

**Recitation 9**  
**October 2, 2008**

1. Problem 3.1, page 184 of the text.

Let  $X$  be uniformly distributed in the unit interval  $[0, 1]$ . Consider the random variable  $Y = g(X)$ , where

$$g(x) = \begin{cases} 1, & \text{if } x \leq 1/3; \\ 2, & \text{if } x > 1/3. \end{cases}$$

Find the expected value of  $Y$  by first deriving its PMF. Verify the result using the expected value rule.

2. The random variable  $X$  is exponentially distributed with parameter  $\lambda$ , i.e.,

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Calculate  $\mathbf{E}[X]$ ,  $\text{var}(X)$  and find  $\mathbf{P}(X \geq \mathbf{E}[X])$ .

*Hint:*

$$\mathbf{P}(X \geq z) = \int_z^{\infty} f_X(x) dx$$

- (b) Find  $\mathbf{P}(X \geq t + z \mid X > t)$ , where  $t \geq 0$  and  $z \geq 0$ .

3. Problem 3.7, page 186 of the text.

Alvin throws darts at a circular target of radius  $r$  and is equally likely to hit any point in the target. Let  $X$  be the distance of Alvin's hit from the center.

- (a) Find the PDF, the mean, and the variance of  $X$ .
- (b) The target has a concentric inner circle of radius  $t$ , with  $t < r$ . If  $X \leq t$ , Alvin gets a score of  $S = 1/X$ . Otherwise his score is  $S = 0$ . Find the CDF of  $S$ . Is  $S$  a continuous random variable?