

6.041 Fall 2004 Quiz 2

Tuesday 9th of November, 7:30-9:30pm

DO NOT TURN THIS QUIZ OVER UNTIL
YOU ARE TOLD TO DO SO

Your Full Name:	
Recitation Instructor and TA's Name:	

- This quiz has 3 problems, worth 100 points. Parts are not necessarily in order of difficulty.
- Write your solutions in the space provided in this quiz itself. We will not consider any work elsewhere. Blue books are provided for your scratch work. Hand in both this quiz and the blue books.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for two double-sided, 8.5 by 11 formula sheets.

Problem	Your Score	Problem	Your Score
1 (2 points)		3 (43 points)	
2 (55 points)		3.(a) 8	
2.(a) 7		3.(b) 7	
2.(b) 8		3.(c) 8	
2.(c) 7		3.(d) 10	
2.(d) 10		3.(e) 10	
2.(e) 7		Total (100 points)	
2.(f)(i) 8			
2.(f)(ii) 8			

Problem 1. (2 points)

Write the names of your recitation instructor **and** TA on the front of this quiz and the blue book.

Problem 2. (55 points)

Let X and Y be independent uniform random variables with range $[0, 1]$. In particular, the variance of X and Y is $1/12$.

(7 pts) **2(a)** Find the mean and variance of $X - 3Y$.

$$\mathbf{E}[X - 3Y] = \underline{\hspace{10cm}}$$

$$\text{var}(X - 3Y) = \underline{\hspace{10cm}}$$

Show your reasoning and calculations here:

(8 pts) **2(b)** Find the probability that $X \geq 3Y$.

$$\mathbf{P}[X \geq 3Y] = \underline{\hspace{10cm}}$$

Show your reasoning and calculations here:

(7 pts) **2(c)** Find the conditional joint PDF of X and Y , given that the event $X + Y \leq 1$ has occurred.

Show your reasoning and calculations here:

(10 pts) **2(d)** Find the conditional PDF of $(X + Y)^2$, given that the event $X + Y \leq 1$ has occurred.

Show your reasoning and calculations here:

(7 pts) **2(e)** Provide a fully labelled sketch of the conditional PDF of XY given that $Y = 0.5$.

(16 pts) **2(f)** A random variable W is defined as follows. We toss a fair coin (independent of X and Y). If the result is “heads”, we let $W = X$; if it is tails, we let $W = 2Y$.

(8 pts) **2(f)(i)** Provide a fully labelled sketch of the PDF of W .

(8 pts) **2(f)(ii)** Find the probability of “heads” given that $0.5 \leq W \leq 1.5$.

$$\mathbf{P}[\text{heads} | 0.5 \leq W \leq 1.5] = \underline{\hspace{10cm}}$$

Show your reasoning and calculations here:

Problem 3. (43 points) Let X_1, X_2, \dots be independent normal random variables with mean 2 and variance 4. Let N be a geometric random variable which is independent of the X_i , with parameter $p = 2/3$. (In particular, $\mathbf{E}[N] = 3/2$ and $\mathbf{E}[N^2] = 3$.)

(8 pts) **3(a)** If δ is a small positive number, we have $\mathbf{P}(|X_1| \leq \delta) \approx \alpha\delta$, for some constant α . Find the value of α . (Your answer may involve π , no need to evaluate numerically.)

$\alpha =$ _____

Show your reasoning and calculations here:

(8 pts) **3(b)** Find $\mathbf{E}[X_1N]$.

$$\mathbf{E}[X_1N]=\underline{\hspace{10cm}}$$

Show your reasoning and calculations here:

(8 pts) **3(c)** Find the variance of X_1N .

$$\text{var}(X_1N)=\underline{\hspace{10cm}}$$

Show your reasoning and calculations here:

(8 pts) **3(d)** Find $\mathbf{E}[X_1 + \cdots + X_N \mid N \geq 2]$.

$$\mathbf{E}[X_1 + \cdots + X_N \mid N \geq 2] = \underline{\hspace{10cm}}$$

Show your reasoning and calculations here:

(8 pts) **3(e)** Write down the transform associated with $N + X_1 + \cdots + X_N$.

$$M_{N+X_1+\cdots+X_N}(s) = \underline{\hspace{15em}}$$

Show your reasoning and calculations here: