6.041 Fall 2005 Quiz 1
Wednesday, October 19, 7:30-9:30 PM.

DO NOT TURN THIS PAGE OVER UNTIL YOU ARE TOLD TO DO SO

Name: ____________________

Recitation Instructor: ____________________

TA: ____________________

Question	Score	Out of
0		3
1		12
2		14
3		28
4		43
Your Grade		100

- You have 120 minutes to complete the quiz.
- Write your solutions in this exam booklet. We will not consider any work not in the exam booklet.
- This quiz has five problems that are not necessarily in order of difficulty.
- You may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. Expressions like $\binom{8}{3}$ or $\sum_{k=0}^{5}(1/2)^k$ are also fine.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for one double-sided, handwritten, 8.5 by 11 formula sheet.
- Calculators are not allowed.
- Be neat! If we can't read it, we can't grade it.

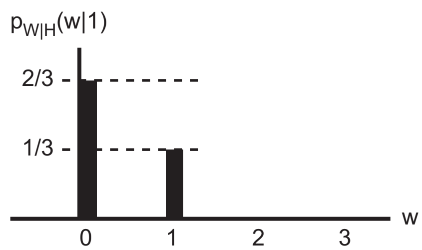
Problem 0: (3 points)

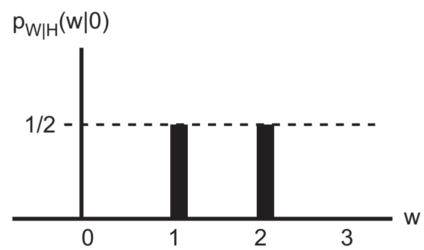
Write your name, your recitation instructor's name, and your TA's name on the front of the booklet.

Problem 1: (12 points)

Random variable H takes on the value 1 with probability 1/3, and the value of 0 with probability 2/3. Random variable W is described by the conditional probabilities as follows:

$$p_{W|H}(w|1) = \begin{cases} 2/3, & w = 0 \\ 1/3, & w = 1 \\ 0, & \text{otherwise} \end{cases} \qquad p_{W|H}(w|0) = \begin{cases} 1/2, & w = 1 \\ 1/2, & w = 2 \\ 0, & \text{otherwise} \end{cases}$$





(a) (4 points) Determine the joint PMF of H and W, i.e., $p_{H,W}(h, w)$.

$p_{H,W}(h, w) =$

(b) (4 points) Determine $p_{H|W}(0|1)$.

$p_{H|W}(0|1) =$

(c) (4 points) Are W and H independent? Explain why or why not.

Answer: Yes / No

Problem 2: (14 points)

Consider three events A, B and C. You know the following information about these events:

- $\mathbf{P}(A) = \mathbf{P}(B) = \mathbf{P}(C) = \frac{1}{10}$
- $\mathbf{P}(A \cap B \cap C) = \frac{1}{1000}$
- $\mathbf{P}(A \cap B^c) = \frac{9}{100}$
- $\mathbf{P}(B^c|C) = \frac{4}{5}$

(a) (4 points) Are events A and B independent? Explain your answer.

Answer: Yes / No

(b) (5 points) Are events A, B and C independent? Explain your answer.

Answer: Yes / No

(c) (5 points) Compute $\mathbf{P}(A|B \cap C)$.

$\mathbf{P}(A|B \cap C) =$

Problem 3: (28 points)

You are playing a simple card game with another friend, using a standard 52 card deck: four suits (Diamonds, Hearts, Clubs and Spades), 13 cards of each suit (numbers from 2 to 10, Jack, Queen, King and Ace). An honest card dealer, different from you two, picks 8 cards out of 52 cards at random and divides them evenly between you two, 4 cards each.

(a) (5 points) What is the probability that you get Ace, King, Queen and Jack of the Diamond suit?

P(You get Ace, King, Queen and Jack of the Diamond suit) =

(b) (5 points) What is the probability that you get Ace, King, Queen and Jack of same suit?

