1. (a) The first part can be completed without reference to anything other than the die roll:

\[ p_N(n) \]

\[
\begin{array}{c|cccc}
   & k = 0 & k = 1 & k = 2 & k = 3 \\
\hline
n = 0 & 1/4 & 0 & 0 & 0 \\
n = 1 & 1/8 & 1/8 & 0 & 0 \\
n = 2 & 1/16 & 1/8 & 1/16 & 0 \\
n = 3 & 1/32 & 3/32 & 3/32 & 1/32 \\
\end{array}
\]

(b) When \( N = 0 \), the coin is not flipped at all, so \( K = 0 \). When \( N = n \) for \( n \in \{1, 2, 3\} \), the coin is flipped \( n \) times, resulting in \( K \) with a distribution that is conditionally binomial. The binomial probabilities are all multiplied by \( 1/4 \) because \( p_N(n) = 1/4 \) for \( n \in \{0, 1, 2, 3\} \). The joint PMF \( p_{N,K}(n,k) \) thus takes the following values and is zero otherwise:

\[
\begin{align*}
    p_{K|N}(k|2) &= \begin{cases} 
        1/4, & \text{if } k = 0, \\
        1/2, & \text{if } k = 1, \\
        1/4, & \text{if } k = 2, \\
        0, & \text{otherwise.} \end{cases} \\
\end{align*}
\]

This is a normalized row of the table in the previous part.

(d) To get \( K = 2 \) heads, there must have been at least 3 coin tosses, so only \( N = 3 \) and \( N = 4 \) have positive conditional probability given \( K = 2 \).

\[
p_{N|K}(2 | 2) = \frac{P\{N = 2 \cap \{K = 2\}\}}{P\{K = 2\}} = \frac{1/16}{1/16 + 1/32 + 1/32 + 1/32} = 2/5.
\]

Similarly, \( p_{N|K}(3 | 2) = 3/5. \)
2. (a) \( x = 0 \) maximizes \( \mathbb{E}[Y \mid X = x] \) since

\[
\mathbb{E}[Y \mid X = x] = \begin{cases} 
2, & \text{if } x = 0, \\
3/2, & \text{if } x = 2, \\
3/2, & \text{if } x = 4, \\
\text{undefined}, & \text{otherwise}.
\end{cases}
\]

(b) \( y = 3 \) maximizes \( \text{var}(X \mid Y = y) \) since

\[
\text{var}(X \mid Y = y) = \begin{cases} 
0, & \text{if } y = 0, \\
8/3, & \text{if } y = 1, \\
1, & \text{if } y = 2, \\
4, & \text{if } y = 3, \\
\text{undefined}, & \text{otherwise}.
\end{cases}
\]

(c) By traversing the points top to bottom and left to right, we obtain

\[
\mathbb{E}[XY] = \frac{1}{8} (0 \cdot 3 + 4 \cdot 3 + 2 \cdot 2 + 4 \cdot 2 + 0 \cdot 1 + 2 \cdot 1 + 4 \cdot 1 + 4 \cdot 0) = \frac{15}{4}.
\]

Conditioning on \( A \) removes the point masses at \((0, 1)\) and \((0, 3)\). The conditional probability of each of the remaining point masses is thus \(1/6\), and

\[
\mathbb{E}[XY \mid A] = \frac{1}{6} (4 \cdot 3 + 2 \cdot 2 + 4 \cdot 2 + 2 \cdot 1 + 4 \cdot 1 + 4 \cdot 0) = 5.
\]

3. See the textbook, Example 2.17, pages 105–106.