# 6.041/6.431 Lecture 2 – Organizational

9/8/2008

**1) <u>New Students:</u> If you did not pick up the course handout and hand in the registration sheet at the first lecture last Wednesday, please get one from head TA, Paul Njoroge, or from me after lecture. Please tear off the last page, fill it out, and hand it in <u>immediately after lecture today</u>.**

**2) ADD DAY is Friday, October 3.**

**3) Enough students have enrolled for 6.431 that we have altered the arrangements so we can give two 6.431 recitations. Dr. Shivani Agarwal will teach them Tuesdays and Thursdays at 10:00 and at 11:00. TA Bill Richoux will teach the 6.431 tutorials. <u>This has resulted in changes in the other recitation and tutorial assignments.</u> Please check the course website for your (possibly altered) recitation and tutorial assignments: <u>http://web.mit.edu/6.041/www</u>**

**4) If you wish to change recitation section or tutorial, e-mail the head TA, Paul Njoroge, <u>njoroge@mt.edu</u>. He will respond this Friday, 9/12, to change requests received by midnight Thursday, 9/11. He will respond next Friday, 12/19, to change requests received later than 9/11 and before midnight Thursday 9/18. Meanwhile attend the assigned recitations and tutorials where possible.**

**5) To receive credit for Problem Set #1, you must turn it this Wednesday, Sept. 10, between 11:55 and 12:15 in front of this lecture hall. If you are unable to attend lecture on Wednesday, you may make arrangements with your TA to submit it before 12:15 in person; by e-mail, FAX, Fed Ex or carrier pigeon. For Harry Potter fans, delivery via a *snowy, spotted, great horned, great gray or barn owl* is preferred. No credit will be given for problem sets received after 12:15 Wednesdays.**

# LECTURE 2

- **Readings:** Sections 1.3-1.4

## Lecture outline

- Review
- Conditional probability
- Three **important** tools:
  - Multiplication rule
  - Total probability theorem
  - Bayes' rule

## Review of probability models

- Sample space
  - Mutually exclusive
  - Collectively exhaustive
  - Right granularity

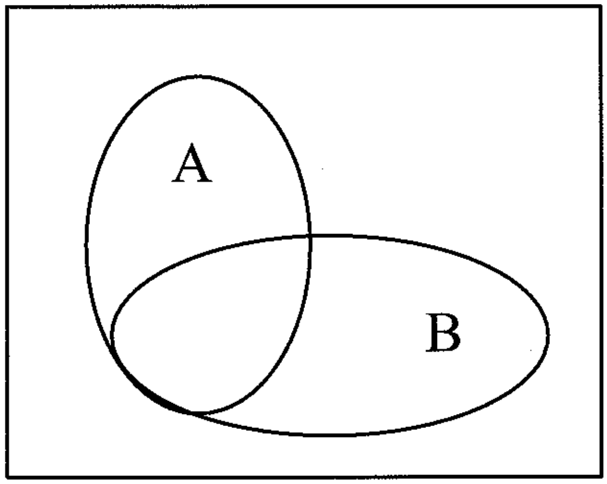
- Allocation of probabilities to events

1. $\mathbf{P}(A) \geq 0$
2. $\mathbf{P}(\text{universe}) = 1$
3. If $A \cap B = \emptyset$,
   then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$

3'. If $A_1, A_2, \ldots$ are disjoint events, then:
$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$

- Problem solving:
  - Setup sample space
  - Define probability law
  - Identify event of interest
  - Calculate...

## Conditional probability

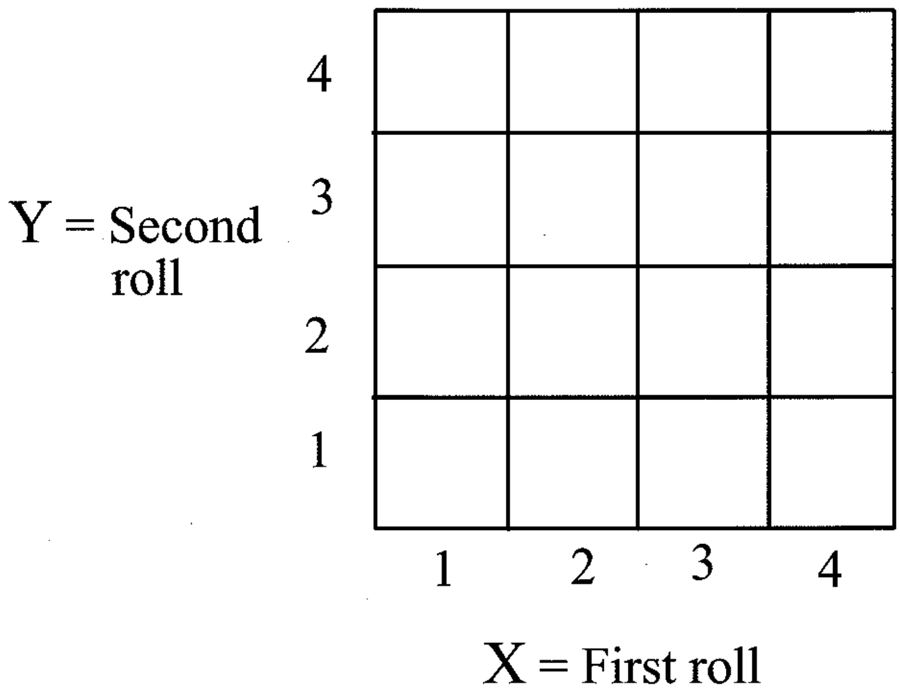



- $\mathbf{P}(A \mid B) =$ probability of $A$, given that $B$ occurred
  - $B$ is our new universe

- **Definition:** Assuming $\mathbf{P}(B) \neq 0$,

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

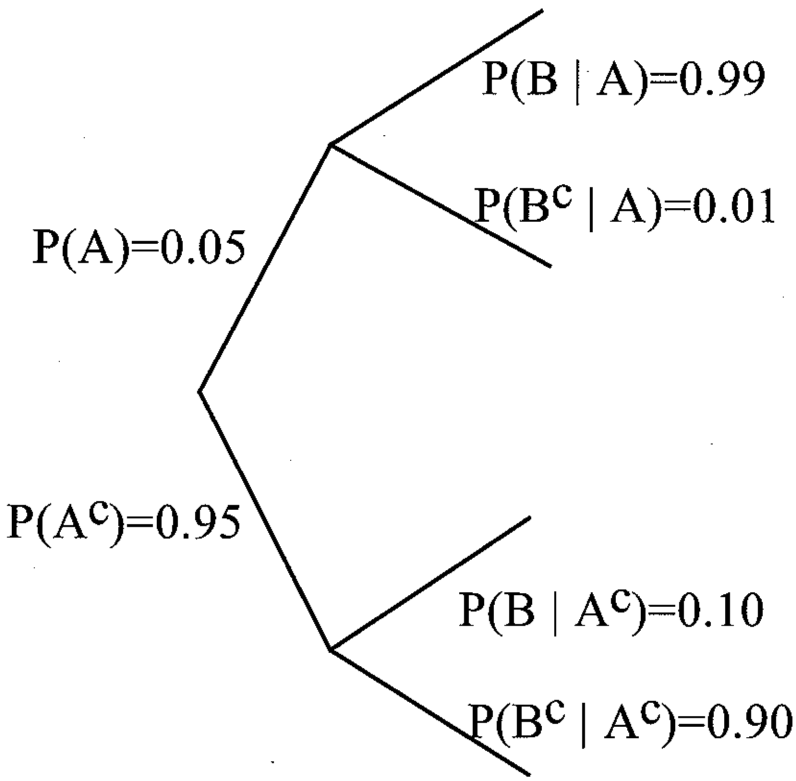
# Die roll example




- Let $B$ be the event: $\min(X, Y) = 2$
- Let $M = \max(X, Y)$
- $\mathbf{P}(M = 1 \mid B) =$
- $\mathbf{P}(M = 2 \mid B) =$

