

Recitation 4
September 16, 2008

1. **The birthday problem.** Problem 1.50, page 67 in the text.

Consider n people who are attending a party. We assume that every person has an equal probability of being born on any day during the year, independent of everyone else, and ignore the additional complication presented by leap years (i.e., assume that nobody is born on February 29). What is the probability that each person has a distinct birthday?

2. Variation on Example 1.31, page 48 in text.

Consider a group of n persons. A club consists of a special person from the group (the club leader) and a number (possibly zero) of additional club members.

- (a) Explain why the number of possible clubs is $n2^{n-1}$.
(b) Find an alternate way of counting the number of possible clubs and show the identity

$$\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}.$$

3. **Hypergeometric probabilities.** Problem 1.61, page 69 in the text.

An urn contains n balls, out of which m are red. We select k of the balls at random, without replacement (i.e., selected balls are not put back into the urn before the next selection). What is the probability that i of the selected balls are red?

4. Consider the different cases that arise from the problem of selecting/sampling k balls from an urn containing n numbered balls, numbered 1 through n :

- Sampling with replacement and ordering
- Sampling without replacement and ordering
- Sampling without replacement and without ordering
- Sampling with replacement and without ordering

The objective of this problem is to study the fourth case. A distinct solution may be expressed in terms of the vector of nonnegative integers (N_1, N_2, \dots, N_n) , where N_i is the number of times that ball i is selected.

- (a) Explain why we must have $N_1 + N_2 + \dots + N_n = k$.
(b) How many distinct solutions does the equation above have? Explain why this is the answer to the number of distinct results of sampling with replacement and without ordering.

G1[†]. Imagine a mouse living in an $n \times n$ grid:

