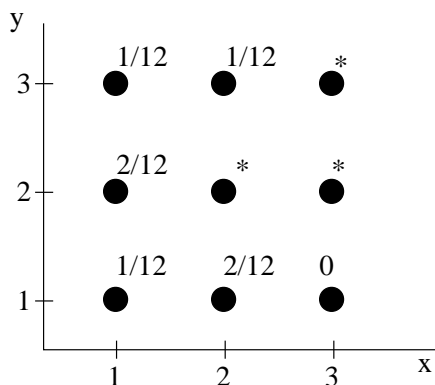


**Recitation 7**<sup>1</sup>  
**September 25, 2008**

1. Random variables  $X$  and  $Y$  can take any value in the set  $\{1, 2, 3\}$ . We are given the following information about their joint PMF, where the entries indicated by a \* are left unspecified:



- (a) What is  $p_X(1)$ ?
- (b) Compute the conditional PMF of  $Y$  given that  $X = 1$ .
- (c) What is  $\mathbf{E}[Y \mid X = 1]$ ?
2. **Joe Lucky and the lottery.** Joe Lucky plays the lottery on any given week with probability  $p$ , independently of whether he played on any other week. Each time he plays, he has a probability  $q$  of winning, again independently of everything else. During a fixed time period of  $n$  weeks, let  $X$  be the number of weeks that he played the lottery and  $Y$  the number of weeks that he won.
- (a) What is the probability that he played the lottery any particular week, given that he did not win anything that week?
- (b) Find the conditional PMF  $p_{Y|X}(y \mid x)$ .
- (c) Find the joint PMF  $p_{X,Y}(x, y)$ .
- (d) Find the marginal PMF  $p_Y(y)$ .
- (e) Find the conditional PMF  $p_{X|Y}(x \mid y)$ . Do this algebraically using previous answers.
- (f) Based on an intuitive interpretation of part (a), rederive your answer to part (e).

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<sup>1</sup>Published September 23, 2008

3. Problem 2.32, page 128 of the text.

**D. Bernoulli's problem of joint lives.**

Consider  $2m$  persons forming  $m$  couples who live together at a given time. Suppose that at some later time, the probability of each person being alive is  $p$ , independently of other persons. At that later time, let  $A$  be the number of persons that are alive and let  $S$  be the number of couples in which both partners are alive. For any number of total surviving persons  $a$ , find  $\mathbf{E}[S|A = a]$ .