1. There are $n$ fish in a lake, some of which are green and the rest blue. Each day, Helen catches 1 fish. She is equally likely to catch any one of the $n$ fish in the lake. She throws back all the fish, but paints each green fish blue before throwing it back in. Let $G_i$ denote the event that there are $i$ green fish left in the lake.

(a) Show how to model this fishing exercise as a Markov chain, where $\{G_i\}$ are the states. Explain why your model satisfies the Markov property.
(b) Find the transition probabilities $\{p_{ij}\}$.
(c) List the transient and the recurrent states.

2. Problem 5.02, from *Fundamentals of Applied Probability* (Drake).
Consider the following three-state discrete-transition Markov chain:

\[
\begin{align*}
S_1 & \rightarrow 0.6 \quad S_2 \\
S_2 & \rightarrow 0.7 \quad S_3 \\
S_3 & \rightarrow 0.4
\end{align*}
\]

Determine the three-step transition probabilities $r_{11}(3)$, $r_{12}(3)$, and $r_{13}(3)$ both from a sequential sample space and by using the equation $r_{ij}(n + 1) = \sum_k r_{ik}(n)p_{kj}$ in an effective manner.

3. Consider the following Markov chain, with states labelled from $s_0, s_1, \ldots, s_5$:

Given that the above process is in state $s_0$ just before the first trial, determine by inspection the probability that:

(a) The process enters $s_2$ for the first time as the result of the $k$th trial.
(b) The process never enters $s_4$.
(c) The process enters $s_2$ and then leaves $s_2$ on the next trial.
(d) The process enters $s_1$ for the first time on the third trial.
(e) The process is in state $s_3$ immediately after the $n$th trial.