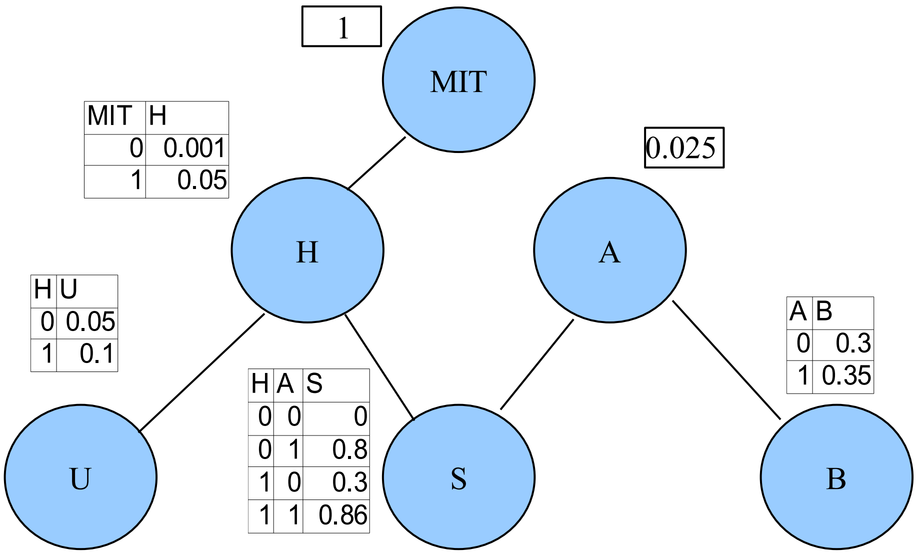
Recitation 2009 November 13.



Jargon:

H is a **direct cause** of U
MIT is an **indirect cause** of U
H is a **common cause** of U and S
H and A have a **common effect** S
H and A are **independent**

Information about this world:
P(MIT) = 1
P(U|H) = 0.1
P(H|MIT) = 0.05; P(H|¬MIT) = 0.001

Now compute things
P(H) = P(¬MIT) * P(H|¬MIT) + P(MIT) * P(H|MIT) = 0 * 0.001 + 1 * 0.05 = 0.05
P(U) = P(¬H) * P(U|¬H) + P(H) * P(U|H) = .95 * .05 + .05 * .1 = .0525

P(S) is more complex: two parents; **summing out** A:
P(S) = P(S|H)P(H) + P(S|¬H) P(¬H)
P(S|H) = P(S|H,A) * P(A) + P(S|H,¬A) * P(¬A)
= (.86 *.025) + (.3 *.975) = .314
P(S|¬H) = P(S|¬H,¬A) * P(¬A) + P(S|¬H,A) * P(A)
= (0 * .975) + (.8*.025) = .02
P(S) = .314*.05 + .02*.95 = .0347

P(S) **directly**: sum over all possibilities, of the likelihood of that possibility times the likelihood of S given that possibility:
P(S|¬H,¬A)P(¬H,¬A) + P(S|¬H,A)P(¬H,A) + P(S|H,¬A)P(H,¬A) + P(S|H,A)P(H,sA) =

____________ + ____________ + _____________ + ______________ = .0347

(.95 * .975 * 0) + (.95 * .025 * .8) + (.05 * .975 * .3) + (.05 * .025 * .86)

But what if we want to "go backwards," i.e., compute $P(H|S)$?

Because we know that $P(A,B) = P(A|B)P(B) = P(B|A)P(A)$, we can divide by $P(A)$ or $P(B)$ to find **Bayes' Rule**: $P(A|B) = P(B|A)*P(A) / P(B)$

$P(H|S) = P(S|H) * P(H) / P(S) = .314 * 0.05 / .0347 = .45$
So knowing there's a sculpture raises the possibility of a hack from .05 to .45.

How is the probability of an art show affected?
$P(A|S) = P(S|A) * P(A) / P(S) =$ ________________ $= .58$
$P(S|A) = P(S|A,H)*P(H) + P(S|A,\neg H)*P(\neg H) = .86*.05 + .8*.95 = .803$

So knowing there's a sculpture raises the possibility of an art show from .025 to .58.

Explaining away
Now what if we know that S and A are both true? How strong is our belief in H, compared to when we knew only that S was true, and the value was .45?

Three variable version of Bayes' Rule (see wikipedia article):
$P(H|S,A) = P(S|H,A)P(H|A) / P(S|A)$
$= .86 * .05 / .803$
$= .054$

So knowing there there is both sculpture and an art show drastically lowers our belief that there was a hack. The art show has "explained away" the sculpture, which no longer needs to be explained by a hack.

If we know that there is no art show to explain the sculpture, what can we say about the probability of a hack?
$P(H|S,\neg A) = P(S|H,\neg A)P(H|\neg A)/P(S|\neg A)$
$P(S|\neg A) = P(S|\neg A,\neg H) * P(\neg H) + P(S|\neg A,H) * P(H)$
$= 0 * .95 + .3 * .05 = .015$
$= .3 * .05 / .015 = 1$

Additional exercises:

$P(H|S,\neg B) = \sim .4696$ which is $> P(H|S)=.45$ --- why?

$P(H|S,\neg B,U) = \sim .639$ explain: