Recitation 8: Answers
September 30, 2008

1. (a) Suppose X and Y are independent. Then we must have $p_{X,Y}(x, y) = p_X(x)\, p_Y(y)$ for every (x, y) pair with $x = 1, 2, 3$ and $y = 1, 2, 3$. In particular, since $p_{X,Y}(3, 1) = 0$ and $p_Y(1)$ is nonzero, we must have $p_X(3) = 0$. This in turn implies $p_{X,Y}(3, 2) = 0$ and $p_{X,Y}(3, 3) = 0$. The only remaining unknown value is $p_{X,Y}(2, 2)$. We must have $p_{X,Y}(2, 2) = 5/12$ for the joint PMF to sum to 1. However,

$$p_{X,Y}(2, 2) = \frac{5}{12} \neq \frac{2}{3} \cdot \frac{7}{12} = p_X(2) \cdot p_Y(2).$$

This contradicts X and Y being independent, so it is not possible for X and Y to be independent.

A simpler explanation is also possible. For X and Y to be independent, $p_{X,Y}(1, 3)/p_{X,Y}(1, 1)$ and $p_{X,Y}(2, 3)/p_{X,Y}(2, 1)$ must be equal because both of these would equal $p_Y(3)/p_Y(1)$. This necessary equality does not hold, so X and Y are not independent.

(b) Knowing that X and Y are conditionally independent given B, we must have

$$\frac{p_{X,Y}(1, 1)}{p_{X,Y}(1, 2)} = \frac{p_{X,Y}(2, 1)}{p_{X,Y}(2, 2)}$$

since the (X, Y) pairs in the equality are all in B. Thus

$$p_{X,Y}(2, 2) = \frac{p_{X,Y}(1, 2)p_{X,Y}(2, 1)}{p_{X,Y}(1, 1)} = \frac{(2/12)(2/12)}{1/12} = \frac{4}{12} = \frac{1}{3}.$$

(c) Since $\mathbf{P}(B) = 9/12 = 3/4$, we normalize to obtain $p_{X,Y|B}(2, 2) = \frac{p_{X,Y}(2,2)}{\mathbf{P}(B)} = 4/9$.

2. Problem 2.35, page 130 of the text. See page 131 of the text.

3. Problem 2.33, page 128 of the text. See pages 128–129 of the text.