P(You get Ace, King, Queen and Jack of the same suit) =

(c) (6 points) What is the probability that you get Ace, King, Queen and Jack?

P(You get Ace, King, Queen and Jack) =

(d) (6 points) What is the probability that all four of your cards are of the Diamond suit?

P(All four of your cards are of the Diamond suit) =

(e) (6 points) You have Ace, King, Queen and Jack of Diamonds. What is the probability that all of the other player's cards are of the Diamond suit?

P(Other player has all diamonds | You have Ace, King, Queen and Jack of Diamonds) =

Problem 4: (43 points)

Oscar is a used car salesman who is always trying to maximize his earnings, but often loses customers by trying to rush their decision. His manager decides to help him spend more time with each customer by limiting the number of customers Oscar is allowed to interact with during the day. He presents Oscar with two different options:

Strategy 1: Have one distinct customer interaction per 15 minute slot, where there is a total of 32 slots per day. The probability of a sale for each interaction is 1/4, and all customer interactions and sales are independent of each other. Each sale yields a commission $c_1 = \$80$.

Strategy 2: Have one distinct customer interaction per 30 minute slot, where there is a total of 16 slots per day. The probability of a sale for each interaction is 1/2, and all customer interactions and sales are independent of each other. Each sale yields a commission $c_2 = \$100$.

Let K be the number of cars Oscar sells on a particular day using Strategy 1. His earnings on that day are $Q_1 = K \cdot c_1$. Let M be the number of cars Oscar sells on a particular day using Strategy 2. His earnings on that day are $Q_2 = M \cdot c_2$.

Question (f) can be solved independently of (d) and (e). Also, question (g) can be solved independently of (d), (e) and (f).

(a) (5 points) Determine the PMF of K.

$p_K(k) =$

(b) (5 points) Find the expectation and the variance of K. Provide numerical answers.

$E[K] =$

$Var(K) =$

(c) (5 points) Determine the expected earnings for each strategy and state which choice Oscar should take to maximize his expected earnings.

$E[Q_1] =$

$E[Q_2] =$

Which strategy should Oscar take? Strategy 1 / Strategy 2

(d) (7 points) Despite the above analysis, Oscar can't make up his mind of which choice to take. He decides to flip a fair coin and follow Strategy 1 if it turns up heads, or follow Strategy 2 if it turns up tails. Let L be the number of cars he sells using this strategy. Determine the PMF and the expectation of L. Provide a numerical answer for the expectation.

$p_L(l) =$

$E[L] =$

(e) (7 points) Oscar is still using his randomized strategy from part d. Given that he sold 4 cars one day, what is the probability that the coin turned up heads? If he had instead sold 18 cars, what would be the probability that the coin turned up heads?

$\mathbf{P}(\text{Head} \mid \text{Oscar sold 4 cars}) =$

$\mathbf{P}(\text{Head} \mid \text{Oscar sold 18 cars}) =$

(f) (7 points) This question can be solved independently of (d) and (e). Suppose that on one particular day, Oscar decides to follow strategy one. On this same day, Oscar's manager decides to raise the commission for strategy one to either \$90 or \$110, and since he is as indecisive as Oscar, he flips a faircoin once to decide which raise to use for the entire day. Let C be the random variable for the new commission. Find the expectation and the variance of Oscar's earnings Q = KC that day. Provide numerical answers.

$E[Q] =$

$Var(Q) =$

(g) (7 points) This question can be solved independently of (d), (e) and (f). Suppose Oscar now decides to always follow strategy one. Let X be the number of unsuccessful customer interactions (i.e. interactions which did not result in a sale) BEFORE Oscar makes his first sale. Find the PMF, and numerical values for the expectation, and the variance of X (Careful, there is zero probability that a standard geometric random variable takes on the value of zero, but X will be zero if Oscar sells to his first customer).

$p_X(x) =$

$E[X] =$

$Var(X) =$