

Recitation 10
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1. Let X and Y be two random variables having the following joint PDF:

$$f_{X,Y}(x, y) = \begin{cases} cxy^2, & \text{for } y \in [0, 1], x \in [0, 2 - 2y]; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) What is the value of c ?
 - (b) What is the marginal PDF of Y , $f_Y(y)$?
 - (c) What is the conditional PDF of X given Y , $f_{X|Y}(x | y)$?
 - (d) Are X and Y independent?
2. Any computer chip will eventually fail, creating a random lifetime. Suppose that a manufacturing process produces a mix of “good” and “bad” chips. For some positive number α , the lifetimes of good chips have the exponential distribution with parameter α and the lifetimes of bad chips have the exponential distribution with parameter 1000α . Assume that the fraction of good chips is p and the fraction of bad chips $1 - p$.
- (a) Find the probability that a randomly selected chip is still functioning after t time units of operation.
 - (b) Find the PDF of the time of failure of a randomly selected chip.
 - (c) To weed out bad chips, each chip is tested for t time units, and only chips that do not fail during the testing period are shipped to customers. Find a formula for the probability that a customer receives a bad chip (as a function of the constants α , p , and t). If $p = 0.9$, how long should the testing be to make the probability of shipping bad product be below 1%?
3. Suppose X_1, X_2, \dots, X_n are independent and identically distributed (i.i.d.) random variables with the uniform distribution over $[0, 1]$.
- (a) Let $Y = \max(X_1, X_2, \dots, X_n)$. Find the CDF of Y .
 - (b) Let $Z = \min(X_1, X_2, \dots, X_n)$. Find the CDF of Z .
 - (c) Find the joint CDF of random variables Y and Z .