6.041 Fall 2004 Quiz 1
Wednesday 13th of October, 12:05-12:55pm

DO NOT TURN THIS QUIZ OVER UNTIL YOU ARE TOLD TO DO SO

- You have 50 minutes to complete the quiz.
- This quiz has 3 problems, worth 100 points. Parts are not necessarily in order of difficulty.
- Write your solutions in the answer booklet. We will not consider any work not in the answer booklet.
- You may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. Expressions like $\binom{8}{3}$ or $\sum_{k=0}^{5}(1/2)^k$ are also fine.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for one double-sided, 8.5 by 11 formula sheet.
- Be neat! If we can't read it, we can't grade it.
- Write your name on this quiz and the answer sheet. At the end of the quiz, turn in your solutions along with this quiz (this piece of paper).

Problem 1. (2 points)
Write the names of your recitation instructor **and** TA.

Problem 2. (34 points)
The joint PMF of the random variables X and Y is given by the following table:

y=3	c	c	2c
y=2	2c	0	4c
y=1	3c	c	6c
	x=1	x=2	x=3

(4) (a) Find the value of the constant c.

(5) (b) Find $p_Y(2)$.

(9) (c) Consider the random variable $Z = YX^2$. Find $\mathbf{E}[Z \mid Y = 2]$.

(8) (d) Conditioned on the event that $X \neq 2$, are X and Y independent? Give a one-line justification.

(8) (e) Find the conditional variance of Y given that $X = 2$.

Problem 3. (64 points)
Consider 10 independent tosses of a coin. The probability of "heads" in each toss is p. We define the event B={the first head occurs in the 4th toss **and** the last head occurs in the 7th toss}, and the random variables F (recording on which toss the first head occurred) and L (recording on which toss the last head occurred).

(8) (a) Find $\mathbf{P}(B)$, the probability that B occurs.

(9) (b) Are the events $F = 4$ and $L = 7$ independent? (Provide a brief justification — 1-2 lines.)

(7) (c) Are the random variables F and L independent? (Provide a brief justification — 1-2 lines.)

(7) (d) Given that B occurred, find the conditional PMF of the total number of heads observed in these 10 tosses. (Your answer can consist of either a formula or a fully labelled sketch.)

(11) (e) A reward of 1 is obtained at each time k such that the k-th and $(k-1)$st tosses were heads. For example, if the sequence is

$$HHHTTHTHHT$$

then the sequence of rewards is

$$0, 1, 1, 0, 0, 0, 0, 0, 1, 0$$

and the total reward is 3. Find the expected value of the total reward.

(11) (f) Find the conditional probability that there were exactly two heads in the first 4 tosses, given that there were exactly 5 heads in the 10 tosses.

(11) (g) Find $\mathbf{E}[N^2]$, where N is the total number of heads in the 10 tosses.