1. If $A \subset B$, then $P(B \cap A) = P(A)$ But we know that in order for $A$ and $B$ to be independent, $P(B \cap A) = P(A)P(B)$. Therefore, $A$ and $B$ are independent if and only if $P(B) = 1$ or $P(A) = 0$. This could happen, for example, if $B$ is the universe or if $A$ is empty.

2. This problem is similar in nature to Example 1.24, page 40. In order to compute the success probability of individual sub-systems, we make use of the following two properties, derived in that example:

- If a serial sub-system contains $m$ components with success probabilities $p_1, p_2, \ldots, p_m$, then the probability of success of the entire sub-system is given by
  \[ P(\text{whole system succeeds}) = p_1p_2p_3\ldots p_m \]

- If a parallel sub-system contains $m$ components with success probabilities $p_1, p_2, \ldots, p_m$, then the probability of success of the entire sub-system is given by
  \[ P(\text{whole system succeeds}) = 1 - (1 - p_1)(1 - p_2)(1 - p_3)\ldots(1 - p_m) \]

Let $P(X \rightarrow Y)$ denote the probability of a successful connection between node $X$ and $Y$. Then,

\[
\begin{align*}
P(A \rightarrow B) &= P(A \rightarrow C)P(C \rightarrow E)P(E \rightarrow B) \quad \text{(since they are in series)} \\
P(A \rightarrow C) &= p \\
P(C \rightarrow E) &= 1 - (1 - p)(1 - P(C \rightarrow D)P(D \rightarrow E)) \\
P(E \rightarrow B) &= 1 - (1 - p)^2
\end{align*}
\]

The probabilities $P(C \rightarrow D)$, $P(D \rightarrow E)$ can be similarly computed as

\[
\begin{align*}
P(C \rightarrow D) &= 1 - (1 - p)^3 \\
P(D \rightarrow E) &= p
\end{align*}
\]

The probability of success of the entire system can be obtained by substituting the subsystem success probabilities:

\[
P(A \rightarrow B) = p (1 - (1 - p)(1 - (1 - (1 - p)^3)p) (1 - (1 - p)^2).
\]
3. The Chess Problem.

(a) 
   i. $P(\text{2nd Rnd Req}) = (0.6)^2 + (0.4)^2 = 0.52$
   ii. $P(\text{Bo Wins 1st Rnd}) = (0.6)^2 = 0.36$
   iii. $P(\text{Al Champ}) = 1 - P(\text{Bo Champ}) - P(\text{Ci Champ})$
       $= 1 - (0.6)^2 \times (0.5)^2 - (0.4)^2 \times (0.3)^2 = 0.8956$

(b) 
   i. $P(\text{Bo Challenger}|\text{2nd Rnd Req}) = \frac{(0.6)^2}{0.52} = \frac{0.36}{0.32} = 0.6923$
   ii. $P(\text{Al Champ}|\text{2nd Rnd Req})$
       $= P(\text{Al Champ}|\text{Bo Challenger, 2nd Rnd Req}) \times P(\text{Bo Challenger}|\text{2nd Rnd Req})$
       $+ P(\text{Al Champ}|\text{Ci Challenger, 2nd Rnd Req}) \times P(\text{Ci Challenger}|\text{2nd Rnd Req})$
       $= (1 - (0.5)^2) \times 0.6923 + (1 - (0.3)^2) \times 0.3077$
       $= 0.7992$

(c) $P(\{(\text{Bo Challenger})|(\text{2nd Rnd Req}) \cap (\text{One Game})\}) = \frac{(0.6)^2 \times (0.5)}{0.2920} = 0.6164$