzzle rest) { ... }

    private void checkRep() { ... }

    @Override public boolean occupied(int cell) { ... }

    @Override public Stream<Car> cars() { ... }

    ... other observers ...
}

/** Mutable Mini Rush Hour game. */
public class RushHourGame {
    // the VIP car in its current position
    private Car vipCar;
    // all cars on the game board, indexed by both cells they occupy
    private final Map<Integer, Car> carsMap;

    /**
     * Make a new Mini Rush Hour game starting from the given puzzle layout.
     * @param starting puzzle to play
     */
    public RushHourGame(RushHourPuzzle starting) {
        // find the VIP car
        this.vipCar = starting.cars().filter(car -> car.topLeft() == 11)
            .findFirst().orElseThrow();

        // populate the cars map
        this.carsMap = Collections.synchronizedMap(new HashMap<>());
        starting.cars().forEach(car -> carsMap.put(car.topLeft(), car));
        starting.cars().forEach(car -> carsMap.put(car.bottomRight(), car));
    }

    /**
     * Move the car that occupies cellNum as far as it can move in the given direction.
     * @param cellNum indicates car to move, a car must currently occupy that cell
     * @param direction direction to move
     * @return true if-and-only-if the VIP car is in the game-winning position
     */
*/

public boolean move(int cellNum, Direction direction) {
    // find the old Car in carsMap
    final Car oldCar = carsMap.get(cellNum);

    // determine how far it can move
    final int newTopLeft = ...

    // create a new Car in the destination position
    final Car newCar = new Car(newTopLeft, oldCar.direction());

    // remove old Car from carsMap, add new Car
    carsMap.remove(oldCar.topLeft());
    carsMap.remove(oldCar.bottomRight());
    carsMap.put(newCar.topLeft(), newCar);
    carsMap.put(newCar.bottomRight(), newCar);

    // if we moved the VIP car, reassign vipCar
    if (oldCar == vipCar) { vipCar = newCar; }

    // return true iff the game is won
    return vipCar.topLeft() == 14;
}