MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2008)

Recitation 2
September 9, 2008

1. (Problem 1.14, page 56 of the text.) We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

   (a) Find the probability that doubles are rolled.

   (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.

   (c) Find the probability that at least one die roll is a 6.

   (d) Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 6.

2. (Problem 1.18, page 57 of the text.) Let $A \subset \Omega$ and $B \subset \Omega$ be events in some sample space Ω. Show that $\mathbf{P}(A \cap B \mid B) = \mathbf{P}(A \mid B)$, assuming that $\mathbf{P}(B) > 0$.

   Also show that $\mathbf{P}(A \mid B) = \mathbf{P}(A \mid B \cap C)\mathbf{P}(C \mid B) + \mathbf{P}(A \mid B \cap C^c)\mathbf{P}(C^c \mid B$, for any event $C \subset \Omega$. Note that this is a conditional version of the total probability theorem (conditioned on the event B in this case). Breaking event B up like this into the disjoint partition $B \cap C$ and $B \cap C^c$ can sometimes be useful in calculating $\mathbf{P}(A|B)$.

3. (Example 1.18, page 33 of the text.)

   A test for a certain rare disease is assumed to be correct 95% of the time: if a person has the disease, the test results are positive with probability 0.95, and if the person does not have the disease, the test results are negative with probability 0.95. A random person drawn from a certain population has probability 0.001 of having the disease. Given that the person just tested positive, what is the probability of having the disease? Explain this result intuitively.