Review Problems – Derived, Markov, Chebyshev, Convergence, Bernoulli Process

1. Xavier and Yolanda enter a frisbee-throwing contest. The distance (in meters) that Xavier throws is uniformly distributed between 0 and 100, and the distance that Yolanda throws (in meters) is exponential with $\lambda = 1/60$.

   (a) What is the expected distance of each competitor's throw?

   (b) Which of the two competitors is more likely to throw farther?

   (c) Let W be the distance Yolanda's frisbee lands past Xavier's. Find the PDF of W.

2. Let $X_1, X_2, \ldots$ be independent, identically distributed random variables with (unknown but finite) mean μ and variance σ^2 where $\sigma^2 > 0$. For $i = 1, 2, \ldots$, let

$$Y_i = \frac{1}{3}X_i + \frac{2}{3}X_{i+1}.$$

   (a) Are the random variables Y_i independent?

   (b) Are they identically distributed?

   (c) Let

$$M_n = \frac{1}{n}\sum_{i=1}^{n} Y_i.$$

   Is M_n convergent in probability to μ?

3. The test scores of 900 students had the following sample statistics:

   Mean: 83   Variance: 36

   Use Chebyshev's inequality to bound the probability that a randomly selected student received a test score between 71 and 95 inclusive. Is it likely that at least 600 students scored between 71 and 95 inclusive? Why or why not?

4. For each night, the probability of a robbery attempt at the local warehouse is $\frac{1}{5}$. A robbery attempt is successful with probability $\frac{3}{4}$, independent of the night. After any particular SUCCESSFUL robbery, the robber celebrates by taking off either the next 2 or 4 nights (with equal probability), during which time there will be no robbery attempts. After that, the robber returns to his original routine.

   (a) Let K be the number of robbery attempts up to (and including) the first successful robbery. Find the PMF of K.

   (b) Let D be the number of days until (and including) the second successful robbery, including the days of celebration after the first robbery. Find the PMF of D, or its transform (whichever you find more convenient).