Recitation 3
September 11, 2008

1. A coin is tossed four times independently. The probability of heads at each toss is p. Let A be the event that the first and second tosses result in heads. Let B be the event that the second and third tosses result in heads. Let C be the event that the third and fourth tosses result in heads.

   (a) Are the events A and B independent?

   (b) Are the events A and C independent?

   Do your answers depend on p?

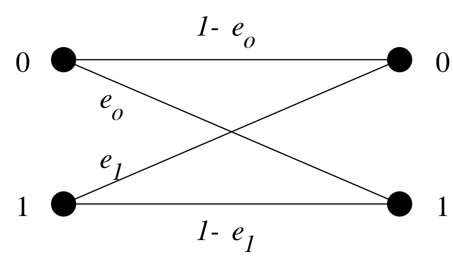
2. **Do longer games favor the stronger player? What does "stronger player" mean?**[1]

   Turtle and Hare compete in a game that may have multiple independent rounds, where the player with highest total score wins. Turtle scores 2 points in every round, while Hare scores 3 points with probability p or 0 points with probability $1 - p$.

   (a) What is the probability Hare wins a two-round game?

   (b) Clearly, Hare wins a one-round game with probability p. Find a value of p such that Hare *loses* a two-round game with probability p, i.e., playing two rounds exactly reverses the likelihoods of winning for the two players.

3. **Communication through a noisy channel.** (Problem 1.31, page 60 of the text.) A source transmits a message (a string of symbols) through a noisy communication channel. Each symbol is 0 or 1 with probability p and $1 - p$, respectively, and is received incorrectly with probability e_0 and e_1, respectively (see below). Errors in different symbol transmissions are independent.



   (a) What is the probability that the kth symbol is received correctly?

   (b) What is the probability that the string of transmitted symbols 1011 is received correctly?

   (c) In an effort to improve reliability, each symbol is transmitted three times and the received string is decoded by a majority rule. In other words, a 0 (or 1) is transmitted as a 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a 0 is correctly decoded?

[1] The example is drawn from T. M. Cover, "Do Longer Games Favor the Stronger Player?," *The American Statistician*, vol. 43, no. 4, pp. 277–278, November 1989.

(d) For what values of e_0 is there an improvement in the probability of correct decoding of a 0 when the scheme of part (c) is used?

(e) Suppose that the scheme of part (c) is used. What is the probability that a symbol was 0 given that the received string is 101?