# Models based on conditional probabilities

- Event $A$: Airplane is flying above
  Event $B$: Something registers on radar screen



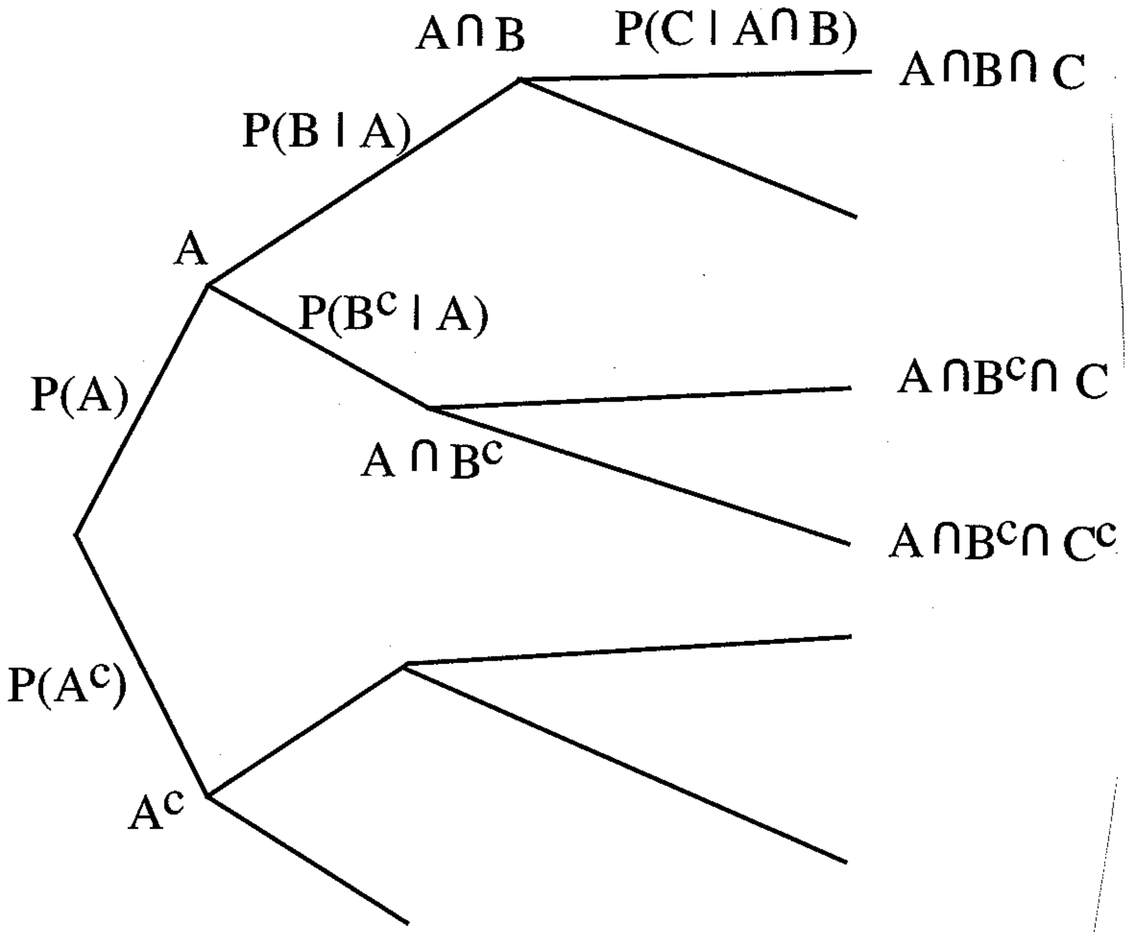

$\mathbf{P}(A \cap B) =$

$\mathbf{P}(B) =$

$\mathbf{P}(A \mid B) =$

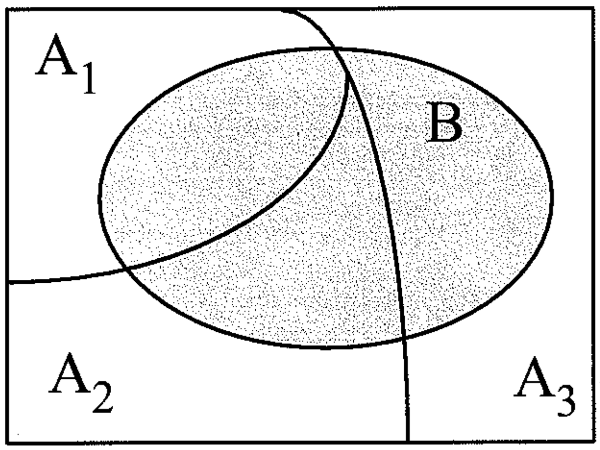
## Multiplication rule

$$P(A \cap B \cap C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \cap B)$$



## Total probability theorem

- Divide and conquer
- Partition of sample space into $A_1, A_2, A_3$




- One way of computing $\mathbf{P}(B)$:

$$\begin{aligned}\mathbf{P}(B) = \quad & \mathbf{P}(A_1)\mathbf{P}(B \mid A_1) \\ + \quad & \mathbf{P}(A_2)\mathbf{P}(B \mid A_2) \\ + \quad & \mathbf{P}(A_3)\mathbf{P}(B \mid A_3)\end{aligned}$$

# BAYES' RULE

# AND ADDITIONAL INFORMATION

Example: You have 100 coins. Of these, 99 are normal fair coins with a probability of ½ of landing heads and a probability of ½ of landing tails. One special coin has heads on both sides: it always comes up heads.

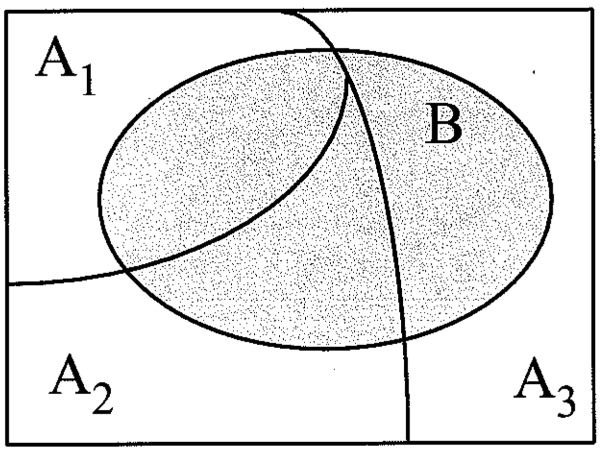
A) Your friend picks a coin at random. The probability that it is the special one is 1/100.

B) Your friend then flips the coin he picked 5 times and on every flip it comes up heads. Using this additional information, what is the probability that the coin he picked is the special one?

## Bayes' rule

- Rules for combining evidence

- "Prior" probabilities $\mathbf{P}(A_i)$
- We know $\mathbf{P}(B \mid A_i)$ for each $i$
- Wish to compute $\mathbf{P}(A_i \mid B)$




$$\begin{aligned}
\mathbf{P}(A_i \mid B) &= \frac{\mathbf{P}(A_i \cap B)}{P(B)} \\
&= \frac{\mathbf{P}(A_i)\mathbf{P}(B \mid A_i)}{\mathbf{P}(B)} \\
&= \frac{\mathbf{P}(A_i)\mathbf{P}(B \mid A_i)}{\sum_j \mathbf{P}(A_j)\mathbf{P}(B \mid A_j)}
\end{aligned}$$