Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2008)

Recitation 8
September 30, 2008

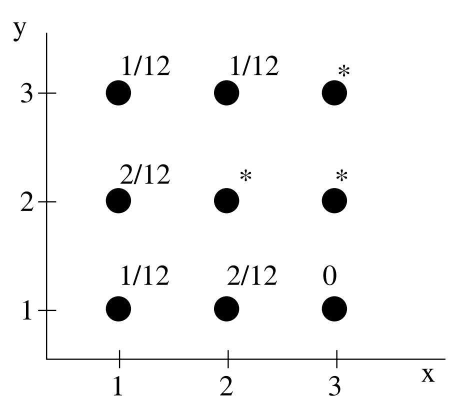
Review/discussion:

The following five types of discrete random variables arise frequently in applications and in the remainder of the course. Their properties are tabulated on pages 116–117 of the text. Make sure you understand how these random variables arise and how to derive their means and variances.

- Discrete uniform over $[a, b]$ (or uniform over $\{a, a+1, \ldots, b\}$)
- Bernoulli with parameter p
- Binomial with parameters n and p
- Geometric with parameter p
- Poisson with parameter λ

Problems:

1. Random variables X and Y each take values in the set $\{1, 2, 3\}$. We are given the following information about their joint PMF, where the entries indicated by a * are left unspecified:



(a) Is there a choice for the unspecified entries that would make X and Y independent?

Let B be the event that $X \leq 2$ and $Y \leq 2$. We are told that conditioned on B, the random variables X and Y are independent.

(b) What is $p_{X,Y}(2, 2)$?

(c) What is $p_{X,Y|B}(2, 2)$?

2. Part of Problem 2.35, page 130 of the text.
   Use the definition of expectation for single random variables and jointly-distributed random variables to formally justify the formula

$$\mathbf{E}[aX + bY + c] = a\mathbf{E}[X] + b\mathbf{E}[Y] + c$$

   for jointly-distributed random variables X and Y and constants a, b, and c.

3. Problem 2.33, page 128 of the text.
   A coin that has probability of heads equal to p is tossed successively and independently until a head comes up twice in a row or a tail comes up twice in a row. Find the expected value of the number of tosses.