1. **Problem 2.35, page 130 in the text.** Verify the expected value rule

\[
\mathbb{E}[g(X, Y)] = \sum_x \sum_y g(x, y)p_{X,Y}(x, y),
\]

using the expected value rule for a function of a single random variable. Then, use the rule for the special case of a linear function, to verify the formula

\[
\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y],
\]

where \(a\) and \(b\) are given scalars.

2. Random variables \(X\) and \(Y\) can take any value in the set \(\{1, 2, 3\}\). We are given the following information about their joint PMF, where the entries indicated by a * are left unspecified:

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1/12 & 2/12 & 0 \\
1/12 & 1/12 & * \\
2/12 & * & * \\
0 & 1/12 & 2/12 \\
\end{array}
\]

(a) What is \(p_X(1)\)?
(b) Provide a clearly labeled sketch of the conditional PMF of \(Y\) given that \(X = 1\).
(c) What is \(\mathbb{E}[Y \mid X = 1]\)?
(d) Is there a choice for the unspecified entries that would make \(X\) and \(Y\) independent?

Let \(B\) be the event that \(X \leq 2\) and \(Y \leq 2\). We are told that conditioned on \(B\), the random variables \(X\) and \(Y\) are independent.

(e) What is \(p_{X,Y}(2, 2)\)?
   (If there is not enough information to determine the answer, say so.)
(f) What is \(p_{X,Y\mid B}(2, 2 \mid B)\)?
   (If there is not enough information to determine the answer, say so.)

3. **Problem 2.33, page 128 in the text.** A coin that has probability of heads equal to \(p\) is tossed successively and independently until a head comes twice in a row or a tail comes twice in a row. Find the expected value of the number of tosses.